



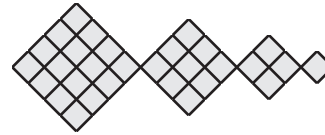
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2026 : DIVISION S
WEDNESDAY 6 MAY 2026

OLYMPIAD
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Total Time: **30 Minutes**. Calculators NOT Permitted.

- 1A.** A tile company's logo consists of unit squares: shaded, arranged, and rotated 45° to form 4×4 , 3×3 , 2×2 , and 1×1 squares, as shown.



The entire logo is then placed in a large white square (not shown).

Find the smallest possible area of the large white square, in square units.

Write your answers in the boxes on the back.

←
Keep your answers hidden by folding backwards on this line.

- 1B.** Given that

$$2 \times 3 = 6, 6 \div 3 = 2, \text{ and } \frac{1}{2} \times 6 = 3$$

are equivalent number sentences, find the value of:

$$(1 \div \frac{1}{2000}) + (1 \div \frac{1}{30}) - (1 \div 0.25).$$

- 1C.** A whole number which reads the same forwards as backwards is called a palindromic number.

For example, 21912 is a five-digit palindrome.

Find the positive difference between the next palindrome greater than 21912 and the previous palindrome less than 21912.

- 1D.** $66^2 + 88^2 = n^2$.

Find the whole number n .

- 1E.** The variables x and y are whole numbers (non-negative integers).

$$7x + 3y = 2026.$$

Find the greatest possible value of $x + y$.



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1A.

Student Name:

1B.

1C.

1D.

1E.

Fold here. Keep your answers hidden.



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Solutions and Answers

(Items in parentheses are not required)

1A: 100

1B: 2026

1C: 210

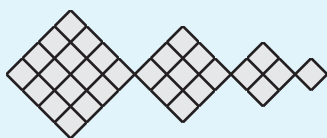
1D: 110

1E: 674

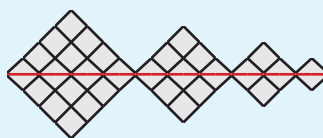
1A. Find the smallest possible area of the large white square, in square units.

Strategy 1: Draw a Diagram

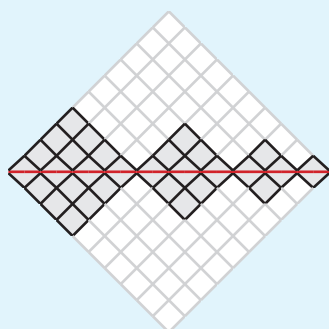
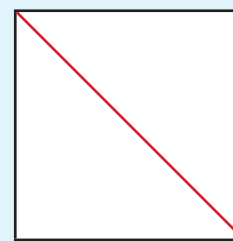
The logo consists of unit squares that are rotated 45° to form 4×4 , 3×3 , 2×2 , and 1×1 squares.



To place the logo wholly within a large white square, we note that the square must be able to accommodate a length that is equal to the distance between two opposite ends of the logo, as shown.



The greatest length that can be accommodated within a square is the length of the diagonal.



When we align the diagonal of the square with the line between two opposite ends of the logo, the logo is contained wholly within the square.

This must therefore be the smallest possible square to contain the logo.

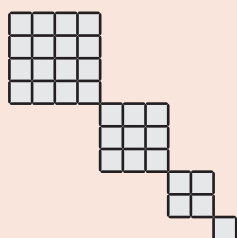
The logo is made up of 4×4 , 3×3 , 2×2 , and 1×1 squares.

The large white square is made up of 10×10 squares.

The smallest possible area of the large white square is $10 \times 10 = 100$ square units.

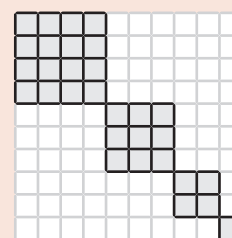
Strategy 2: Draw a Diagram (Alternative Approach)

Since the logo consists of unit squares that are rotated 45° , we can begin by rotating them back to their original orientation.



In this configuration, the logo is 10 squares wide, and 10 squares high.

The smallest number of square units in the area of the large white square is $10 \times 10 = 100$.



Follow-Up: Suppose that the 4×4 , 3×3 , 2×2 , and 1×1 squares can be rearranged. Each square is then placed without overlap in a larger $n \times n$ square. Find the least possible value of n . [7]



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1B. Find the value of $(1 \div \frac{1}{2000}) + (1 \div \frac{1}{30}) - (1 \div 0.25)$.

Strategy 1: Reason Algebraically

We begin by noting that $2 \times 3 = 6$, $6 \div 3 = 2$, and $\frac{1}{2} \times 6 = 3$ are equivalent number sentences.

Let $a = 1 \div \frac{1}{2000}$.

• $1 \div \frac{1}{2000} = a$ and

• $a \times \frac{1}{2000} = 1$

are equivalent number sentences.

This sentence is true if $a = 2000$.

Let $b = 1 \div \frac{1}{30}$.

• $1 \div \frac{1}{30} = b$ and

• $b \times \frac{1}{30} = 1$

are equivalent number sentences.

This sentence is true if $b = 30$.

Let $c = 1 \div 0.25$.

• $1 \div 0.25 = c$ and

• $c \times 0.25 = 1$

are equivalent number sentences.

This sentence is true if $c = 4$.

$$\begin{aligned} (1 \div \frac{1}{2000}) + (1 \div \frac{1}{30}) - (1 \div 0.25) \\ = a + b - c \\ = 2000 + 30 - 4 \\ = 2026. \end{aligned}$$

Strategy 2: Reason Arithmetically

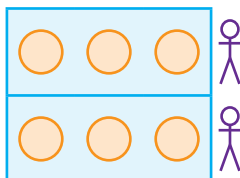
Division can be thought of in different ways.

For example, to conceptualise $6 \div 2 = 3$:

Method 1: Partition

There are 6 biscuits and 2 children.

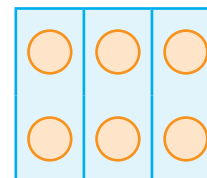
How many biscuits does each child receive?



Method 2: Quotition

There are 6 biscuits.

How many packets of 2 biscuits can I make?



When dividing by a fraction, it makes sense to consider division as quotition.

$1 \div \frac{1}{2000}$:

How many $\frac{1}{2000}$ s are there in 1?

There are 2000 lots of $\frac{1}{2000}$ in 1.

$1 \div \frac{1}{30}$:

How many $\frac{1}{30}$ s are there in 1?

There are 30 lots of $\frac{1}{30}$ in 1.

$1 \div 0.25$:

How many 0.25s are there in 1?

There are 4 lots of 0.25 in 1.

The value of $(1 \div \frac{1}{2000}) + (1 \div \frac{1}{30}) - (1 \div 0.25)$ is $2000 + 30 - 4 = 2026$.

Strategy 3: Multiply by the Reciprocal

Division by a value is equivalent to multiplication by its reciprocal.

Therefore, $(1 \div \frac{1}{2000}) + (1 \div \frac{1}{30}) - (1 \div 0.25) = (1 \times 2000) + (1 \times 30) - (1 \times 4) = 2026$.

Follow-Up: What whole number equals $1 \div 0.002 + 1 \div 0.001 + 1 \div 0.0005 + 1 \div 0.00025$? [7500]



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- 1C. Find the positive difference between the next palindrome greater than 21912, and the previous palindrome less than 21912.

Strategy 1: Build a Table

10000s	1000s	100s	10s	1s
2	1	9	1	2

We can use a place value table to express the value 21912.

To maintain it as a palindrome, if we increase the value in the 1s place, we would also need to increase the value in the 10000s place.

10000s	1000s	100s	10s	1s
2	1	9	1	2
3	1	9	1	3

Likewise, if we increase the value in the 10s place, we would also need to increase the value in the 1000s place.

10000s	1000s	100s	10s	1s
2	1	9	1	2
3	1	9	1	3
2	2	9	2	2

Since the value in the 100s place is 9, it is not possible to increase its value without impacting the value in the 1000s place.

If we change the value in the 1000s place, we will also need to make the same change to the value in the 10s place.

We can see that the next palindrome greater than 21912 is 22022.

10000s	1000s	100s	10s	1s
2	1	9	1	2
3	1	9	1	3
2	2	9	2	2
2	2	0	2	2

Using similar logic to find the previous palindrome less than 21912, we should decrease the value in the 100s place.

The previous palindrome less than 21912 is 21812.

10000s	1000s	100s	10s	1s
2	1	9	1	2
3	1	9	1	3
2	2	9	2	2
2	2	0	2	2
2	1	8	1	2

The difference between the two palindromes is $22022 - 21812 = 210$.

Strategy 2: Make an Organised List, and Reason Logically

To find the next palindrome that is greater than 21912, we can count up by ones from 21912 until we find another palindrome.

	21912	...	21919	21920
21921	21922	...	21929	21930

It is clear that we will not find another palindrome while the number is 21XXX.

We can therefore start with 22000.

For a number 22XXX to be a palindrome, the 10s and 1s place values must both be 2.

The least possible number that fits this format is 22022.

Likewise, to find the previous palindrome that is less than 21912, we can count down by ones from 21912 until we find another palindrome.

			21912	21911
21910	21909	...	21902	21901
21900	21899	...	21892	21891

If the number 218XX is to be a palindrome, the 10s and 1s place values must be 1 and 2.

The previous lesser palindrome is therefore 21812.

The difference between 22022 and 21812 is 210.

Follow-Up: Find the pair of five-digit palindromes with the least sum, whose positive difference is 44,355. [15651, 60006]

Find the pair of five-digit palindromes with the greatest sum, whose positive difference is 44,355. [49994, 94349]



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1D. Find the whole number n , such that $66^2 + 88^2 = n^2$.

Strategy 1: Reason Arithmetically

We can begin by finding the value of $66^2 + 88^2$.

$$\begin{array}{r} 66 \\ \times 66 \\ \hline 396 \\ 3960 \\ \hline 4356 \end{array}$$

$$\begin{array}{r} 88 \\ \times 88 \\ \hline 704 \\ 7040 \\ \hline 7744 \end{array}$$

$$\begin{array}{r} 4356 \\ + 7744 \\ \hline 12100 \end{array}$$

$$\begin{aligned} n^2 &= 12100 \\ &= 121 \times 100. \end{aligned}$$

Listing square numbers:

n	1	2	3	4	5	6	7	8	9	10	11
n^2	1	4	9	16	25	36	49	64	81	100	121

$$\begin{aligned} \text{Since } n^2 &= 121 \times 100 \\ &= 11^2 \times 10^2 \\ &= 11 \times 11 \times 10 \times 10, \\ n &= 11 \times 10 \\ &= 110. \end{aligned}$$

Strategy 2: Find a Common Factor

We begin by noting that 66 and 88 are both multiples of 22.

Therefore,

$$\begin{aligned} 66^2 &= 66 \times 66 \\ &= 3 \times 22 \times 3 \times 22 \\ &= 3^2 \times 22^2. \end{aligned}$$

$$\begin{aligned} 88^2 &= 88 \times 88 \\ &= 4 \times 22 \times 4 \times 22 \\ &= 4^2 \times 22^2. \end{aligned}$$

$$66^2 + 88^2 = 3^2 \times 22^2 + 4^2 \times 22^2.$$

Using an area model (not to scale):

$$\begin{array}{c} 3^2 \\ 22^2 \end{array} + \begin{array}{c} 4^2 \\ 22^2 \end{array} = \begin{array}{c} 3^2 + 4^2 \\ 22^2 \end{array}$$

$$3^2 \times 22^2 + 4^2 \times 22^2 = (3^2 + 4^2) \times 22^2.$$

$$\begin{aligned} n^2 &= 66^2 + 88^2 \\ &= (3^2 + 4^2) \times 22^2 \\ &= 25 \times 22^2 \\ &= 5 \times 5 \times 22 \times 22 \\ &= (5 \times 22)^2. \end{aligned}$$

$$\begin{aligned} \text{Since } n^2 &= (5 \times 22)^2, \\ n &= 5 \times 22 \\ &= 110. \end{aligned}$$

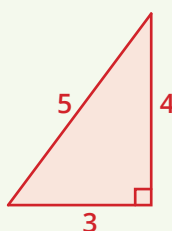
Checking:

$$\begin{array}{r} 66 \\ \times 66 \\ \hline 396 \\ 3960 \\ \hline 4356 \end{array} \quad \begin{array}{r} 88 \\ \times 88 \\ \hline 704 \\ 7040 \\ \hline 7744 \end{array} \quad \begin{array}{r} 110 \\ \times 110 \\ \hline 1100 \\ 11000 \\ \hline 12100 \end{array} \quad \begin{array}{r} 4356 \\ + 7744 \\ \hline 12100 \end{array}$$

Strategy 3: Identify a Pythagorean Triad

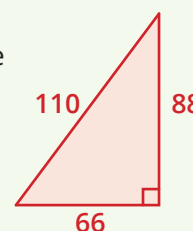
66:88 is equivalent to 3:4.

By recognising that (3, 4, 5) is a Pythagorean triad, we can begin by constructing a right-angled triangle with side lengths 3, 4, and 5 units long.



A similar right-angled triangle can therefore be constructed with side lengths $22 \times 3 = 66$, $22 \times 4 = 88$, and $22 \times 5 = 110$ units long.

The whole number n is 110.



Follow-Up: Find the whole number n if $28^2 + n^2 = 100^2$. [96]



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- 1E.** Find the greatest possible value of $x + y$, where $7x + 3y = 2026$ and x and y are whole numbers.

Strategy 1: Reason Algebraically

Since x and y are whole numbers, the least possible value of either variable is 0.

If $x = 0$, then $3y = 2026$

$y = 675$ with a remainder of 1.

If $y = 0$, then $7x = 2026$

$x = 289$ with a remainder of 3.

$$\begin{array}{r} 675 \\ 3 \overline{) 2026} \\ \underline{- 1800} \\ 226 \\ \underline{- 210} \\ 16 \\ \underline{- 15} \\ 1 \end{array}$$

$$\begin{array}{r} 675 \\ 7 \overline{) 2026} \\ \underline{- 1400} \\ 626 \\ \underline{- 560} \\ 66 \\ \underline{- 63} \\ 3 \end{array}$$

We want to maximise the value of $x + y$.

To maintain the condition that $7x + 3y = 2026$, increasing the value of x will decrease the value of y , and vice versa.

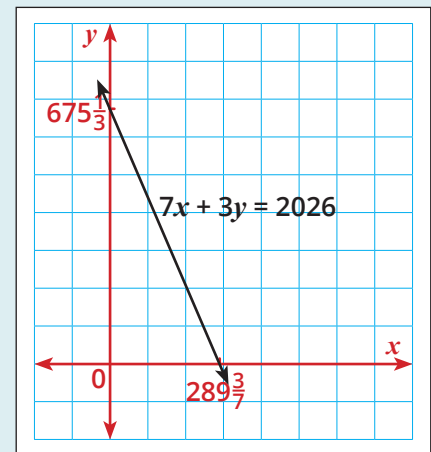
However, because x is multiplied by 7 and y is multiplied by 3, increasing x by 1 will cause y to decrease by more than 1.

x	0	1	2	...	287	288	289
y	$675\frac{1}{3}$	673	$670\frac{2}{3}$	...	$5\frac{2}{3}$	$3\frac{1}{3}$	1

We can see that the greatest possible value of $x + y$ will occur when x is minimised.

As seen in the table, $7x + 3y = 2026$ has whole number values for x and y when $x = 1$ and $y = 673$.

The greatest possible value of $x + y$ is $1 + 673 = 674$.



Strategy 2: Reason Algebraically (Alternative Approach)

Since $7x + 3y = 2026$,

$$4x + 3x + 3y = 2026$$

$$4x + 3(x + y) = 2026.$$

Therefore, to maximise the value of $(x + y)$, we should minimise the value of x .

$$7x + 3y = 2026$$

$$3y = 2026 - 7x.$$

Both x and y are whole numbers, so we need to find the least value of x for which $2026 - 7x$ is divisible by 3.

This occurs when $x = 1$ and $y = 673$.

The greatest possible value of $x + y$ is $1 + 673 = 674$.

x	$3y = 2026 - 7x$	y
0	2026	$675\frac{1}{3}$
1	2019	673

Follow-Up: What is the least possible value of $x + y$? [290]