

2026 Maths Olympiads Division S Preparation Kit



Preparing for the APSMO Maths Olympiads

The purpose of this Preparation Kit is to provide students with an opportunity to familiarise themselves with the concepts, and terminology, that will subsequently be used in the four competition papers for 2026:

- Wednesday, 6 May 2026
- Wednesday, 10 June 2026
- Wednesday, 29 July 2026
- Wednesday, 9 September 2026

For each of the problems in this kit, a number of different solution methods are suggested, so that students can be exposed to multiple ways of approaching mathematical problems.

The kit additionally includes a reference sheet for relevant skills and terminology. This reference sheet can also be found in the Contest Preparation section of the Members' Portal.

Examples of how this kit may be used include:

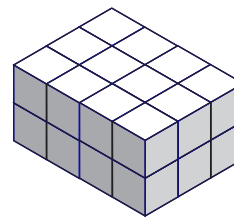
- Reinforcing previously learned concepts and terminology
- Introducing new or different solution methods
- Providing diagrams that support a teacher's or student's explanations
- Offering problem-solving homework
- Supporting students' own study as a standalone resource

Further questions and solution methods can also be found in the APSMO resource books, available from www.apsmo.edu.au.

- A.** Rory has four clear plastic pockets on his pencil case. The pockets contain letter tiles that form "RORY".
One day, Rory rearranged the tiles to form "YRRO".
How many different four-letter arrangements can be made using his tiles, including "RORY" and "YRRO"?

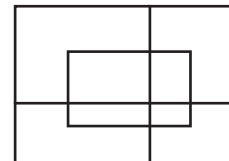
Write your answers in the boxes on the back.

- B.** How many different rectangular prisms can be constructed using exactly 24 identical unit cubes?
[One example is shown with dimensions $2 \times 3 \times 4$.]



← *Keep your answers hidden by folding backwards on this line.*

- C.** In the diagram, all angles are right angles.
How many different rectangles can be traced along the lines in this diagram?



- D.** A cryptarithm is a mathematical puzzle where the digits have been replaced by letters.
The same letter represents the same digit.
Different letters represent different digits.
In the cryptarithm at the right, the letter *E* represents the digit 0.
What is the 6-digit number represented by the letters *TWENTY*?

$$\begin{array}{r}
 S \ E \ V \ E \ N \\
 + \ S \ E \ V \ E \ N \\
 + \ S \ E \ V \ E \ N \\
 \hline
 T \ W \ E \ N \ T \ Y
 \end{array}$$

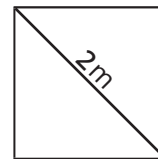
- E.** There are 25 prime numbers less than 100.
Some of these are two-digit prime numbers.
What fraction of these two-digit prime numbers contain the digit 9?
Express the answer in its most simplified form.

A.	<i>Fold here. Keep your answers hidden.</i>
B.	
C.	
D.	
E.	

- F.** What is the probability that a randomly selected three-digit positive integer has no repeated digits?
Express your answer as a fraction in its lowest terms, in decimal form, and as a percentage.

Write your answers in the boxes on the back.

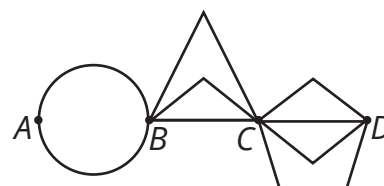
- G.** Mrs Hicks marked out a square in the quadrangle. The diagonals of this square are 2 metres in length. What is the area of the square, in square metres?



← *Keep your answers hidden by folding backwards on this line.*

- H.** There is more than one possible value for X so that the mean and the median of the set $\{2.3, 3.1, 1.6, 4.0, 2.7, X\}$ are equal. Find the greatest possible value for X .

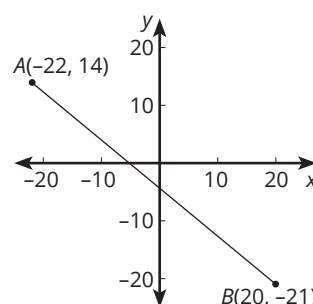
- I.** The diagram shows a number of edges that connect vertices A , B , C and D .
Following only the edges shown, what is the number of different paths that go from A to B to C to D ?
You must visit each vertex exactly once.



- J.** A lattice point is a point on a Cartesian plane where both the x and y co-ordinates are integers.

The interval $\overline{AB}$ joins the points $A(-22, 14)$ and $B(20, -21)$.

Including endpoints A and B , how many lattice points lie on $\overline{AB}$?





F.	<i>Fold here. Keep your answers hidden.</i>
G.	
H.	
I.	
J.	

Example Solution A

Rory has four clear plastic pockets on his pencil case. The pockets contain letter tiles that form "RORY".

One day, Rory rearranged the tiles to form "YRRO".

How many different four-letter arrangements can be made using his tiles, including "RORY" and "YRRO"?

Strategy: Make an Organised List

We can list the options alphabetically.

Beginning with arrangements starting with **O**:

- We have **R, R, Y** left.

O	R	R	Y
---	---	---	---

- As there is only one **Y**, we just need to think about the different places the **Y** can be.

O	R	Y	R
---	---	---	---

O	Y	R	R
---	---	---	---

Next, starting with **R**:

- We have **O, R, Y** left.

- What options are possible if the second letter is **O**?

R	O	R	Y
---	---	---	---

R	O	Y	R
---	---	---	---

- What options are possible if the second letter is the other **R**?

R	R	O	Y
---	---	---	---

R	R	Y	O
---	---	---	---

- What options are possible if the second letter is **Y**?

R	Y	O	R
---	---	---	---

R	Y	R	O
---	---	---	---

Finally, starting with **Y**:

- We have **O, R, R** left.

Y	O	R	R
---	---	---	---

- Since the only one that is different is **O**, we just need to think about the different places the **O** can be.

Y	R	O	R
---	---	---	---

Y	R	R	O
---	---	---	---

Since we counted them in an organised way, we can be sure that we have found all of the possible arrangements.

Rory can use his tiles to make **12** different four-letter arrangements.

Strategy: Count in an Organised Way

We can begin by pretending that all of the letter tiles are different. So, instead of **RORY**, we will imagine that the letter tiles have the letters "**RORY**".

There are four clear plastic pockets on Rory's pencil case.

--	--	--	--

Since we have **4** different tiles, there would be **4** choices for the first pocket.

1st pocket could be:

R			
---	--	--	--

or

O			
---	--	--	--

or

r			
---	--	--	--

or

Y			
---	--	--	--

Suppose we placed the **r** in the first pocket.

Then there would be **3** remaining options for the second pocket.

Whichever letter we chose for the first pocket, there would be **3** options remaining for the second pocket.

2nd pocket could be:

r	R		
---	---	--	--

or

r	O		
---	---	--	--

or

r	Y		
---	---	--	--

There would then be **2** different options remaining for the 3rd pocket,

3rd pocket could be:

r	Y	R	
---	---	---	--

or

r	Y	O	
---	---	---	--

and then just **1** option left for the last pocket.

r	Y	R	O
---	---	---	---

With **4** ways to choose the first tile, **3** ways for the second, **2** ways for the third and **1** way for the fourth, there are $4 \times 3 \times 2 \times 1 = 24$ ways to arrange these tiles.

However, the **24** combinations would include both **RORY** and **rORY**, and **YRRO** and **yRRO**, and so on. With two different Rs, we have ended up double-counting each arrangement.

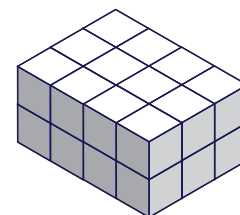
There are $24 \div 2 = 12$ different four-letter arrangements for Rory's tiles.

Answer: 12

Example Solution B

How many different rectangular prisms can be constructed using exactly 24 identical unit cubes?

[One example is shown with dimensions $2 \times 3 \times 4$.]



Strategy: List All Possible Cases

The three dimensions of the rectangular prism must be positive integers that multiply to 24.

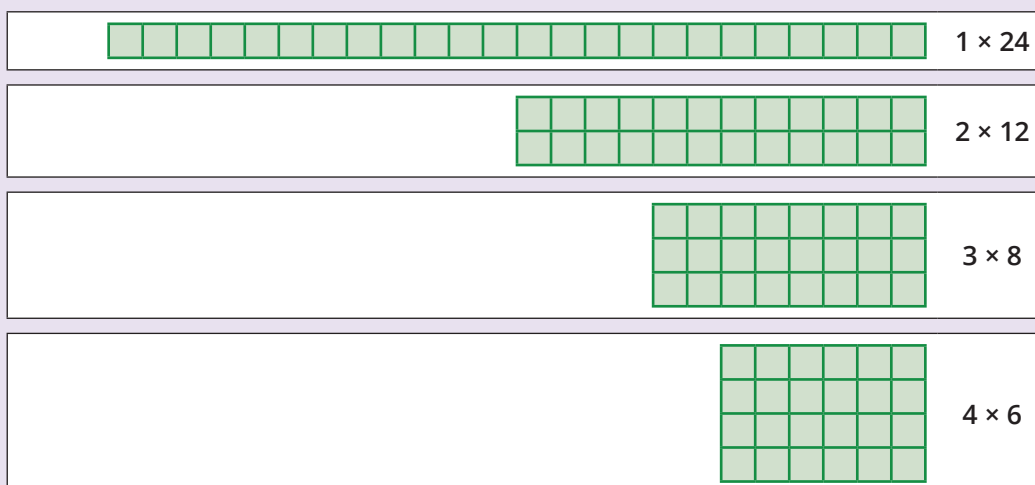
The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

Case 1: Shortest dimension is 1 unit long.

The product of the other two dimensions must be $24 \div 1 = 24$.

With a shortest dimension of 1 unit, the possible dimensions for the rectangular prism are:

- $1 \times 1 \times 24$,
- $1 \times 2 \times 12$,
- $1 \times 3 \times 8$, and
- $1 \times 4 \times 6$.

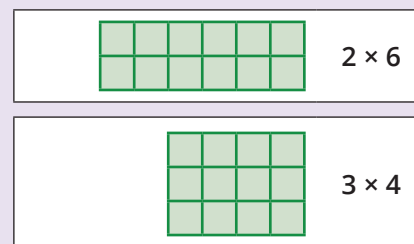


Case 2: Shortest dimension is 2 units long.

The product of the other two dimensions must be $24 \div 2 = 12$.

With a shortest dimension of 2 units, the possible dimensions for the rectangular prism are:

- $2 \times 2 \times 6$, and
- $2 \times 3 \times 4$.



Case 3: Shortest dimension is 3 units long.

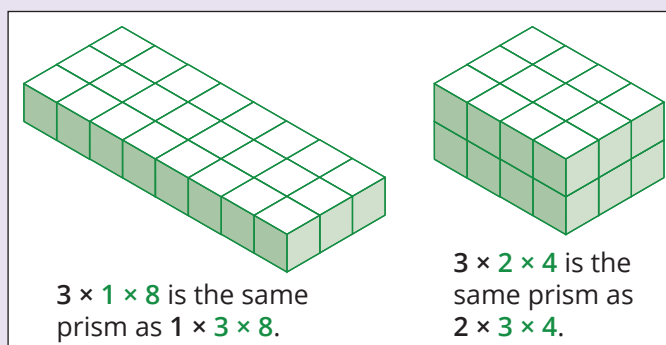
The product of the other two dimensions must be $24 \div 3 = 8$.

If one dimension is 3 units long, the possible dimensions for the rectangular prism are $3 \times 1 \times 8$, and $3 \times 2 \times 4$.

In both cases, one of the other dimensions is shorter than 3 units.

We counted those prisms already, when we listed prisms with a shortest dimension of 1 or 2 units.

We cannot construct a rectangular prism using 24 unit cubes, where the shortest dimension is 3 units long.



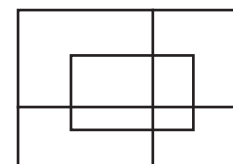
All together, there are $4 + 2 = 6$ possible rectangular prisms constructible with 24 unit cubes.

Answer: 6

Example Solution C

In the diagram, all angles are right angles.

How many different rectangles can be traced along the lines in this diagram?

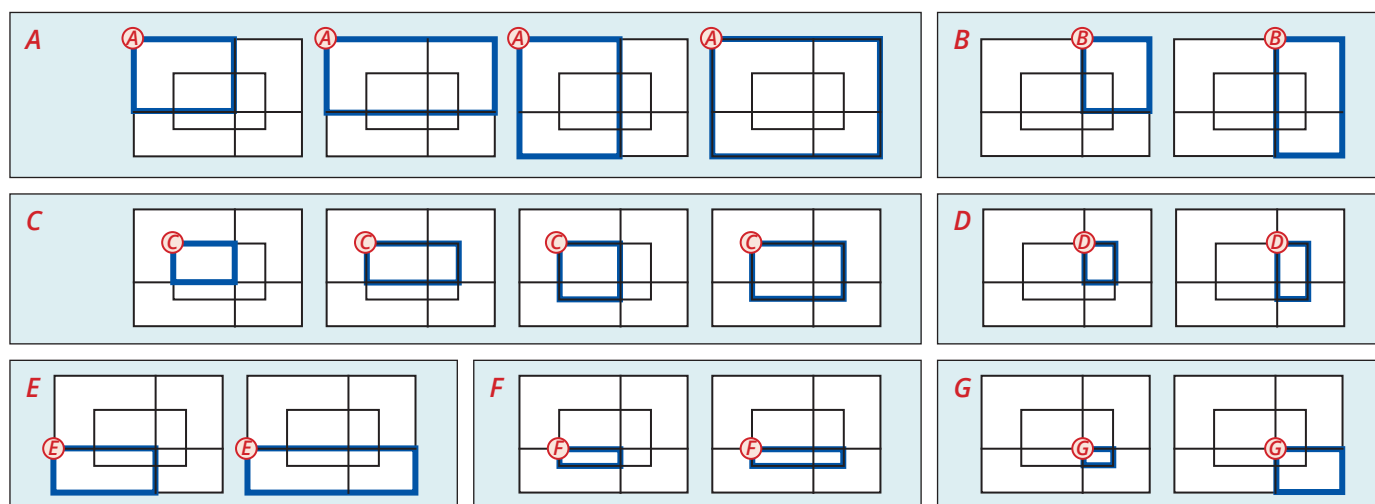
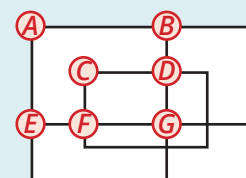


Strategy 1: Make an Organised List

We could begin by marking all of the positions where we might have the top left corner of a rectangle.

Then, we can list all of the possible rectangles with top left corner **A**, **B**, and so on.

Working our way around systematically through all of the different positions, we can be sure that we have found them all.

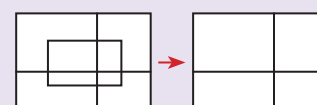


We can trace **18** different rectangles on the lines in this diagram.

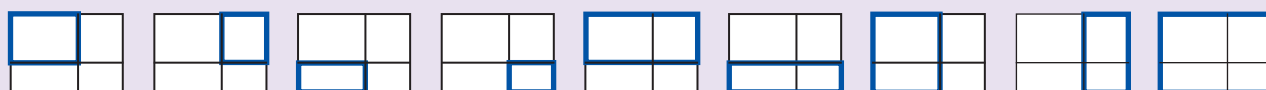
Strategy 2: Solve a Simpler Related Problem

We can make the problem easier to solve by first removing some of the lines in the diagram.

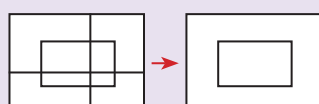
For example, we might begin by removing the inner rectangle.



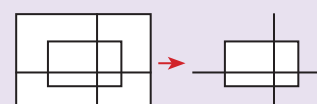
The following **9** rectangles can be traced on the resulting diagram.



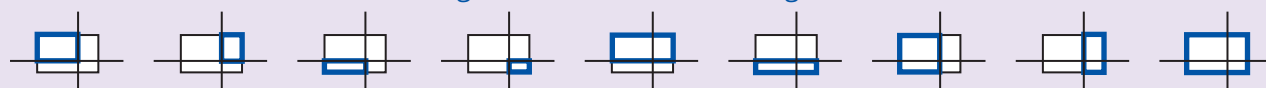
It is impossible for a rectangle to include sides from both the inner and outer rectangles.



However, we can remove the outer rectangle, and trace another **9** rectangles on the resulting diagram.



We can trace $9 + 9 = 18$ different rectangles on the lines in this diagram.



Answer: 18

Example Solution D

A cryptarithm is a mathematical puzzle where the digits have been replaced by letters.

The same letter represents the same digit. Different letters represent different digits.

In the cryptarithm at the right, the letter *E* represents the digit 0.

What is the 6-digit number represented by the letters *TWENTY*?

$$\begin{array}{r} S \ E \ V \ E \ N \\ + \ S \ E \ V \ E \ N \\ + \ S \ E \ V \ E \ N \\ \hline T \ W \ E \ N \ T \ Y \end{array}$$

Strategy: Eliminate All But One Possibility

As noted in the question, the letter *E* represents the digit 0.

$$\begin{array}{r} S \ 0 \ V \ 0 \ N \\ + \ S \ 0 \ V \ 0 \ N \\ + \ S \ 0 \ V \ 0 \ N \\ \hline T \ W \ 0 \ N \ T \ Y \end{array}$$

None of the other letters can represent the digit 0.

0	1	2	3	4	5	6	7	8	9
E									

With 0s for every value in the thousands place, we can see that there can be no trading from the thousands place.

This means that we can consider the digits in the ten-thousands and hundred-thousands place values as a separate addition problem.

$$\begin{array}{r} S \ 0 \ V \ 0 \ N \\ + \ S \ 0 \ V \ 0 \ N \\ + \ S \ 0 \ V \ 0 \ N \\ \hline T \ W \ 0 \ N \ T \ Y \end{array}$$



$$\begin{array}{r} S \\ + \ S \\ + \ S \\ \hline T \ W \end{array} \quad \begin{array}{r} 0 \ V \ 0 \ N \\ + \ 0 \ V \ 0 \ N \\ + \ 0 \ V \ 0 \ N \\ \hline 0 \ N \ T \ Y \end{array}$$



Similarly, we can also consider the digits in the hundreds and thousands place values as yet another separate addition problem.

$$\begin{array}{r} S \\ + \ S \\ + \ S \\ \hline T \ W \end{array} \quad \begin{array}{r} 0 \ V \\ + \ 0 \ V \\ + \ 0 \ V \\ \hline 0 \ N \end{array} \quad \begin{array}{r} 0 \ N \\ + \ 0 \ N \\ + \ 0 \ N \\ \hline T \ Y \end{array}$$

Let's begin by considering the equation $V + V + V = N$.

Since *N* must be a one-digit number, the only possible values for *V* are 1, 2, and 3; and so *N* can only be 3, 6 or 9.

$\begin{array}{r} 0 \ V \\ + \ 0 \ V \\ + \ 0 \ V \\ \hline 0 \ N \end{array}$	$\begin{array}{r} 0 \ 1 \\ + \ 0 \ 1 \\ + \ 0 \ 1 \\ \hline 0 \ 3 \end{array}$	$\begin{array}{r} 0 \ 2 \\ + \ 0 \ 2 \\ + \ 0 \ 2 \\ \hline 0 \ 6 \end{array}$	$\begin{array}{r} 0 \ 3 \\ + \ 0 \ 3 \\ + \ 0 \ 3 \\ \hline 0 \ 9 \end{array}$	$\begin{array}{r} 0 \ 4 \\ + \ 0 \ 4 \\ + \ 0 \ 4 \\ \hline \cancel{0} \ 2 \end{array}$
--	--	--	--	---

Given that *N* can only be 3, 6 or 9, for the equation $N + N + N = TY$, we have the following options.

0 has already been used, so *TY* cannot represent 09.

Therefore, *N* can only be 6 or 9.

Since $V + V + V = N$, *V* can only be 2 or 3.

$\begin{array}{r} 0 \ N \\ + \ 0 \ N \\ + \ 0 \ N \\ \hline T \ Y \end{array}$	$\begin{array}{r} 0 \ 3 \\ + \ 0 \ 3 \\ + \ 0 \ 3 \\ \hline \cancel{0} \ 9 \end{array}$	$\begin{array}{r} 0 \ 6 \\ + \ 0 \ 6 \\ + \ 0 \ 6 \\ \hline 1 \ 8 \end{array}$	$\begin{array}{r} 0 \ 9 \\ + \ 0 \ 9 \\ + \ 0 \ 9 \\ \hline 2 \ 7 \end{array}$
--	---	--	--

Let's guess that *V* represents 2 and *N* represents 6.

If so, *TY* represents 18, and 3, 4, 5, 7 and 9 are possible for *S* and *W*.

0	1	2	3	4	5	6	7	8	9
E	T	V				N		Y	

$\begin{array}{r} S \\ + \ S \\ + \ S \\ \hline 1 \ W \end{array}$	$\begin{array}{r} 3 \\ + \ 3 \\ + \ 3 \\ \hline \cancel{0} \ 9 \end{array}$	$\begin{array}{r} 4 \\ + \ 4 \\ + \ 4 \\ \hline 1 \ 2 \end{array}$	$\begin{array}{r} 5 \\ + \ 5 \\ + \ 5 \\ \hline 1 \ 5 \end{array}$	$\begin{array}{r} 7 \\ + \ 7 \\ + \ 7 \\ \hline \cancel{2} \ 1 \end{array}$	$\begin{array}{r} 9 \\ + \ 9 \\ + \ 9 \\ \hline \cancel{2} \ 7 \end{array}$
--	---	--	--	---	---

We can see that none of these options will work.

Let's guess that *V* represents 3 and *N* represents 9.

If so, *TY* represents 27, and 1, 4, 5, 6 and 8 are possible for *S* and *W*.

0	1	2	3	4	5	6	7	8	9
E		T	V				Y	N	

$\begin{array}{r} S \\ + \ S \\ + \ S \\ \hline 2 \ W \end{array}$	$\begin{array}{r} 1 \\ + \ 1 \\ + \ 1 \\ \hline \cancel{0} \ 3 \end{array}$	$\begin{array}{r} 4 \\ + \ 4 \\ + \ 4 \\ \hline \cancel{1} \ 2 \end{array}$	$\begin{array}{r} 5 \\ + \ 5 \\ + \ 5 \\ \hline \cancel{1} \ 5 \end{array}$	$\begin{array}{r} 6 \\ + \ 6 \\ + \ 6 \\ \hline \cancel{1} \ 8 \end{array}$	$\begin{array}{r} 8 \\ + \ 8 \\ + \ 8 \\ \hline 2 \ 4 \end{array}$
--	---	---	---	---	--

It works if *S* represents 8, and *W* represents 4.

Therefore *TWENTY* must represent 240927.

Answer: 240927

Example Solution E

There are 25 prime numbers less than 100.

Some of these are two-digit prime numbers.

What fraction of two-digit prime numbers contain the digit 9?

Strategy: Make an Organised List

We can begin by considering two-digit numbers that contain the digit 9.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Eliminate multiples of 2 and 3.

1	2	3	X	5	X	7	X	X	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Eliminate multiples of 5 and 7.

1	2	3	X	5	X	7	X	X	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

All composite numbers under 10^2 must have at least one factor that is less than 10.

By considering every prime number less than 10, and eliminating all of their multiples, we can be sure that we have eliminated all of the two-digit composite numbers.

All of the two-digit numbers that remain on the grid must be prime.

There are 21 two-digit prime numbers.

There are 6 two-digit primes that contain the digit 9.

1	2	3	X	5	X	7	X	X	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The fraction of two-digit prime numbers that contain the digit 9 is $\frac{6}{21} = \frac{2}{7}$.

Answer: $\frac{2}{7}$

Example Solution F

What is the probability that a randomly selected three-digit positive integer has no repeated digits?
Express your answer as a fraction in its lowest terms, in decimal form, and as a percentage.

Strategy: Count in an Organised Way

We begin by finding the number of 3-digit positive integers.

With 9 possible values in the hundreds place,

~~0~~ 1 2 3 4 5 6 7 8 9

10 possible values in the tens place,

0 1 2 3 4 5 6 7 8 9

and 10 possible values in the ones place,

0 1 2 3 4 5 6 7 8 9

there are $9 \times 10 \times 10 = 900$ different 3-digit positive integers.

To find how many of these 3-digit positive integers have no repeated digits:

There are 9 possible values in the 100s place.

~~0~~ 1 2 3 4 5 6 7 8 9

Let's suppose that the hundreds digit is 3.

~~0~~ 1 2 3 4 5 6 7 8 9

If we want no repeated digits, there are now just 9 possible values in the tens place.

3 0 3 1 3 2 ~~3 3~~ 3 4 3 5 3 6 3 7 3 8 3 9

Let's suppose that the tens digit is 1.

3 0 3 1 3 2 ~~3 3~~ 3 4 3 5 3 6 3 7 3 8 3 9

If we want no repeated digits, there are now just 8 possible values in the ones place.

3 1 0 ~~3 1 1~~ 3 1 2 ~~3 1 3~~ 3 1 4 3 1 5 3 1 6 3 1 7 3 1 8 3 1 9

There are $9 \times 9 \times 8 = 648$ 3-digit positive integers that have no repeated digits.

The probability that a randomly selected 3-digit positive integer has no repeated digits, is $\frac{9 \times 9 \times 8}{9 \times 10 \times 10} = \frac{648}{900}$.

Fraction in Lowest Terms

$$\begin{aligned} \frac{9 \times 9 \times 8}{9 \times 10 \times 10} &= \frac{9}{9} \times \frac{9 \times 2 \times 2 \times 2}{5 \times 2 \times 5 \times 2} \\ &= 1 \times \frac{9 \times 2}{5 \times 5} \times \frac{2 \times 2}{2 \times 2} \\ &= 1 \times \frac{18}{25} \times 1 \\ &= \frac{18}{25} \end{aligned}$$

Decimal and Percentage

$$\begin{aligned} \frac{9 \times 9 \times 8}{9 \times 10 \times 10} &= \frac{9}{9} \times \frac{72}{100} \\ &= 1 \times 72\% \\ &= 1 \times 0.72 \\ &= 0.72 \end{aligned}$$

As a fraction, the probability is $\frac{18}{25}$.

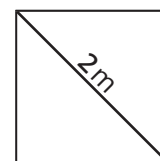
As a decimal, the probability is 0.72.

As a percentage, the probability is 72%.

Answer: $\frac{18}{25}$; 0.72, 72%

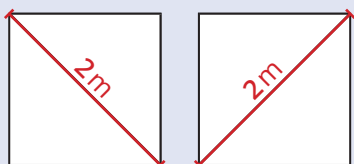
Example Solution G

Mrs Hicks marked out a square in the quadrangle.
The diagonals of this square are 2 metres in length.
What is the area of the square, in square metres?

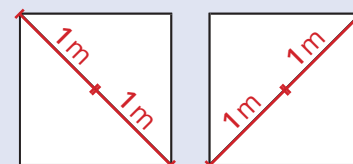


Strategy: Divide The Shape

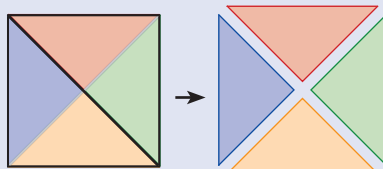
The length of each diagonal, from one vertex to the opposite vertex, is **2 metres**.



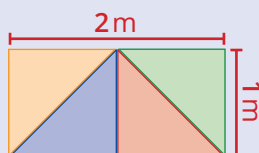
So the length of each line from the vertex to the centre of the square is $2 \div 2 = 1$ metre.



We can begin by dividing the square into four triangles, along the two diagonals.



These four triangles can be reassembled into a rectangle that is **2 metres** long and **1 metre** high.



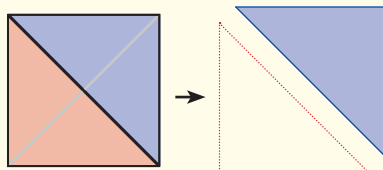
The area of the resulting rectangle is $2 \times 1 = 2$ square metres.

Since we have used all of the pieces from the original square, this rectangle must have the same area.

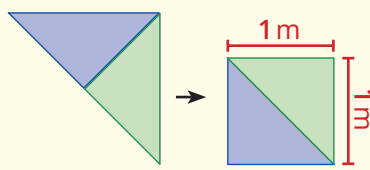
Therefore the area of the original square is **2 square metres**.

Strategy: Divide The Shape (Alternate Approach)

Suppose we divide the square in half, along a diagonal.



This half can be divided into two triangles, and then rearranged to form a square.



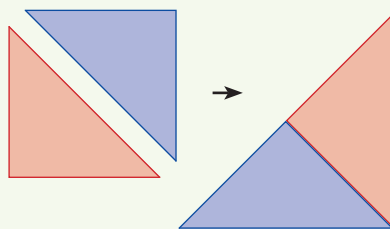
The area of the resulting square is $1 \times 1 = 1$ square metre.

This square is half of the area of the original square.

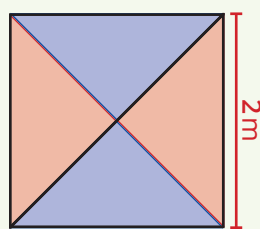
Therefore the area of the original square is $2 \times 1 = 2$ square metres.

Strategy: Divide The Shape (Another Approach)

If we divide the square in half along a diagonal, and rearrange it to form a triangle,



doubling that triangle forms a square with sides of length **2 metres**.



The area of the resulting square is $2 \times 2 = 4$ square metres.

This square is double the area of the original square.

Therefore the area of the original square is $4 \div 2 = 2$ square metres.

Answer: 2 (m²)

Example Solution H

There is more than one possible value for X so that the mean and the median of the set $\{2.3, 3.1, 1.6, 4.0, 2.7, X\}$ are equal.

Find the greatest possible value for X .

Strategy: Find the value of the median

By rearranging the values in the set in ascending order, the following arrangements are possible.

$\{X, 1.6, 2.3, 2.7, 3.1, 4.0\}$
 $\{1.6, X, 2.3, 2.7, 3.1, 4.0\}$
 $\{1.6, 2.3, X, 2.7, 3.1, 4.0\}$
 $\{1.6, 2.3, 2.7, X, 3.1, 4.0\}$
 $\{1.6, 2.3, 2.7, 3.1, X, 4.0\}$
 $\{1.6, 2.3, 2.7, 3.1, 4.0, X\}$

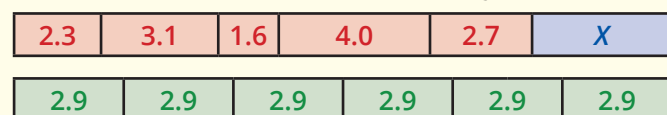
The greatest value for X will occur with the greatest possible median value.

We see here that the greatest possible median is **2.9**, midway between **2.7** and **3.1**.

Having determined that the median of the set will be **2.9**, we now know that the mean will likewise be **2.9**.

Method 1: Work backwards from the formula for the mean.

The mean of a set of values is found by:



- Summing all of the values, and
- Dividing the sum by the number of values in the set.

$$\frac{2.3 + 3.1 + 1.6 + 4.0 + 2.7 + X}{6} = 2.9$$

The same idea can be set up as an equation.

Working backwards, we can:

$$2.9 \times 6 = 17.4$$



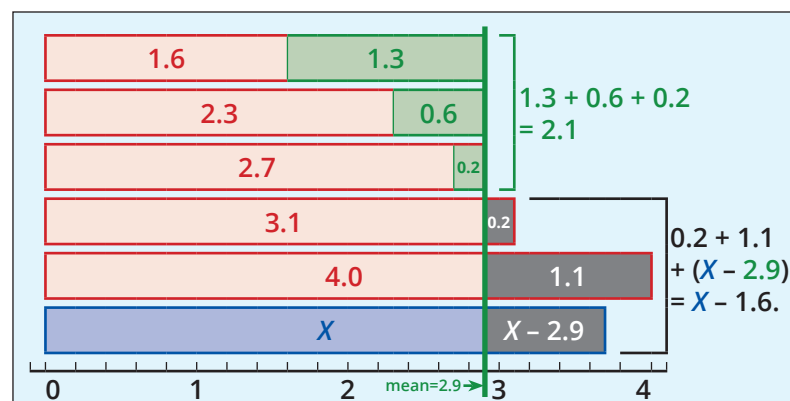
- Multiply the mean by the number of values in the set, and
- Subtract all of the other values to find the value of X .

$$2.3 + 3.1 + 1.6 + 4.0 + 2.7 + X = 2.9 \times 6$$

$$= 17.4$$

Since $2.3 + 3.1 + 1.6 + 4.0 + 2.7 = 13.7$, we find that the value of X will be $17.4 - 13.7 = 3.7$.

Method 2: Work backwards from a conceptual understanding of the mean.



Another way to think about the mean might be as a measure of central tendency - as the result of sharing a set of quantities equally.

To equalise the quantities, we would transfer $(X - 1.6)$ from the three large values to the three small values.

Therefore: $X - 1.6 = 2.1$

$$X = 2.1 + 1.6$$

$$= 3.7$$

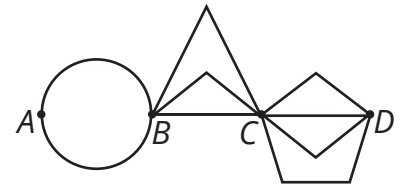
Answer: 3.7

Example Solution I

The diagram shows a number of edges that connect vertices A , B , C and D .

Following only the edges shown, what is the number of different paths that go from A to B to C to D ?

You must visit each vertex exactly once.

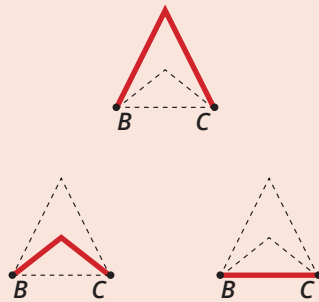


Strategy: Make an Organised List

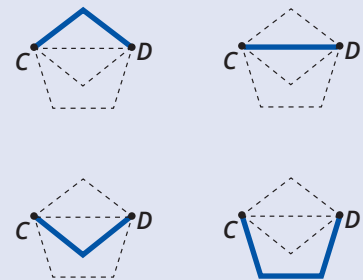
There are 2 paths from A to B .



There are 3 paths from B to C .

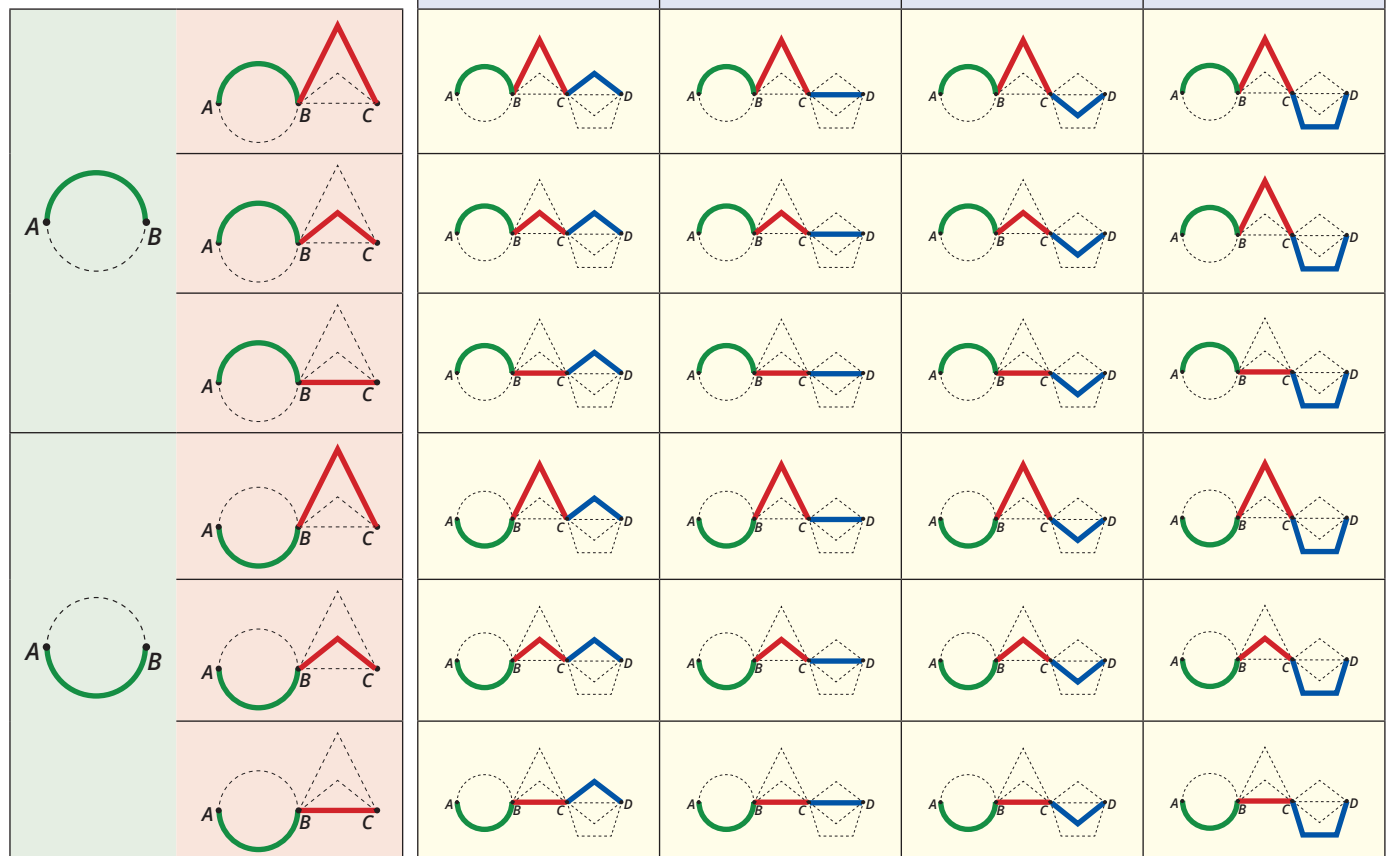


There are 4 paths from C to D .



With 2 paths from A to B , and 3 paths from B to C , there are $2 \times 3 = 6$ paths from A to C .

For each path from A to C , there are 4 possible paths from C to D .



There are $2 \times 3 \times 4 = 24$ different paths from A to D .

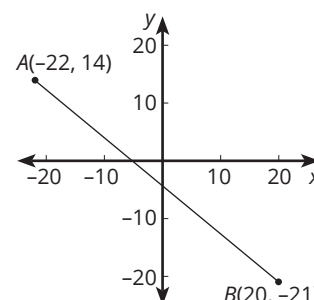
Answer: 24

Example Solution J

A lattice point is a point on a Cartesian plane where both the x and y co-ordinates are integers.

The interval $\overline{AB}$ joins the points $A(-22, 14)$ and $B(20, -21)$.

Including endpoints A and B , how many lattice points lie on $\overline{AB}$?

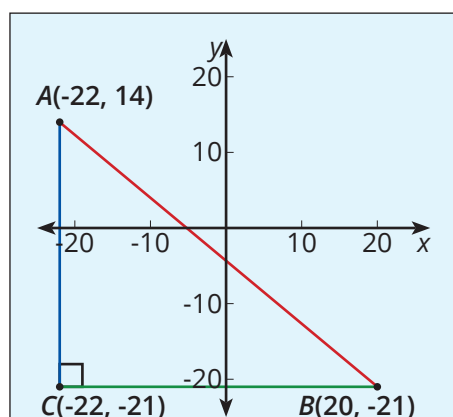


Strategy 1: Consider the Gradient of $\overline{AB}$

We begin by constructing the point C that has the same x -value as A , and the same y -value as B .

This results in a right-angled triangle ABC .

We can use this triangle to determine the gradient of $\overline{AB}$.

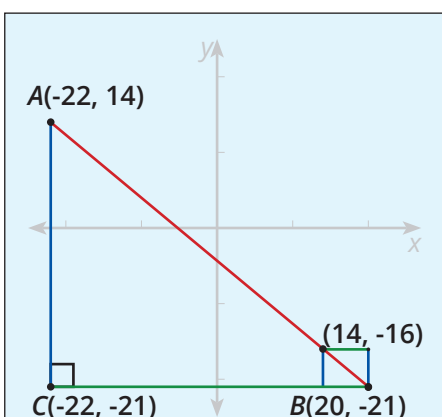


The rise CA is $14 - (-21) = 35$.

The run BC is $-22 - 20 = -42$.

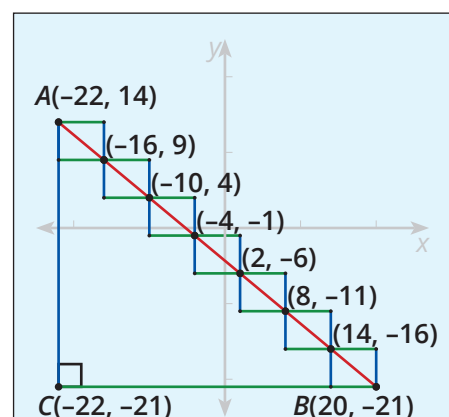
Therefore the gradient of $\overline{AB}$ is

$$\frac{CA}{BC} = \frac{35}{-42} = -\frac{5}{6}.$$



Since the gradient of $\overline{AB}$ is $-\frac{5}{6}$, then there will be a lattice point **6** units to the left and **5** units up from $B(20, -21)$.

This means that there is a lattice point at $(14, -16)$.



We can now continue to locate lattice points that are **6** units to the left and **5** units up from the previous lattice point.

We can see that, including A and B , there are **8** lattice points on $\overline{AB}$.

Strategy 2: Build a Table, and Find a Pattern

The gradient of the line through $A(-22, 14)$ and $B(20, -21)$ is

$$\frac{(-21) - 14}{20 - (-22)} = -\frac{5}{6}.$$

Since this line passes through $A(-22, 14)$, the equation of the line would be:

$$y - 14 = -\frac{5}{6}(x - (-22))$$

$$y = -\frac{5}{6}x - \frac{13}{3}$$

To find lattice points for $-22 \leq x \leq 20$ on the line $y = -\frac{5}{6}x - \frac{13}{3}$, consider the y -value for every integer value of x .

x	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9
y	14	$13\frac{1}{6}$	$12\frac{1}{3}$	$11\frac{1}{2}$	$10\frac{2}{3}$	$9\frac{5}{6}$	9	$8\frac{1}{6}$	$7\frac{1}{3}$	$6\frac{1}{2}$	$5\frac{2}{3}$	$4\frac{5}{6}$	4	$3\frac{1}{6}$

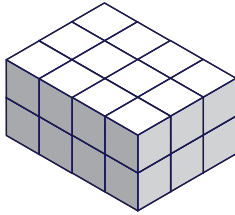
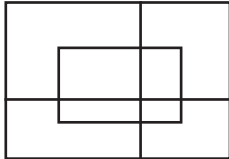
We can see that there is a pattern. A lattice point is occurring for every 6th integral x -value.

x	-22	...	-16	...	-10	...	-4	...	2	...	8	...	14	...	20
y	14	...	9	...	4	...	-1	...	-6	...	-11	...	-16	...	-21

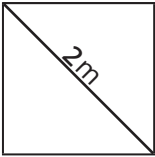
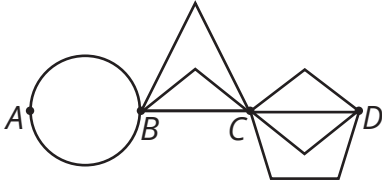
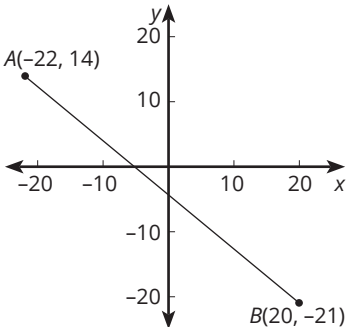
Therefore there are **8** lattice points on $\overline{AB}$.

Answer: 8

Answers

A. Answer: 12	Rory has four clear plastic pockets on his pencil case. The pockets contain letter tiles that form "RORY". One day, Rory rearranged the tiles so that they formed "YRRO". How many different four-letter arrangements can be made using his tiles, including "RORY" and "YRRO"?
B. Answer: 6	How many different rectangular prisms can be constructed using exactly 24 identical unit cubes? [One example is shown with dimensions $2 \times 3 \times 4$.] 
C. Answer: 18	In the diagram, all angles are right angles. How many different rectangles can be traced along the lines in this diagram? 
D. Answer: 240927	A cryptarithm is a mathematical puzzle where the digits have been replaced by letters. The same letter represents the same digit. Different letters represent different digits. In the cryptarithm at the right, the letter <i>E</i> represents the digit 0. What is the 6-digit number represented by the letters <i>TWENTY</i>? $\begin{array}{r} S \ E \ V \ E \ N \\ + \ S \ E \ V \ E \ N \\ + \ S \ E \ V \ E \ N \\ \hline T \ W \ E \ N \ T \ Y \end{array}$
E. Answer: $\frac{2}{7}$	There are 25 prime numbers less than 100. Some of these are two-digit prime numbers. What fraction of these two-digit prime numbers contain the digit 9? Express the answer in its most simplified form.

Answers

F. Answer: $\frac{18}{25}$, 0.72, 72%	What is the probability that a randomly selected three-digit positive integer has no repeated digits? Express your answer as a fraction in its lowest terms, in decimal form, and as a percentage.
G. Answer: 2 (m²)	Mrs Hicks marked out a square in the quadrangle. The diagonals of this square are 2 metres in length. What is the area of the square, in square metres? 
H. Answer: 3.7	There is more than one possible value for X so that the mean and the median of the set $\{2.3, 3.1, 1.6, 4.0, 2.7, X\}$ are equal. Find the greatest possible value for X.
I. Answer: 24	The diagram shows a number of edges that connect vertices A, B, C and D. Following only the edges shown, what is the number of different paths that go from A to B to C to D? You must visit each vertex exactly once. 
J. Answer: 8	A lattice point is a point on a Cartesian plane where both the x and y co-ordinates are integers. The interval $\overline{AB}$ joins the points $A(-22, 14)$ and $B(20, -21)$. Including endpoints A and B, how many lattice points lie on $\overline{AB}$? 

Basic Terms

- | | | | |
|---------------------------|--------------------|-----------------------|-------------|
| • Sum | • Difference | • Product | • Quotient |
| • Value | • Multiple | • Factor | • Remainder |
| • Fraction | • Decimal Fraction | • Percentage | • Ratio |
| • Square / Perfect Square | • Square Root | • Cube / Perfect Cube | • Cube Root |

• "Or" is inclusive: "**a** or **b**" means "**a** or **b** or both".

• An ellipsis (...) indicates that some information has been omitted intentionally.

Read " $1 + 2 + 3 + \dots$ " as "one plus two plus three and so on, without end."

Read " $1 + 2 + 3 + \dots + 10$ " as "one plus two plus three and so on up to ten."

Units of Measurement

Familiarity with units of measurement is assumed, including conversions from one unit to another:

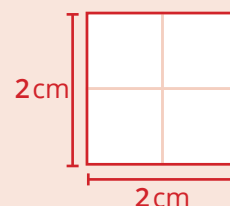
- **Time:** seconds $\leftrightarrow$ minutes $\leftrightarrow$ hours $\leftrightarrow$ days
- **Length:** millimetres $\leftrightarrow$ centimetres $\leftrightarrow$ metres $\leftrightarrow$ kilometres
- **Area:** $\text{mm}^2 \leftrightarrow \text{cm}^2 \leftrightarrow \text{m}^2 \leftrightarrow \text{km}^2$
- **Volume/Capacity:** $\text{mm}^3 \leftrightarrow \text{cm}^3 \leftrightarrow \text{m}^3$; millilitres $\leftrightarrow$ litres
- **Mass:** grams $\leftrightarrow$ kilograms
- **Angles:** degrees ($^\circ$)

Units of measurement must be correct if given in an answer.

To avoid confusion, read cm^2 as "square centimetres", not "centimetres squared".



2 square
centimetres



Possible
misinterpretation

Presenting Answers

Unless otherwise specified in a problem, **equivalent numbers or expressions are acceptable**.

- For example, $3\frac{1}{2}$, $\frac{7}{2}$, and 3.5 are equivalent. $3\frac{2}{4}$ and $\frac{70}{20}$ are not in lowest terms and will not be accepted.

After reading a problem, it is useful to indicate the nature of the answer, before commencing the solution strategy. For example:

- "**A** = ___, **B** = ___."
- "The largest number is ___."
- "The [sum | difference | product | quotient] is ___."
- "The probability, as a [fraction | decimal | percentage], is ___."
- "The perimeter is ___ centimetres."
- "The area is ___ square units."
- "The average speed is ___ kilometres per hour."

Digits and Integers

A **digit** is any one of the ten numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

- 358 is a three-digit number.

The **lead digit** (leftmost digit) of a number is not counted as a digit if it is 0.

- 0358 is a three-digit number.

Terminal zeroes of a number are the zeroes to the right of the last nonzero digit.

- 30 500 has two terminal zeroes.

Whole numbers: { 0, 1, 2, 3, ... }.

Counting numbers, or **Positive Integers:** { 1, 2, 3, ... }.

Integers: { ..., -2, -1, 0, 1, 2, 3, ... }.

- Positive numbers, negative numbers, non-negative numbers, and non-positive numbers are terms that may appear in Division S problems.

Consecutive Numbers are counting numbers that differ by 1.

- 83, 84, 85, 86, 87.

Consecutive Even Numbers are multiples of 2 that differ by 2.

- 36, 38, 40, 42.

Consecutive Odd Numbers are non-multiples of 2 that differ by 2.

- 57, 59, 61, 63.

Factors and Divisibility

Suppose $A = B \times C$, and A , B , and C are all **counting numbers** (1, 2, 3, ...).

- $6 = 2 \times 3$.

Then, A is divisible by B , and A is a multiple of B .

- 6 is divisible by 2.
- 6 is a multiple of 2

Likewise, A is divisible by C , and A is a multiple of C .

- 6 is divisible by 3.
- 6 is a multiple of 3

Both B and C are factors of A .

- 2 and 3 are factors of 6.

A **prime number** is a counting number with exactly two factors, 1 and itself.

- 2, 3, 5, 7, 11, 13, ...

A **composite number** is a counting number which has at least three different factors.

- 4, 6, 8, 9, 10, 12, ...

The number 1 is neither prime nor composite since it has exactly one factor.

A number is **factored completely** when it is expressed as a product of only prime numbers.

- $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
 $= 2^4 \times 3^2$.

The **Highest Common Factor (HCF)** of two counting numbers is the largest counting number that divides each of the two numbers, and the remainder is zero.

If the **HCF** of two numbers is 1, then we say that the numbers are **relatively prime**.

- $\text{HCF}(12, 18) = 6$.

The **Lowest Common Multiple (LCM)** of two counting numbers is the smallest number that each of the given numbers divides, and the remainder is zero.

- $\text{LCM}(12, 18) = 36$.

Fractions

For **common** or **simple fractions** $\frac{a}{b}$,

- a (the **numerator**) and b (the **denominator**) are both integers, and
- $b \neq 0$.

In a **unit fraction**, the numerator is 1.

- $\frac{1}{2}$ and $\frac{1}{100}$ are both unit fractions.

In a **proper fraction**, $a < b$.

- $\frac{1}{2}$ and $\frac{5}{6}$ are both proper fractions.

In an **improper fraction**, $a \geq b$.

- $\frac{3}{2}$ and $\frac{11}{8}$ are both improper fractions.

A **complex fraction** is a fraction whose numerator or denominator contains a fraction.

- $\frac{\frac{2}{3}}{5}$, $\frac{2}{\frac{3}{5}}$, $\frac{\frac{2}{3}}{\frac{5}{2}}$, $\frac{2+\frac{3}{5}}{5-\frac{1}{2}}$ are complex fractions.

The fraction $\frac{a}{b}$ is **simplified** (in **lowest terms**) if a and b have no common factor other than 1, i.e. $\text{HCF}(a,b) = 1$.

- $\frac{1}{2}$ and $\frac{3}{2}$ are both expressed in lowest terms. $\frac{2}{4}$ and $\frac{30}{20}$ are not in lowest terms.
- Unless otherwise specified, fraction answers to Olympiad problems must be expressed in lowest terms.

A **decimal** or **decimal fraction** is a fraction whose denominator is a power of ten.

The decimal is written using decimal point notation.

- $0.07 = \frac{7}{100}$, $0.153 = \frac{153}{1000}$, $6.4 = 6\frac{4}{10}$ or $\frac{64}{10}$.

A **recurring decimal**, or **repeating decimal**, is a decimal fraction with a digit, or group of digits, that repeats forever.

- $\frac{1}{3} = 0.333... = 0.\dot{3} = 0.\overline{3}$
- $\frac{1}{6} = 0.1666... = 0.1\dot{6} = 0.1\overline{6}$
- $\frac{1}{7} = 0.142857142857... = 0.1\dot{4}285\dot{7} = 0.\overline{142857}$

A **percentage** is a fraction whose denominator is 100. The **percent sign** represents the division by 100.

- $9\% = \frac{9}{100}$, $125\% = \frac{125}{100}$, $0.3\% = \frac{0.3}{100}$ or $\frac{3}{1000}$.

Order of Operations

When an expression has more than one arithmetic symbol, certain operations occur before others.

There are a few ways to remember the order of operations, and mnemonics are often used (e.g. **BIDMAS**; **PEMDAS**).

However, it can also be useful to consider the intent when an arithmetic expression is constructed.

By convention, we observe the following priorities:

1. Perform operations in **parentheses**, **braces**, or **brackets**. The **vinculum** (line in a fraction) is also considered as a grouping symbol, similar to parentheses.
2. Evaluate **exponents** (**indices**).
3. Evaluate **multiplication** and **division**, from left to right.
4. Evaluate **addition** and **subtraction**, from left to right.

Example 1

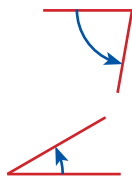
$$\begin{aligned} & 30 + 6 \div 2 - 5 \times (9 - 7) \\ &= 30 + 6 \div 2 - 5 \times 2 \\ &= 30 + 3 - 10 \\ &= 23 \end{aligned}$$

Example 2

$$\begin{aligned} & 20 - (8 + (1 + 2)^2) \\ &= 20 - (8 + 3^2) \\ &= 20 - (8 + 9) \\ &= 20 - 17 \\ &= 3 \end{aligned}$$

Two-Dimensional Figures

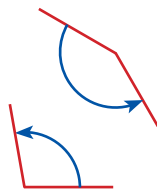
Acute angle
between 0° and 90°



Right angle
 90°



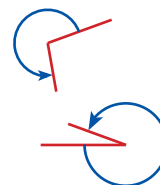
Obtuse angle
between 90° and 180°



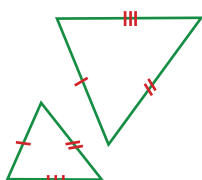
Straight angle
 180°



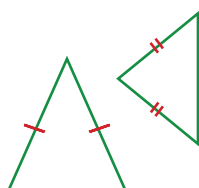
Reflex angle
between 180° and 360°



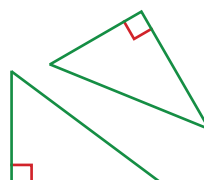
Scalene triangle



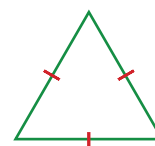
Isosceles triangle



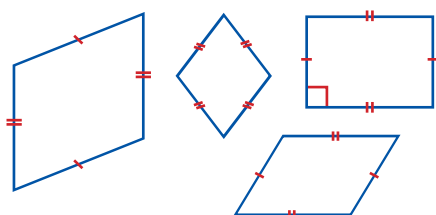
Right-angled triangle



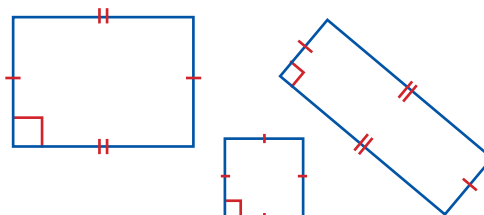
Equilateral triangle



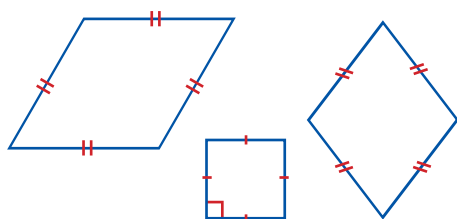
Parallelogram



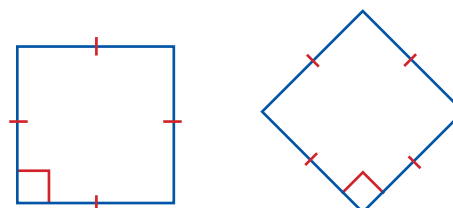
Rectangle



Rhombus



Square



Pentagon



Hexagon



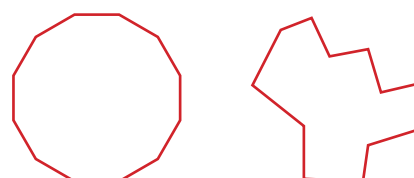
Octagon



Decagon

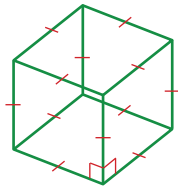


Dodecagon

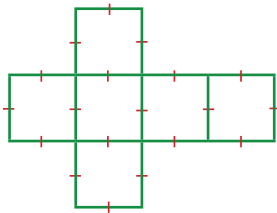


Three-Dimensional Objects

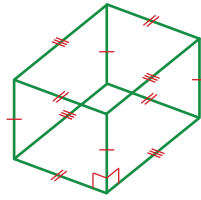
Cube



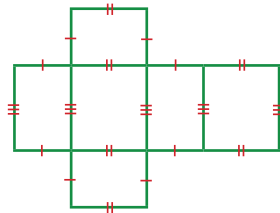
One possible net of a cube



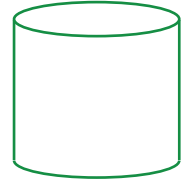
Rectangular Prism



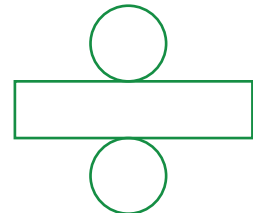
One possible net of a rectangular prism



Right Cylinder



One possible net of a right cylinder



Congruence and Similarity

Two geometric figures are **congruent** if they are identical.

- Congruent triangles coincide exactly when one is superimposed upon the other.
- Congruent plane figures have corresponding pairs of sides that are equal, and corresponding pairs of angles that are the same.

Two geometric figures are **similar** if their shape is the same, even though their size may be different.

- All squares are similar, and all circles are similar.

Classification of Geometric Figures

All equilateral triangles are isosceles, but only some isosceles triangles are equilateral.

A square is a rectangle with all sides congruent.

A square is also a rhombus with all angles congruent.

Within the USA/Canada, a trapezium is an irregular quadrilateral.

Outside the USA/Canada, a trapezium is a quadrilateral with at least one pair of parallel sides (known as a "trapezoid" within the USA/Canada).

Calendar Conventions

There was no year 0. The first century spanned the years 1 to 100 inclusive.

- The **20th century** spanned the years 1901 to 2000 inclusive.
- The **21st century** spans the years 2001 to 2100 inclusive.

Measures of Centre

The **mean**, **arithmetic mean**, or **average**, of a set of values is

- the sum of the values, divided by
 - the number of values.
- For the set $\{5, 5, 7, 11\}$, the mean is $\frac{5+5+7+11}{4} = 28 \div 4 = 7$.
 - For the set $\{7, 11, 23, 5, 5\}$, the mean is $\frac{7+11+23+5+5}{5} = 51 \div 5 = 10\frac{1}{5}$.

The **median** is the value that is exactly in the middle of the set when it is ordered.

If there are an even number of values, then the median is the mean of the two middle values.

- For the set $\{5, 5, 7, 11\}$, the median is $(5 + 7) \div 2 = 6$.
- For the set $\{7, 11, 23, 5, 5\}$, we begin by ordering the set of values: $\{5, 5, 7, 11, 23\}$.
The median is the middle value, 7.

The **mode** is the value that occurs the greatest number of times.

- For the set $\{5, 5, 7, 11\}$, the mode is 5.

A set with every value listed an equal number of times is said to have no mode.

- For the set $\{5, 5, 7, 7, 8, 8\}$, there is no mode.

Probability

The probability of an event is a value that expresses how likely an event is to occur.

- If the event is impossible, then the probability is 0.
- If the event is certain, then the probability is 1.
- All probabilities are between 0 and 1 inclusive.

The probability is found by dividing the number of times an event does occur, by the total number of times the event can possibly occur.

- The probability of rolling an odd number on a die is $\frac{3}{6}$ or $\frac{1}{2}$.