



APSMO

2025 : DIVISION S

WEDNESDAY 7 MAY 2025

OLYMPIAD

1

Total Time Allowed: **30 Minutes**. Calculators NOT Permitted.

1A. The value of $\sqrt{9}$ is 3, because 3 is the positive square root of 9.

What integer is equal to $\sqrt{2025} - \sqrt{1600} - \sqrt{25}$?

1B. Consider the following sequence of whole numbers:

1, 2, 4, 7, 11, 16, ...

The positive difference between each pair of consecutive terms increases by one each time.

Find the 20th term in this sequence.

1C. The 1st century CE encompassed the years 1 through to 100.

What is the second year in the 21st century CE that is exactly divisible by 43?

1D. x and y are two positive integers.

$$2x + xy + y = 32.$$

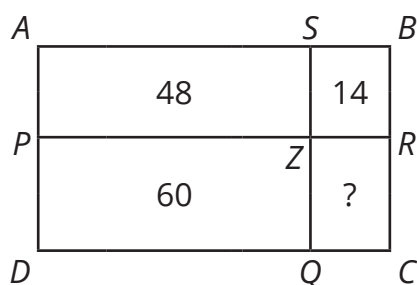
Find the value of $x + y$.

1E. Rectangle $ABCD$ is partitioned into smaller rectangles by line segments PR and QS .

PR and QS meet at interior point Z .

The areas of three of the four regions are shown, in square units.

Find the exact area of rectangle $ZRCQ$, in square units.



Write your answers in the boxes on the back.

← Keep your answers hidden by folding backwards on this line.



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1A.

Student Name:

1B.

1C.

1D.

1E.

Fold here. Keep your answers hidden.



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Solutions and Answers
(Items in parentheses are not required)
For teacher use only. Not for Distribution.

1A: 0

1B: 191

1C: 2064

1D: 16

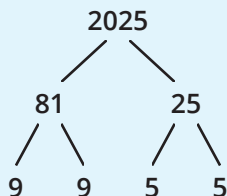
1E: 17.5 or $\frac{35}{2}$ (u^2)

1A. What integer is equal to $\sqrt{2025} - \sqrt{1600} - \sqrt{25}$?

Strategy 1: Convert to a more convenient form.

To find factors of 2025, we might notice that:

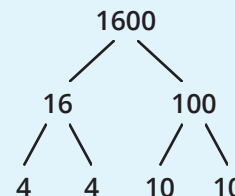
$$\begin{aligned}100 &= 4 \times 25 \\2000 &= 20 \times 100 \\&= 20 \times 4 \times 25 \\&= 80 \times 25 \\2025 &= 81 \times 25 \\&= 9 \times 9 \times 5 \times 5 \\&= 45 \times 45.\end{aligned}$$



Therefore, $\sqrt{2025} = 45$.

To find factors of 1600, we might notice that:

$$\begin{aligned}1600 &= 16 \times 100 \\&= 4 \times 4 \times 10 \times 10 \\&= 40 \times 40.\end{aligned}$$



Therefore, $\sqrt{1600} = 40$.

Finally, we know that $\sqrt{25} = 5$.

$$\sqrt{2025} - \sqrt{1600} - \sqrt{25} = 45 - 40 - 5 = 0.$$

Strategy 2: Build a table.

To find the value of $\sqrt{2025}$, we could build a table to list the squares of different integers, until we find the integer whose square is 2025.

Integer	1	2	3	...									
Square	1	4	9										

Since 2025 is quite a large value, it might be worth skipping some integers in our search.

We might, for example, choose to list the squares of multiples of 10.

Through this, we might notice that $40^2 = 1600$.

Integer	1	2	3	...	10	...	20	...	30	...	40	...	50	
Square	1	4	9		100		400		900		1600		2500	

We can see that the value of $\sqrt{2025}$ is between 40 and 50.

Upon testing 45, we find that $45^2 = 2025$.

Integer	1	2	3	...	10	...	20	...	30	...	40	45	50	
Square	1	4	9		100		400		900		1600	2025	2500	

$$\text{We can see that } \sqrt{2025} - \sqrt{1600} - \sqrt{25} = 45 - 40 - 5 = 0.$$

Follow-Up: What is the value of $\sqrt{1369} - \sqrt{1089} - \sqrt{9}$? [1. Note that 1369 and 1089 both occur between 30^2 and 40^2 . With 9 in the ones place, the ones digit of the square root must be 3 or 7. $37 - 33 - 3 = 1$.]



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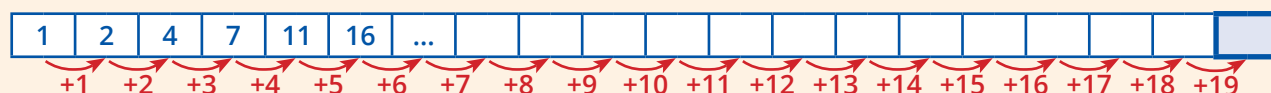
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1B. Find the 20th term in the sequence 1, 2, 4, 7, 11, 16, ...

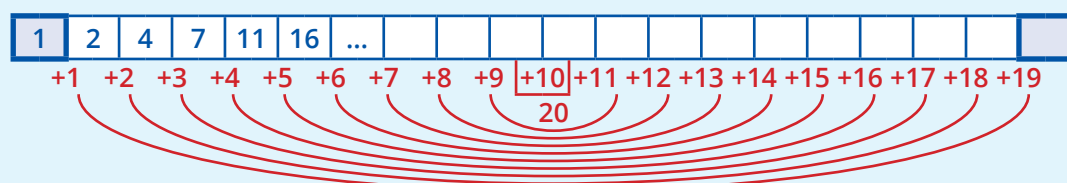
Strategy: Find a pattern.

To find the twentieth term, we need to add 19 values that continue to increase by one.



Method 1:
Find "friends of 20".

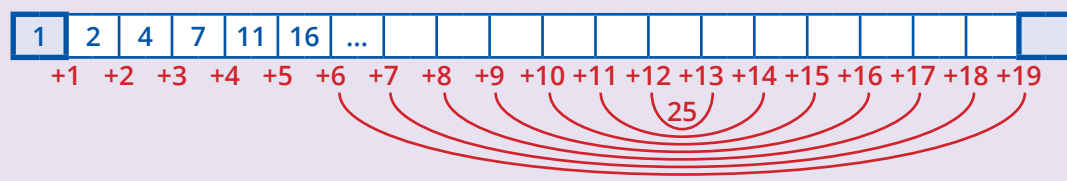
To add $1 + 2 + 3 + \dots + 19$, we can pair values that together result in a sum of 20.



The twentieth term is equal to $1 + (9 \times 20) + 10 = 191$.

Method 2:
Find "friends of 25".

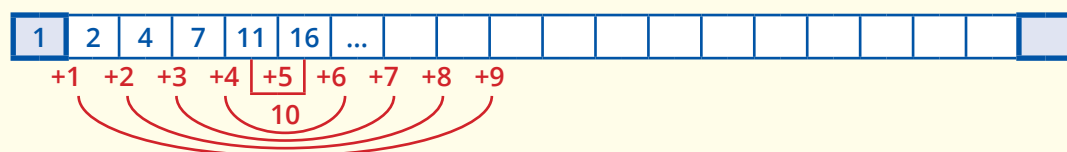
We can also begin with 16 and add $6 + 7 + 8 + \dots + 19$.



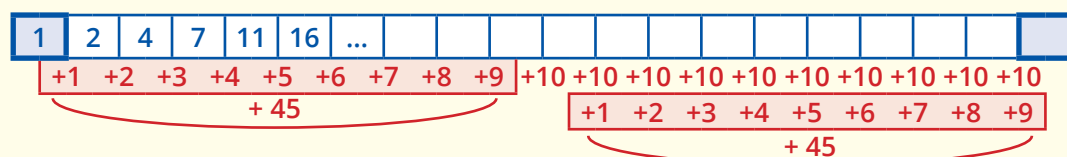
The twentieth term is equal to $16 + (7 \times 25) = 191$.

Method 3:
Find the sum of $1 + 2 + \dots + 9$.

We begin by finding that $1 + 2 + \dots + 9 = (4 \times 10) + 5 = 45$.



Then, by recognising that $11 = 10 + 1$, $12 = 10 + 2$, and so on, we can use place value partitioning to find the value of $1 + 2 + 3 + \dots + 19$.



The twentieth term is equal to $1 + 45 + (10 \times 10) + 45 = 191$.

Follow-Up: A sequence begins 2, 3, 5, 9, 17, The positive differences between each two consecutive terms continues to double. What is the 10th term of the sequence? [513]



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1C. What is the second year in the 21st century CE that is exactly divisible by 43?

The 1st century CE encompassed the years 1 – 100.

The 2nd century CE would then be 101 – 200.

We can see that the century is defined by its final year.

Using this idea, we can work out the range of years for the 21st century CE.

Century	First Year	Final Year
1 CE	1	100
2 CE	101	200
10 CE	901	1000
20 CE	1901	2000
21 CE	2001	2100

Strategy 1: Convert to a more convenient form.

To find values that are exactly divisible by 43, we can begin by listing convenient multiples of 43.

Using multiples of $10 \times 43 = 430$, we find that the range of values lies between $40 \times 43 = 1720$ and $50 \times 43 = 2150$.

This range can be further refined to $47 \times 43 = 2021$ and $48 \times 43 = 2064$.

These are the only two years in the 21st century CE that are exactly divisible by 43.

The second year in the 21st century CE that is exactly divisible by 43, is 2064.

$$1 \times 43 = 43$$

$$10 \times 43 = 430$$

$$20 \times 43 = 860$$

$$40 \times 43 = 1720$$

$$50 \times 43 = 2150$$

$$49 \times 43 = 2150 - 43 = 2107$$

$$48 \times 43 = 2107 - 43 = 2064$$

$$47 \times 43 = 2064 - 43 = 2021$$

$$46 \times 43 = 2021 - 43 = 1978$$

Strategy 2: Use a written algorithm.

We know that the last year in the 21st century CE is 2100.

Dividing 2100 by 43:

$$\begin{array}{r} 48 \\ 43 \overline{) 2100} \\ \underline{172} \\ 380 \\ \underline{344} \\ 36 \end{array}$$

$2100 \div 43 = 48$, with a remainder of 36.

Therefore, we know that $43 \times 48 < 2100$.

The value of 43×48 can be found in many different ways.

$$\begin{array}{r} 40 \quad 8 \\ 43 \times 48 \\ \hline 1600 \quad 320 \\ \hline 1600 \quad 320 \\ + \quad 120 \quad 24 \\ \hline 2064 \end{array}$$

The question is asking for the second year in the 21st century CE that is exactly divisible by 43.

We can see that:

- The year $2064 - 43 = 2021$ is also divisible by 43.
- There can be no other years in the 21st century CE that are divisible by 43.

The first year in the 21st century CE that is exactly divisible by 43, is 2021.

The second year in the 21st century CE that is exactly divisible by 43, is 2064.

Follow-Up: How many positive integers between 4301 and 42999 are divisible by 43? [899]



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1D. Find the value of $x + y$, if $2x + xy + y = 32$, and x and y are both positive integers.

Strategy 1: Build a table, and find a pattern.

Since x and y are both positive integers, we can build a table with values for x and y , placing the value of $2x + xy + y$ at each intersection.

$y \backslash x$	1	2	3	4	5
1	4	7	10	13	16
2	6	10	14	18	22
3	8	13	18	23	28
4	10	16	22	28	34
5	12	19	26	33	40

Through doing so, we can see arithmetic sequences in each row and column.

By inspection, the column for $x = 1$ appears to include every even number greater than 2.

Therefore, 32 must occur in the column associated with $x = 1$.

The question indicates that there is exactly one value for $x + y$ for which $2x + xy + y = 32$, so setting $x = 1$ must lead to the answer.

$$\begin{aligned}
 \text{Substituting } x = 1: \quad & 2(1) + (1)y + y = 32 \\
 & 2y = 30 \\
 & y = 15 \\
 & x + y = 1 + 15 \\
 & = 16
 \end{aligned}$$

$y \backslash x$	1	2	3	4	5	6	7	8	9	10	11
1	4	7	10	13	16	19	22	25	28	31	34
2	6	10	14	18	22	26	30	34			
3	8	13	18	23	28	33					
4	10	16	22	28	34						
5	12	19	26	33	40						
6	14	22	30								
7	16	25	34								
8	18	28									
9	20	31									
10	22	34									
11	24										
12	26										
13	28										
14	30										
15	32										

Alternatively, we can use the table to situate values for $2x + xy + y$ that are close to 32.

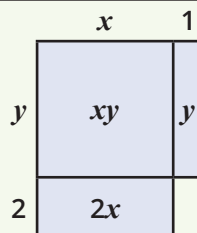
The arithmetic sequences can facilitate a quicker calculation of $2x + xy + y$ in each case.

Since $2x + xy + y = 32$ when $x = 1$ and $y = 15$, the value of $x + y$ is $1 + 15 = 16$.

Strategy 2: Draw a diagram.

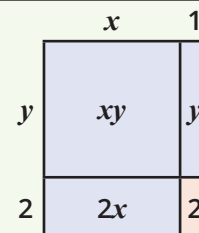
We can represent $2x + xy + y$ diagrammatically, using an area model.

This entire area is 32 square units.



If we add an extra 2 square units, we can turn the area into a rectangle.

The area of the entire rectangle is 34 square units.



The area of the rectangle is $(x + 1) \times (y + 2)$.

Since the entire area is 34 square units, and x and y are both positive integers, we know that both $(x + 1)$ and $(y + 2)$ must be factors of 34.

We can see that there is only one rectangle where both x and y are positive integers.

Therefore, the value of $x + y$ is $1 + 15 = 16$.

$x + 1$	$y + 2$	x	y
1	34	0	32
2	17	1	15
17	2	16	0
34	1	33	-1

Follow-Up: Suppose p and q are primes such that $pq + 6p + q = 27$. Find p and q . [$p = 2, q = 5$]



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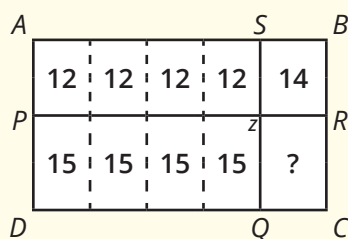
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1E. Find the exact area of rectangle ZRCQ.

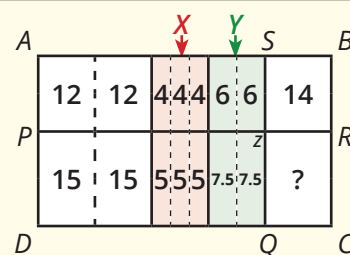
Strategy 1: Divide a complex shape.

If we divide rectangle **ASQD** in quarters, there would be four rectangles with an area of 12 units², and four with an area of 15 units².

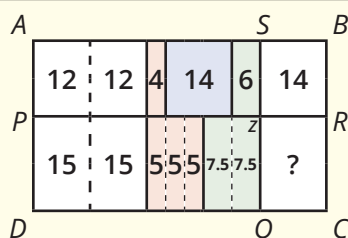


Suppose we select two of these rectangles, **X** and **Y**.

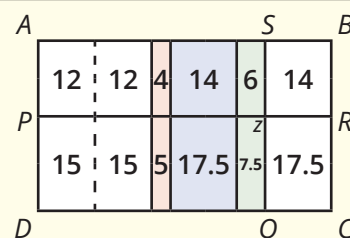
We can divide rectangle **X** into thirds, and **Y** in half.



We can combine three of the rectangles in the top row to make a single rectangle that, like **SBRZ**, has area 14 units².



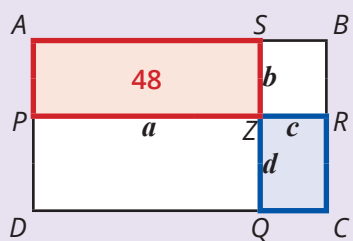
Combining the corresponding rectangles in the bottom row, we have a rectangle that, like **ZRCQ**, has area 17.5 units².



Strategy 2: Reason algebraically.

Let a , b , c , and d represent the lengths of **PZ**, **SZ**, **ZR** and **ZQ**.

$ab = 48$, $bc = 14$, and $ad = 60$.



Our aim is to find the value cd .

Finding c in terms of a :
 $\frac{bc}{ab} = \frac{14}{48}$
 $\frac{c}{a} = \frac{7}{24}$
 $c = \frac{7}{24}a$

Finding d in terms of b :
 $\frac{ad}{ab} = \frac{60}{48}$
 $\frac{d}{b} = \frac{5}{4}$
 $d = \frac{5}{4}b$

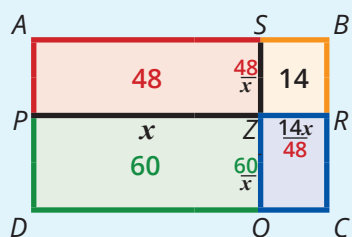
ZRCQ is $\frac{7}{24}$ as wide as **ASZP**, and $\frac{5}{4}$ as high.

Therefore the area of **ZRCQ** is equal to

$$\begin{aligned} & \frac{7}{24} \times \frac{5}{4} \times \text{Area}(\text{ASZP}) \\ &= \frac{7}{24} \times \frac{5}{4} \times 48 \\ &= \frac{7}{24} \times 60 \\ &= 7 \times 2.5 \\ &= 17.5 \text{ units}^2. \end{aligned}$$

Strategy 3: Reason algebraically (Alternative approach).

Let x represent the length of **PZ**. Other lengths can then be expressed in terms of x .



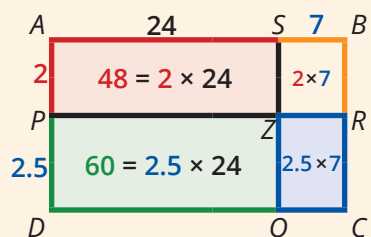
We have $ZR = \frac{14x}{48}$ and $ZQ = \frac{60}{x}$.

The area of **ZRCQ** is equal to $\frac{14x}{48} \times \frac{60}{x} = \frac{35}{2}$ units².

Strategy 4: Use a convenient value.

Suppose the length of **AP** is 2 units.

Other lengths can then be deduced.



The area of **ZRCQ** is $2.5 \times 7 = 17.5$ units².

FOLLOW-UP: Consider the original diagram. Suppose that instead of the numbers being the areas of the smaller rectangles, they are the perimeters of the smaller rectangles. What is the perimeter of **ZRCQ**? [26]