

2025 Maths Olympiads Division J Preparation Kit



Welcome to the APSMO Maths Olympiads for 2025

The format of the Maths Olympiads for 2025 will be:

Preparation Kit: Available from February, 2025

Competition 1: Wednesday May 7, 2025

Competition 2: Wednesday June 11, 2025

Competition 3: Wednesday July 30, 2025

Competition 4: Wednesday September 10, 2025

Preparing for the APSMO Maths Olympiads

The purpose of this Preparation Kit is to provide students with an opportunity to familiarise themselves with the concepts, and terminology, that will subsequently be used in the four competition papers for 2025.

The kit contains:

1) Preparation Questions p.2

- 6 questions (with and without hints)
- Solutions to work through with students

2) Preparation Tasks p.8

- 4 differentiated preparation tasks
- Work samples and solutions for review with students

3) Preparation Paper p.13

- 5 contest style questions in 30 minutes
- Fully worked solutions to review with students

4) Skills and Terminology p.19

An appendix of skills and terminology that students should be familiar with to participate in the Maths Olympiad

This kit may be used to:

- Reinforce previously learned concepts and terminology
- Introduce new or different solution methods
- Provide diagrams and animations that support teacher or student explanations
- Provide opportunity to collaboratively consider student work samples
- Support students' own study as a standalone resource

Please click this video link to watch an introduction to [2025 Maths Olympiad Junior](#).

Further questions and solution methods can also be found in the APSMO resource books, available from www.apsmo.edu.au.

Preparation Questions 1 - 3

- 1 Four volunteers can pack 12 boxes every 30 minutes.
How many additional volunteers are needed to pack 72 boxes every hour?
[Assume all volunteers work at the same pace.]
- 2 Suppose the number of units in each of the length and width of a rectangle are prime numbers and the perimeter is 36 cm.
What is the area of the largest rectangle in square centimetres?
- 3 Grace chooses five different numbers from the list 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
Two of those numbers are 4 and 5, and they are the only two numbers she picks that differ by 1.
What is the greatest possible sum of the five numbers?

Preparation Questions 4 - 6

- 4** Asha has 5 more 40c stamps than 30c stamps.
The total value of her 40c stamps is \$5.20 more than that of her 30c stamps.
How many 40c stamps does Asha have?
- 5** At the end of a power cut, a digital clock resets to 12:00 midnight.
At 9:35 a.m. on the same day the power cut occurred, the digital clock shows 3:50 a.m.
At what time did the power cut end?
(Label your answer a.m. or p.m.)
- 6** The room numbers on one side of a hotel hall are odd.
They are numbered from 11 through 59 inclusive.
Kristen is in one of these rooms.
Express as a fraction the probability that Kristen's room number is divisible by 5.

Preparation Questions 1 - 3 with Hints

- 1** Four volunteers can pack 12 boxes every 30 minutes.
How many additional volunteers are needed to pack 72 boxes every hour?
[Assume all volunteers work at the same pace.]

Hint: How many boxes can 4 volunteers pack in one hour?

- 2** Suppose the number of units in each of the length and width of a rectangle are prime numbers and the perimeter is 36 cm.
What is the area of the largest rectangle in square centimetres?

Hint: The semi-perimeter of the rectangle is 18. Which 2 primes have a sum of 18?

- 3** Grace chooses five different numbers from the list 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
Two of those numbers are 4 and 5, and they are the only two numbers she picks that differ by 1.
What is the greatest possible sum of the five numbers?

Hint: Work your way down from the greatest choice given that 4 and 5 must be included.

Preparation Questions 4 - 6 with Hints

- 4** Asha has 5 more 40c stamps than 30c stamps.
The total value of her 40c stamps is \$5.20 more than that of her 30c stamps.
How many 40c stamps does Asha have?

Hint: What is the value of the extra stamps if you pair each 30c stamp with a 40c stamp?

- 5** At the end of a power cut, a digital clock resets to 12:00 midnight.
At 9:35 a.m. on the same day the power cut occurred, the digital clock shows 3:50 a.m.
At what time did the power cut end?
(Label your answer a.m. or p.m.)

Hint: For how many hours did the power cut last?

- 6** The room numbers on one side of a hotel hall are odd.
They are numbered from 11 through 59 inclusive.
Kristen is in one of these rooms.
Express as a fraction the probability that Kristen's room number is divisible by 5.

Hint: How many rooms are there on that side of the hall?



Preparation Questions Solutions

1: 8	2: 77 cm^2	3: 29	4: 37	5: 5:45 a.m.	6: $\frac{1}{5}$
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1 Strategy 1: Find how much 4 volunteers can do in 1 hour.

If 4 volunteers can pack 12 boxes every 30 minutes, then 4 volunteers can pack 24 boxes every hour (60 minutes).

In order to pack 72 boxes in an hour, since $72 \div 24 = 3$, three times as many volunteers are needed.

So 12 volunteers are needed to do the job; therefore **8 additional volunteers are needed.**

Strategy 2: Find how many boxes 1 volunteer can pack in an hour.

If 4 volunteers can pack 12 boxes every 30 minutes, then 1 volunteer can pack $\frac{1}{4}$ as many in 30 minutes, or 3 boxes.

One volunteer can then pack 6 boxes in an hour.

To pack 72 boxes requires 12 volunteers and so **8 additional volunteers are needed.**

2 Strategy: Find the semi-perimeter.

The sum of the width and length (called the "semi-perimeter") is 18.

We need to find two primes to represent the width (W) and the length (L) with a sum of 18.

The largest area the rectangle could have is 77 cm^2 .

W	L	Area
5	13	65
7	11	77

3 Strategy 1: Start with the greatest number.

To get the greatest sum, start by choosing 10. 9 can't be used as it differs from 10 by 1.

Next we choose 8. This means 7 can't be used.

6 can't be used because it's adjacent to 5 which is given as one of the numbers.

4 and 5 must be included, so 3 can't be used.

Choose 2 as the final number.

The greatest possible sum of Grace's numbers is **$10 + 8 + 5 + 4 + 2 = 29$.**

Strategy 2: Start with the known numbers.

Two of the numbers are 4 and 5.

Neither 3 nor 6 can be used because they differ by 1 from 4 and 5 respectively.

The other three numbers must be chosen from 1, 2, 7, 8, 9, and 10, but not consecutive numbers.

The greatest possible sum of Grace's numbers is **$10 + 8 + 5 + 4 + 2 = 29$.**

Preparation Questions Solutions

4 Strategy 1: Pair each 30c stamp with a 40c stamp.

All but five of the 40c stamps can be paired with 30c stamps.

These five "extra" 40c stamps account for \$2.00 of the given \$5.20.

Then the total difference of all the pairs is the remaining \$3.20.

Since each pair differs by 10c, there are 32 pairs.

Then Asha has thirty-two 30c stamps and **thirty-seven 40c stamps**.

Checking, $(37 \times 40c) - (32 \times 30c) = \$14.80 - \$9.60 = \5.20 .

Strategy 2: Make an organised list of simpler cases and find a pattern.

1	Number of 30c stamps	1	2	3	...	
2	Number of 40c stamps	6	7	8	...	?
3	Value of 30c stamps	\$0.30	\$0.60	\$0.90	...	
4	Value of 40c stamps	\$2.40	\$2.80	\$3.20	...	
5	Difference in value	\$2.10	\$2.20	\$2.30	...	\$5.20

For each additional 30c stamp, the difference in value increases by 10c, as indicated by line 5.

To change the difference in value from \$2.30 to \$5.20, $290c \div 10c = 29$ additional stamps of each type are needed.

Asha has $(3 + 29) =$ thirty-two 30c stamps and $(8 + 29) =$ thirty-seven 40c stamps.

5 Strategy: Use a convenient time, then adjust.

When the power came back on, the clock reset to 12:00 midnight. It now shows the time 3:50 a.m.

This shows us that the power outage ended 3 hours and 50 minutes ago.

To determine the time, subtract 3 hours and 50 minutes from 9:35 a.m.

4 hours is a more convenient time.

Because 4 hours earlier the time was 5:35, then 3 hours and 50 minutes earlier the time was 5:45.

The power outage ended at **5:45 a.m.**

6 Strategy 1: Count the odd numbers from 11 to 59 inclusive.

There are 49 whole numbers from 11 through 59. Since the list starts and ends with an odd number, it contains one more odd number than even number.

Thus there are 25 odd and 24 even numbers.

There are five odd multiples of 5 between 11 and 59: 15, 25, 35, 45, and 55.

The probability of Kristen being in an odd-numbered room is $\frac{5}{25}$ or $\frac{1}{5}$.

Strategy 2: Split the room numbers up into decades.

Consider the five decades 11-19, 21-29, 31-39, 41-49, and 51-59.

Each decade has 5 odd room numbers, for a total of 25 odd-numbered rooms.

In each decade only the room number ending in 5 is divisible by 5.

The probability of Kristen being in an odd-numbered room is $\frac{5}{25}$ or $\frac{1}{5}$.

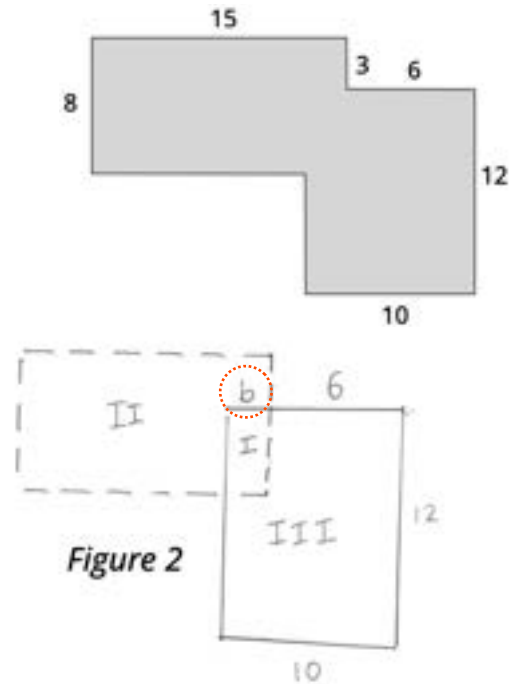
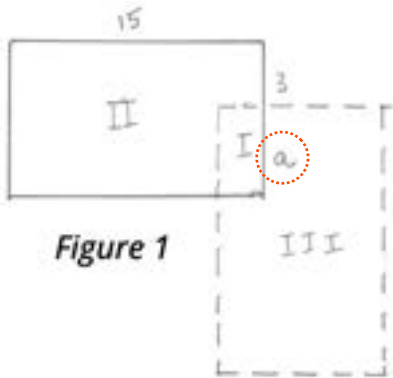
Preparation Task #1

- A** Maisie and Julian have been asked to find the area of this shape.

Their teacher told them that all angles shown were right angles and that all lengths are given in centimetres.

Julian starts by drawing 2 rectangles and places one on top of the other to create the shape.

He wants to find the area of the overlapping region.



Decide how to calculate the missing length for *Figure 1* (labelled *a*) and width for *Figure 2* (labelled *b*).

Add this information to the figures.

- B** Maisie uses the information from *Figures 1* and *2* and **declares** the answer to the problem is 240 cm^2 . She is incorrect.

Describe how she arrived at her answer.

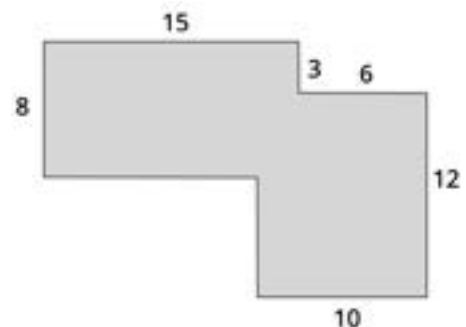
Pinpoint the detail she overlooked in her calculation.

- C** Eloy and his partner solved this problem correctly.

He says to Maisie and Julian, "We solved this differently. We enclosed the shape within one large rectangle."

Use Eloy's method to work out the shaded area.

Record the steps Eloy followed to find the area.



Preparation Task #2

A Patrick and Malee work together to solve this problem:

A teacher surveyed 24 students and discovered that:

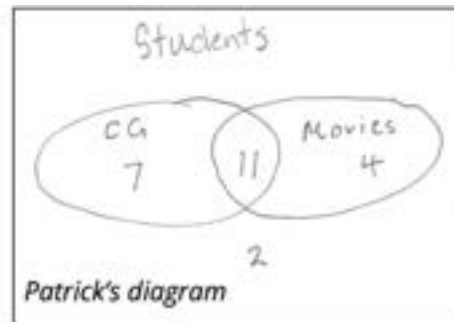
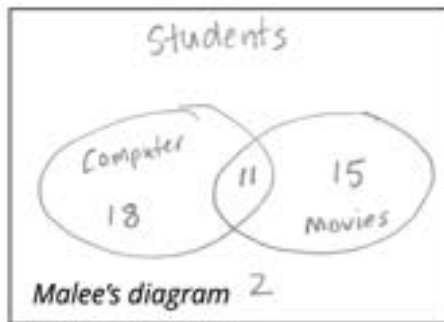
- 18 of them like to play computer games;
- 15 of them like to go to the movies; and
- 2 of them do not like either playing computer games or going to the movies.

How many of the 24 students like both activities?

Patrick and Malee both decide to use a Venn diagram to solve this problem.

Analyse each of the diagrams below.

Identify the student whose diagram is incorrect and **discuss** why.



B Patrick suggests they use another method to **check** their solution.

He rules up a diagram with 24 boxes, representing the 24 students.

Patrick writes an *N* in 2 of the boxes for the students who don't like either activity. Then, he writes *C* in 18 boxes to record the students who like playing computer games in the diagram.

Finally, Patrick writes *M* 15 times to represent the 15 students who like going to the movies.

N	N	C	C	C	C	C	C
C	C	C	C	C	C	C	C
C	C	C	C	M	M	M	M
C	C	M	M	M	M	M	M

Discuss with your partner how this diagram helps Patrick and Marlee check their solution.

C Challenge:

Can you solve this problem by drawing a similar diagram?

Out of all the students at Wantagh Middle School, 80% own computers and 40% are in the band.

However, 10% of all the students neither own computers nor are in the band.

What percentage of students own computers and are in the band?

Preparation Task #3

A Luella and Edison work together to solve this problem:

Takeru has four 1 cm long blocks, three 5 cm long blocks, and three 25 cm long blocks.

By joining these blocks to make different total lengths, how many different lengths of at least 1 cm can Takeru make?

Edison immediately says, "I know how to solve this! $4 \times 3 \times 3 = 36$."

Edison is incorrect.

Identify the method he is recommending.

Provide an example of a problem where this method works.

Explain why Edison's method does not work in this case.

B Luella suggests they solve the problem by establishing the maximum length they can make with the blocks.

"When we know that," Luella says, "we can eliminate the lengths that are impossible to make."

Determine the largest length that can be made with the blocks.

Find and **show** 3 lengths longer than 30 cm that cannot be made with these blocks.

Preparation Task #4 - Challenge

- A** Penny and Brett are solving a problem. They know every digit from 1 through 9 must appear exactly once in the correct addition problem to the right. Penny says, "It's impossible to place 9 or 8 directly below the 7." **Consider** if you agree with her and **explain** your decision.

$$\begin{array}{r}
 \quad 5 \quad 6 \quad 7 \\
 + \quad \square \quad \square \quad \square \\
 \hline
 \quad \square \quad \square \quad \square
 \end{array}$$

- B** Brett is confident that 4 must be in the **addend**. **Explore** the following possibilities to **determine** if he is correct.

$$\begin{array}{r}
 \quad 5 \quad 6 \quad 7 \\
 + \quad \square \quad \square \quad 4 \\
 \hline
 \quad \square \quad \square \quad \square
 \end{array}$$

$$\begin{array}{r}
 \quad 5 \quad 6 \quad 7 \\
 + \quad \square \quad 4 \quad \square \\
 \hline
 \quad \square \quad \square \quad \square
 \end{array}$$

$$\begin{array}{r}
 \quad 5 \quad 6 \quad 7 \\
 + \quad 4 \quad \square \quad \square \\
 \hline
 \quad \square \quad \square \quad \square
 \end{array}$$

Preparation Task Solutions

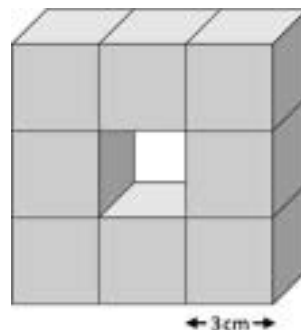
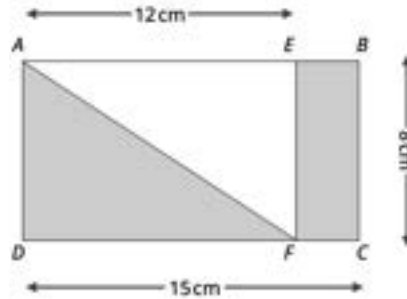
A slide show of student work samples and solutions for each of the Preparation Tasks will be posted in the APSMO Portal on Monday, 10th March.

If you would like to contribute student work to be considered for the slide show, please email it to showcase@apsmo.edu.au by Wednesday, 5th March.



Suggested Time: 30 Minutes

<p>A. When A is divided by B, the result is 5. The product of A and B is 45. What is the sum of A and B if A and B are two positive integers?</p>	<p><i>Write your answers in the boxes on the back.</i></p>	
<p>B. A cyclist takes 2 minutes and 30 seconds to travel a distance of 1 kilometre. At this rate, how many kilometres will the cyclist travel in 1 hour?</p>		<p><i>Keep your answers hidden by folding backwards on this line.</i></p>
<p>C. $ABCD$ and $EBCF$ are both rectangles. The length of CD is 15 cm. The length of BC is 8 cm. The length of AE is 12 cm. Find the total shaded area in square centimetres.</p>		
<p>D. 1 blue marble and 2 green marbles cost 16 cents. 1 red marble and 2 blue marbles also cost 16 cents. 1 green marble and 2 red marbles only cost 13 cents. How much does 1 green marble cost?</p>		
<p>E. Eight cubes are glued together to form the object shown. Each cube has a length of 3 cm. The entire object is dipped in and out of a can of paint. How many square centimetres are covered in paint?</p>		



A.	<i>Fold Here. Keep your answers hidden.</i>
B.	
C.	
D.	
E.	

Solutions and Answers

For teacher use only. Not for Distribution.

A: 18	B: 24km	C: 72cm²	D: 5cents	E: 288cm²
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- A.** The question is:
When A is divided by B, the result is 5.
The product of A and B is 45.
What is the sum of A and B if A and B are two positive integers?

Strategy: *Eliminate all but one possibility.*

List all the factors of 45 in pairs whose product is 45: 1×45 , 3×15 , and 5×9 .
The only pair of numbers that has a quotient of 5 is 3 and 15.
The sum of these is $3 + 15 = \mathbf{18}$

- B.** The question is:
A cyclist takes 2 minutes and 30 seconds to travel a distance of 1 kilometre.
At this rate, how many kilometres will the cyclist travel in 1 hour?

Strategy: *Convert to a more convenient form.*

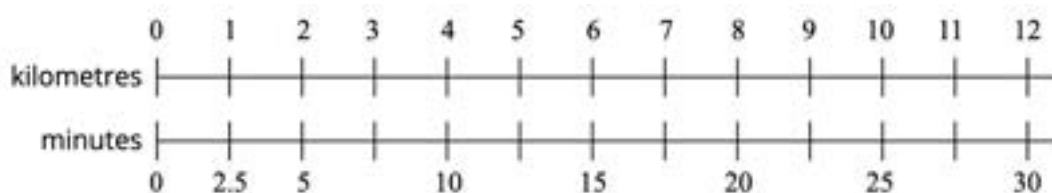
Method 1: *Simplify the time it takes to travel 2 kilometres.*

We can simplify the problem by considering the time it takes the cyclist to travel 2 kilometres.
 $2 \text{ minutes and } 30 \text{ seconds} + 2 \text{ minutes } 30 \text{ seconds} = 5 \text{ minutes.}$

Therefore it takes:

- 5 minutes to ride 2 kilometres
- 10 minutes to ride 4 kilometres
- 30 minutes to ride 12 kilometres
- 60 minutes to ride **24 kilometres**

Method 2: *Plot the distance against time.*



The cyclist travelled 12 kilometres in 30 minutes or **24 kilometres** in 60 minutes which is 1 hour.



C. The question is:

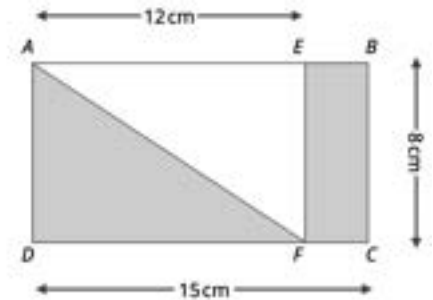
$ABCD$ and $EBCF$ are both rectangles.

The length of CD is 15 cm.

The length of BC is 8 cm.

The length of AE is 12 cm.

Find the total number of square centimetres in the areas of the shaded regions.

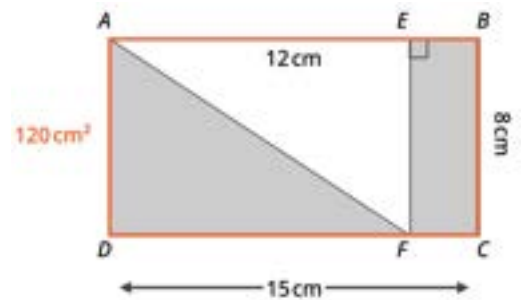


Strategy: Divide a Complex Shape.

Method 1: Find the area of the region that is not shaded.

The area of rectangle $ABCD = 15 \text{ cm} \times 8 \text{ cm} = 120 \text{ cm}^2$.

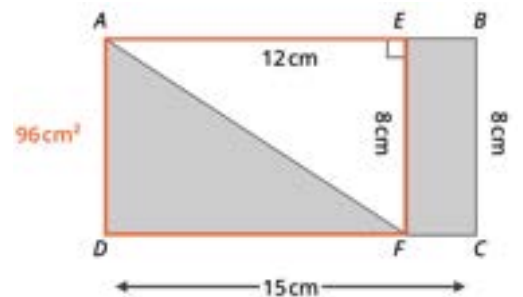
If $EBCF$ is a rectangle, then triangle AEF is a right angled triangle.



The area of unshaded triangle $AEF = \frac{12 \times 8}{2} = 48 \text{ cm}^2$.

The area of the shaded region = area of $ABCD$ – area of AEF .

The total area of the shaded regions is $120 \text{ cm}^2 - 48 \text{ cm}^2 = 72 \text{ cm}^2$.



Method 2: Add the areas of the shaded regions.

Using rectangle $EBCF$:

$EB = 15 \text{ cm} - 12 \text{ cm} = 3 \text{ cm}$ and $BC = 8 \text{ cm}$.

Area $EBCF = 24 \text{ cm}^2$.

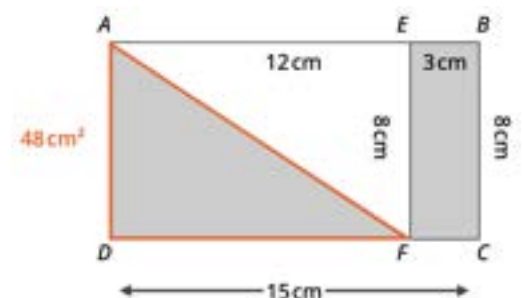
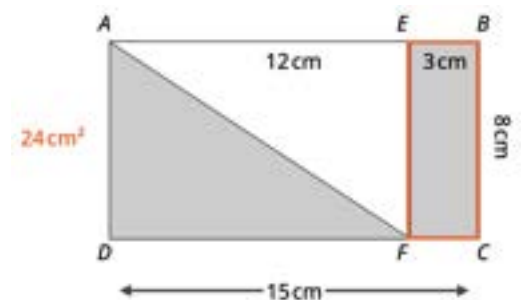
Triangle ADF :

Using area $ADF = \frac{1}{2}$ of the area of rectangle $AEFD$

$= \frac{1}{2}$ of $12 \text{ cm} \times 8 \text{ cm} = 48 \text{ cm}^2$.

The area of the shaded regions is the sum of the areas of rectangle $EBCF$ and triangle ADF .

The total area of the shaded regions is $24 \text{ cm}^2 + 48 \text{ cm}^2 = 72 \text{ cm}^2$.



D. The question is: How much does 1 green marble cost?

Strategy 1: Reason Logically

Suppose all 3 purchases are made. Then 3 green marbles, 3 blue marbles, and 3 red marbles cost a total of 45 cents.

So 1 green marble, 1 blue marble, and 1 red marble cost 15 cents.



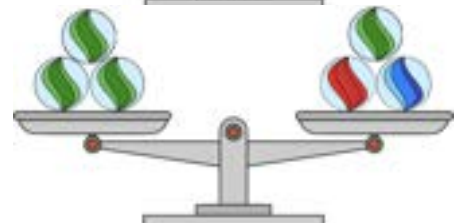
The first and second sentences show that 1 blue and 2 green marbles cost as much as 1 red and 2 blue marbles.



If we remove 1 blue marble from each side we can see that 2 green marbles balance 1 red and 1 blue marble.



If we add 1 green marble to each side, we can see that 3 green marbles balance 1 red, 1 blue, and 1 green, and we know that 1 green marble, 1 blue marble, and 1 red marble cost 15 cents.



So we know that 3 green marbles cost 15 cents, so each **green marble costs 5 cents.**



Strategy 2: Guess, check and refine. Draw a table.

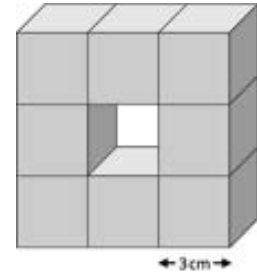
The cost of 1 green plus 2 red marbles is an odd number of cents but the cost of 2 red marbles is **even**.

So the cost of 1 green marble is **odd**.

Suppose one green costs	3c	5c	7c	9c
Then 2 red = 13c - one green, so 2 red cost	10c	8c	6c	4c
Therefore, one red would cost	5c	4c	3c	2c
Next, 2 blue = 16c - one red, so 2 blue cost	11c	12c	13c	14c
Therefore, one blue would cost	Not Possible	6c	Not Possible	7c
Check: one blue + 2 green costs	Not Possible	16c	Not Possible	25c

1 blue and 2 green cost 16c, not 25c. Therefore, a **green marble costs 5 cents.**

- E.** The question is:
 Eight cubes are glued together to form the object shown.
 Each cube has a length of 3 cm.
 The entire object is dipped in and out of a can of paint.
 How many square centimetres are covered in paint?



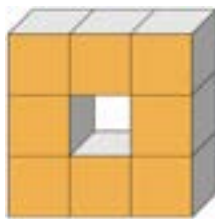
Strategy: Count in an organised way.

Method 1: Count the number of cubes painted on 4 faces.

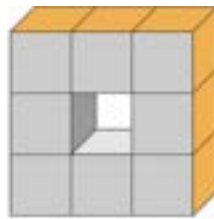
Since every one of the eight cubes is painted on four of its six faces, 32 square faces are painted.
 The area of each face is $3 \times 3 = 9 \text{ cm}^2$, therefore $32 \times 9 = \mathbf{288 \text{ square centimetres}}$ are covered in paint.

Method 2: Count the number of faces that are painted.

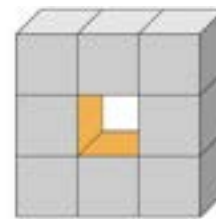
The front of the figure has 8 faces painted and the back has 8 faces painted.
 The top, bottom, and two sides each have 3 faces painted.
 The middle has 4 exposed faces that are painted.
 Therefore, a total of $(2 \times 8) + (4 \times 3) + 4 = 32$ faces are painted.
 The area of each face is 3×3 or 9 cm^2 , so $32 \times 9 = \mathbf{288 \text{ square centimetres}}$ are covered in paint.



8 painted faces on the front and back.



3 painted faces on the top, bottom and 2 sides.



4 exposed faces in the middle hole.

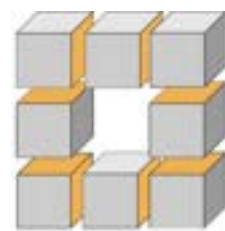
Method 3: Count the number of cube faces that are not painted.

Each cube has 6 faces and there are 8 cubes in the figure and therefore has a total of 48 faces.

There are 8 places where 2 cube faces are glued together; this means 8×2 faces are not painted.

Then $48 - 16 = 32$ faces are painted.

The area of each face is $3 \times 3 = 9 \text{ cm}^2$, so a total of $32 \times 9 = \mathbf{288 \text{ square centimetres}}$ are covered in paint.



8 places where 2 cubes are glued together.

Basic Terms

- Sum
 - Difference
 - Product
 - Quotient
 - Value
 - Multiple
 - Factor
 - Remainder
 - Fraction
 - Decimal Fraction
 - Percentage
 - Ratio
 - Square / Perfect Square
 - Square Root
 - Cube / Perfect Cube
 - Cube Root
- "Or" is inclusive: "*a* or *b*" means "*a* or *b* or both".

- An ellipsis (...) indicates that some information has been omitted intentionally.
 Read " $1 + 2 + 3 + \dots$ " as "one plus two plus three and so on, without end."
 Read " $1 + 2 + 3 + \dots + 10$ " as "one plus two plus three and so on up to ten."

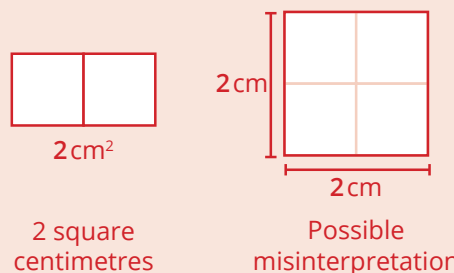
Units of Measurement

Familiarity with units of measurement is assumed, including conversions from one unit to another:

- **Time:** seconds \leftrightarrow minutes \leftrightarrow hours \leftrightarrow days
- **Length:** millimetres \leftrightarrow centimetres \leftrightarrow metres \leftrightarrow kilometres
- **Area:** $\text{mm}^2 \leftrightarrow \text{cm}^2 \leftrightarrow \text{m}^2 \leftrightarrow \text{km}^2$
- **Volume/Capacity:** $\text{mm}^3 \leftrightarrow \text{cm}^3 \leftrightarrow \text{m}^3$; millilitres \leftrightarrow litres
- **Mass:** grams \leftrightarrow kilograms
- **Angles:** degrees ($^\circ$)

Units of measurement must be correct if given in an answer.

To avoid confusion, read cm^2 as "square centimetres", not "centimetres squared".



Presenting Answers

Unless otherwise specified in a problem, **equivalent numbers or expressions are acceptable.**

- For example, $3\frac{1}{2}$, $\frac{7}{2}$, and 3.5 are equivalent. $3\frac{2}{4}$ and $\frac{70}{20}$ are not in lowest terms and will not be accepted.

After reading a problem, it is useful to indicate the nature of the answer, before commencing the solution strategy. For example:

- " $A = \underline{\quad}, B = \underline{\quad}.$ "
- "The largest number is $\underline{\quad}.$ "
- "The [sum | difference | product | quotient] is $\underline{\quad}.$ "
- "The probability, as a [fraction | decimal | percentage], is $\underline{\quad}.$ "
- "The perimeter is $\underline{\quad}$ centimetres."
- "The area is $\underline{\quad}$ square units."
- "The average speed is $\underline{\quad}$ kilometres per hour."

Digits and Integers

A **digit** is any one of the ten numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

- 358 is a three-digit number.

The **lead digit** (leftmost digit) of a number is not counted as a digit if it is 0.

- 0358 is a three-digit number.

Terminal zeroes of a number are the zeroes to the right of the last nonzero digit.

- 30 500 has two terminal zeroes.

Whole numbers: { 0, 1, 2, 3, ... }.

Counting numbers, or Positive Integers: { 1, 2, 3, ... }.

Integers: { ..., -2, -1, 0, 1, 2, 3, ... }.

- Positive numbers, negative numbers, non-negative numbers, and non-positive numbers are terms that may appear in Division 5 problems.

Consecutive Numbers are counting numbers that differ by 1.

- 83, 84, 85, 86, 87.

Consecutive Even Numbers are multiples of 2 that differ by 2.

- 36, 38, 40, 42.

Consecutive Odd Numbers are non-multiples of 2 that differ by 2.

- 57, 59, 61, 63.

Factors and Divisibility

Suppose $A = B \times C$, and A , B , and C are all **counting numbers** (1, 2, 3, ...).

- $6 = 2 \times 3$.

Then, A is divisible by B , and A is a multiple of B .

- 6 is divisible by 2.
- 6 is a multiple of 2

Likewise, A is divisible by C , and A is a multiple of C .

- 6 is divisible by 3.
- 6 is a multiple of 3

Both B and C are **factors** of A .

- 2 and 3 are factors of 6.

A **prime number** is a counting number with exactly two factors, 1 and itself.

- 2, 3, 5, 7, 11, 13, ...

A **composite number** is a counting number which has at least three different factors.

- 4, 6, 8, 9, 10, 12, ...

The number 1 is neither prime nor composite since it has exactly one factor.

A number is **factored completely** when it is expressed as a product of only prime numbers.

- $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
 $= 2^4 \times 3^2$.

The **Highest Common Factor (HCF)** of two counting numbers is the largest counting number that divides each of the two numbers, and the remainder is zero.

If the **HCF** of two numbers is 1, then we say that the numbers are **relatively prime**.

- $\text{HCF}(12, 18) = 6$.

The **Lowest Common Multiple (LCM)** of two counting numbers is the smallest number that each of the given numbers divides, and the remainder is zero.

- $\text{LCM}(12, 18) = 36$.

Fractions

For **common** or **simple fractions** $\frac{a}{b}$,

- a (the **numerator**) and b (the **denominator**) are both integers, and
- $b \neq 0$.

In a **unit fraction**, the numerator is 1.

- $\frac{1}{2}$ and $\frac{1}{100}$ are both unit fractions.

In a **proper fraction**, $a < b$.

- $\frac{1}{2}$ and $\frac{5}{6}$ are both proper fractions.

In an **improper fraction**, $a \geq b$.

- $\frac{3}{2}$ and $\frac{11}{8}$ are both improper fractions.

A **complex fraction** is a fraction whose numerator or denominator contains a fraction.

- $\frac{\frac{2}{3}}{5}$, $\frac{2}{\frac{3}{5}}$, $\frac{\frac{2}{3}}{\frac{5}{7}}$, $\frac{2 + \frac{3}{5}}{5 - \frac{1}{2}}$ are complex fractions.

The fraction $\frac{a}{b}$ is **simplified** (in **lowest terms**) if a and b have no common factor other than 1 - i.e. $\text{HCF}(a,b) = 1$.

- $\frac{1}{2}$ and $\frac{3}{2}$ are both expressed in lowest terms. $\frac{2}{4}$ and $\frac{30}{20}$ are not in lowest terms.
- Unless otherwise specified, fraction answers to Olympiad problems must be expressed in lowest terms.

A **decimal** or **decimal fraction** is a fraction whose denominator is a power of ten.

The decimal is written using decimal point notation.

- $0.07 = \frac{7}{100}$, $0.153 = \frac{153}{1000}$, $6.4 = 6\frac{4}{10}$ or $\frac{64}{10}$.

A **recurring decimal**, or **repeating decimal**, is a decimal fraction with a digit, or group of digits, that repeats forever.

- $\frac{1}{3} = 0.333\dots = 0.\dot{3} = 0.\bar{3}$
- $\frac{1}{6} = 0.1666\dots = 0.1\dot{6} = 0.1\bar{6}$
- $\frac{1}{7} = 0.142857142857\dots = 0.\dot{1}4285\dot{7} = 0.\overline{142857}$

A **percentage** is a fraction whose denominator is 100. The **percent sign** represents the division by 100.

- $9\% = \frac{9}{100}$, $125\% = \frac{125}{100}$, $0.3\% = \frac{0.3}{100}$ or $\frac{3}{1000}$.

Order of Operations

When an expression has more than one arithmetic symbol, certain operations occur before others.

There are a few ways to remember the order of operations, and mnemonics are often used (e.g. **BIDMAS**; **PEMDAS**).

However, it can also be useful to consider the intent when an arithmetic expression is constructed.

By convention, we observe the following priorities:

1. Perform operations in **parentheses**, **braces**, or **brackets**. The **vinculum** (line in a fraction) is also considered as a grouping symbol, similar to parentheses.
2. Evaluate **exponents** (**indices**).
3. Evaluate **multiplication** and **division**, from left to right.
4. Evaluate **addition** and **subtraction**, from left to right.

Example 1

$$\begin{aligned} & 30 + 6 \div 2 - 5 \times (9 - 7) \\ &= 30 + 6 \div 2 - 5 \times 2 \\ &= 30 + 3 - 10 \\ &= 23 \end{aligned}$$

Example 2

$$\begin{aligned} & 20 - (8 + (1 + 2)^2) \\ &= 20 - (8 + 3^2) \\ &= 20 - (8 + 9) \\ &= 20 - 17 \\ &= 3 \end{aligned}$$



Two-Dimensional Figures

<p>Acute angle between 0° and 90°</p>	<p>Right angle 90°</p>	<p>Obtuse angle between 90° and 180°</p>	<p>Straight angle 180°</p>	<p>Reflex angle between 180° and 360°</p>
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<p>Scalene triangle</p>	<p>Isosceles triangle</p>	<p>Right-angled triangle</p>	<p>Equilateral triangle</p>
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<p>Parallelogram</p>	<p>Rectangle</p>
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<p>Rhombus</p>	<p>Square</p>
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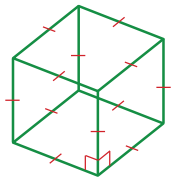
<p>Pentagon</p>	<p>Hexagon</p>	<p>Octagon</p>
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<p>Decagon</p>	<p>Dodecagon</p>
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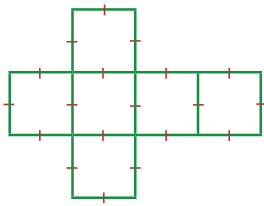


Three-Dimensional Objects

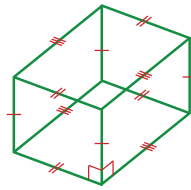
Cube



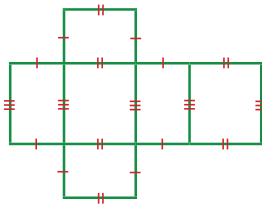
One possible net of a cube



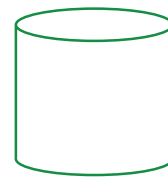
Rectangular Prism



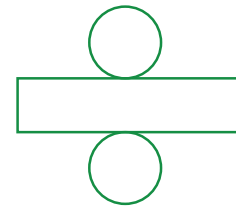
One possible net of a rectangular prism



Right Cylinder



One possible net of a right cylinder



Congruence and Similarity

Two geometric figures are **congruent** if they are identical.

- Congruent triangles coincide exactly when one is superimposed upon the other.
- Congruent plane figures have corresponding pairs of sides that are equal, and corresponding pairs of angles that are the same.

Two geometric figures are **similar** if their shape is the same, even though their size may be different.

- All squares are similar, and all circles are similar.

Classification of Geometric Figures

All equilateral triangles are isosceles, but only some isosceles triangles are equilateral.

A square is a rectangle with all sides congruent.

A square is also a rhombus with all angles congruent.

Within the USA/Canada, a trapezium is an irregular quadrilateral.

Outside the USA/Canada, a trapezium is a quadrilateral with at least one pair of parallel sides (known as a "trapezoid" within the USA/Canada).

Calendar Conventions

There was no year 0. The first century spanned the years 1 to 100 inclusive.

- The **20th century** spanned the years 1901 to 2000 inclusive.
- The **21st century** spans the years 2001 to 2100 inclusive.

Measures of Centre

The **mean**, **arithmetic mean**, or **average**, of a set of values is

- the sum of the values, divided by
 - the number of values.
- For the set $\{5, 5, 7, 11\}$, the mean is $\frac{5+5+7+11}{4} = 28 \div 4 = 7$.
 - For the set $\{7, 11, 23, 5, 5\}$, the mean is $\frac{7+11+23+5+5}{5} = 51 \div 5 = 10\frac{1}{5}$.

The **median** is the value that is exactly in the middle of the set when it is ordered.

If there are an even number of values, then the median is the mean of the two middle values.

- For the set $\{5, 5, 7, 11\}$, the median is $(5 + 7) \div 2 = 6$.
- For the set $\{7, 11, 23, 5, 5\}$, we begin by ordering the set of values: $\{5, 5, 7, 11, 23\}$.
The median is the middle value, 7.

The **mode** is the value that occurs the greatest number of times.

- For the set $\{5, 5, 7, 11\}$, the mode is 5.

A set with every value listed an equal number of times is said to have no mode.

- For the set $\{5, 5, 7, 7, 8, 8\}$, there is no mode.

Probability

The probability of an event is a value that expresses how likely an event is to occur.

- If the event is impossible, then the probability is 0.
- If the event is certain, then the probability is 1.
- All probabilities are between 0 and 1 inclusive.

The probability is found by dividing the number of times an event does occur, by the total number of times the event can possibly occur.

- The probability of rolling an odd number on a die is $\frac{3}{6}$ or $\frac{1}{2}$.