APSMO

2025 OLYMPIADS

IMPORTANT

The information contained in this file is ONLY for the use of registered participants of the 2025 Australasian Problem Solving Mathematical Olympiads.

APSMO
2025 OLYMPIADS

Organisation and Procedures

For full details, see the Members' Area

To ensure the integrity of the competition, the Olympiads must be administered under examination conditions.

DO	DO NOT
• Supervise students at all times • Seat students apart • Maintain silence • Provide blank working paper • Give time warnings when 3 minutes remain, and again when 1 minute remains • Collect, mark and retain the papers	• Print the Olympiad papers prior to the Olympiad Date • Read the questions aloud to the students • Interpret the questions for students • Permit any discussion or movement around the room • Permit the use of calculators or other electronic devices

- Olympiad papers are scored by the PICO using the *Solutions and Answers* sheet provided.
- Results should be submitted in the Members' Area within 7 days of the Olympiad.
- Original student answer sheets should be retained by the PICO until the end of the year.
- *Solutions and Answers sheets* are not to be handed out to students. They are a teaching resource for use in class ***after*** completion of the Olympiad paper.

Timing of the Olympiad

- The *Total Time Allowed* for the Olympiad is **30 minutes**.

Absent Student Policy

A student who is legitimately absent on the Olympiad date, may sit the Olympiad under examination conditions on their first day back at school (if that date is within 2 weeks of the original Olympiad date). If these conditions cannot be met, the student must be marked as absent on the submitted results.

The Absent Student Policy is available in the **Contest Administration** section of the Members' Area.

APSMO

2025 : DIVISION S

WEDNESDAY 10 SEPTEMBER 2025

OLYMPIAD 4

*Total Time Allowed: **30 Minutes.** Calculators NOT Permitted.*

Write your answers in the boxes on the back.

Keep your answers hidden by folding backwards on this line.

4A. The sum of the factors of 54 equals $12 \times N$.

What is the value of the whole number N?

4B. Four students each write their own name on separate slips of paper.

The four slips are placed in a bag.

Then, without looking, the four students each take one slip of paper from the bag.

What is the probability, expressed as a fraction in simplest form, that each student draws his or her own name?

4C. The addition problem shown has an unknown sum.

Each of A, B, C, and D represent different non-zero digits.

How many different sums are possible?

$$\begin{array}{r} A\ B\ C\ D \\ B\ C\ D\ A \\ C\ D\ A\ B \\ +\ D\ A\ B\ C \\ \hline \\ \hline \end{array}$$

4D. x and y are both integers.

$x^2 + y^3 = 0$.

If $0 < x < 100$, what is the greatest possible value for x?

4E. Palindromes are read the same backwards as they are read forwards.

For example, '*MOM*' and the number '12321' are palindromes.

How many 5-digit palindromes are multiples of 15?

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OLYMPIAD 4

4A.

Student Name:

4B.

Fold here. Keep your answers hidden.

4C.

4D.

4E.

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OLYMPIAD 4

MATHS OLYMPIAD

Solutions and Answers

(Items in parentheses are not required)

For teacher use only. Not for Distribution.

4A: 10	**4B:** ¹⁄₂₄	**4C:** 21	**4D:** 64	**4E:** 33

4A. The sum of the factors of 54 equals 12 × *N*. What is the value of the whole number *N*?

Strategy: *Make an organised list.*

We begin by listing the factors of **54**.

There are a number of different methods for doing this.

Method 1: Test all integers that are less than the square root of 54.

We begin by recognising that **7 × 7 < 54**, and **8 × 8 > 54**.

Therefore, every pair of factors that multiply to give **54** must include one factor x that is less than **8**.

Every such factor x can be paired with another factor that has the value **54 ÷ x**.

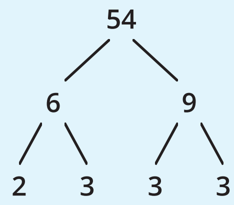
x	1	2	3	~~4~~	~~5~~	6	~~7~~
54 ÷ x	54	27	18	~~13 r. 2~~	~~10 r. 4~~	9	~~7 r. 5~~

The factors of **54** are **1, 2, 3, 6, 9, 18, 27** and **54**.

Method 2: Find the prime factors of 54.

We can use a factor tree to find that the prime factors of **54** are **2 × 3 × 3 × 3**.

Since there are just two distinct prime factors, we can use a 2-way table to identify all of the factors of **54**.



×	3^0	3^1	3^2	3^3
2^0	$2^0 \times 3^0$ **= 1**	$2^0 \times 3^1$ **= 3**	$2^0 \times 3^2$ **= 9**	$2^0 \times 3^3$ **= 27**
2^1	$2^1 \times 3^0$ **= 2**	$2^1 \times 3^1$ **= 6**	$2^1 \times 3^2$ **= 18**	$2^1 \times 3^3$ **= 54**

The factors of **54** are **1, 2, 3, 6, 9, 18, 27** and **54**.

Method 3: Use prior knowledge about number properties.

Recognising that **54 = 6 × 9**, we can construct a 2-way table listing factors of **6** and **9**.

After removing duplicates, we find that the factors of **54** are **1, 2, 3, 6, 9, 18, 27** and **54**.

×	1	2	3	6
1	1 × 1 **= 1**	1 × 2 **= 2**	1 × 3 **= 3**	1 × 6 **= 6**
3	3 × 1 **= 3**	3 × 2 **= 6**	3 × 3 **= 9**	3 × 6 **= 18**
9	9 × 1 **= 9**	9 × 2 **= 18**	9 × 3 **= 27**	9 × 6 **= 54**

The sum of the factors of **54** is **1 + 2 + 3 + 6 + 9 + 18 + 27 + 54 = 120**.

Since **120 = 12 × 10**, the value of the whole number ***N*** is **10**.

Follow-Up: *The sum of the factors of 5^4 equals 11 × N. What is the value of the whole number N?* [71]

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OLYMPIAD 4

4B. What is the probability that each student draws his or her own name?

In the following solution methods, we shall call the four students ***A***, ***B***, ***C*** and ***D***.

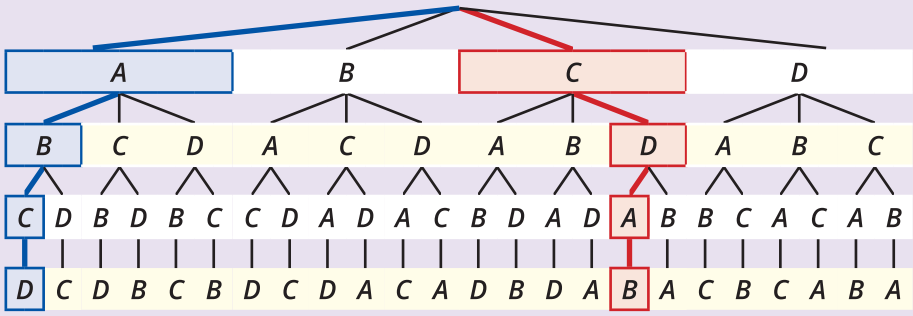
Strategy 1: *Draw a diagram.*

The top horizontal row in the diagram represents the name that might be drawn by ***A***.

After ***A*** draws a name, there will be **3** names left in the bag.

After ***B*** draws a name, there will be **2** names left.

After ***C*** draws a name, there will only be **1** name left for ***D*** to choose.



The path marked by the red lines and boxes represents a circumstance where ***A*** selects the slip of paper with ***C***'s name on it, ***B*** selects ***D***, ***C*** selects ***A***, and ***D*** selects ***B***.

With **24** results in the last row, we can see that there are **24** different ways that the students might choose the slips of paper.

Only one of those ways sees each of the students draw their own name. This path is marked in the above diagram by blue lines and boxes.

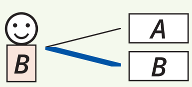
The probability that each student draws his or her own name is $\mathbf{\frac{1}{24}}$.

Strategy 2: *Solve a simpler related problem.*

Scenario 1:
2 students, ***B*** then ***A***.

With **2** slips of paper in the bag, the probability that ***B*** selects their own name is $\frac{1}{2}$.

Since ***A*** has no choice but to take the last slip, the probability that both students draw their own name is $\frac{1}{2}$.

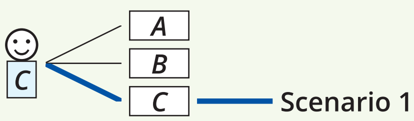


Scenario 2:
3 students, ***C***, ***B***, then ***A***.

With **3** slips of paper, the probability that ***C*** selects their own name is $\frac{1}{3}$.

The situation for ***B*** and ***A*** is now the same as **Scenario 1**, which has a $\frac{1}{2}$ probability of success.

The probability that ***C***, ***B*** and ***A*** all draw their own name is $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

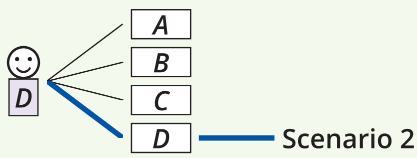


Scenario 3:
4 students, ***D***, ***C***, ***B***, then ***A***.

With **4** slips of paper, the probability that ***D*** selects their own name is $\frac{1}{4}$.

The situation for ***C***, ***B*** and ***A*** is now the same as **Scenario 2**, which has a $\frac{1}{6}$ probability of success.

The probability that ***D***, ***C***, ***B*** and ***A*** all draw their own name is $\frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$.



Follow-Up: *There are 5 slips of paper put into a bag, each with a different number from the set {2, 3, 5, 7, 11} written on it. Four students each pull out a slip and do not replace it. What is the probability that the product of their slips' numbers is not a multiple of 10?* [0.4]

4C. How many different sums are possible?

	A	*B*	*C*	*D*
	B	*C*	*D*	*A*
	C	*D*	*A*	*B*
+	*D*	*A*	*B*	*C*

Strategy 1: *Convert to a more convenient form.*

The 4-digit number ***ABCD*** represents a value that is equal to:

$A \times 1000 + B \times 100 + C \times 10 + D$.

In this problem, we are adding:

$A \times 1000 + B \times 100 + C \times 10 + D$

$+ B \times 1000 + C \times 100 + D \times 10 + A$.

$+ C \times 1000 + D \times 100 + A \times 10 + B$.

$+ D \times 1000 + A \times 100 + B \times 10 + C$.

Addition is commutative - the order in which we add the values does not change the final result.

This means that we would get the same result by adding:

$A \times 1000 + A \times 100 + A \times 10 + A$

$+ B \times 1000 + B \times 100 + B \times 10 + B$

$+ C \times 1000 + C \times 100 + C \times 10 + C$

$+ D \times 1000 + D \times 100 + D \times 10 + D$.

Using an area diagram to represent this value, we can see that the result is equal to

$(A + B + C + D) \times (1000 + 100 + 10 + 1)$

$= (A + B + C + D) \times 1111$.

	1000	100	10	1
A	*A* × 1000	*A* × 100	*A*×10	*A*
B	*B* × 1000	*B* × 100	*B*×10	*B*
C	*C* × 1000	*C* × 100	*C*×10	*C*
D	*D* × 1000	*D* × 100	*D*×10	*D*

Since *A*, *B*, *C* and *D* are all different non-zero digits:

- The smallest possible value for $A + B + C + D$ is $1 + 2 + 3 + 4 = 10$.
- The greatest possible value for $A + B + C + D$ is $6 + 7 + 8 + 9 = 30$.

There are **30** values from **1** to **30**.

There are **9** values from **1** to **9**.

Therefore, from **10** to **30** inclusive there are **30 – 9 = 21** different values.

There are **21** different sums that can result from this addition.

Strategy 2: *Solve a simpler related problem.*

If we lift the restriction that *A*, *B*, *C* and *D* all represent different non-zero digits, then it would be possible for $K = A + B + C + D$ to be a one-digit number.

If so, the result of the sum would be the four-digit number ***KKKK***.

Regardless of whether *K* is a one-digit or a two-digit number, we can express the value of the sum as $1000K + 100K + 10K + K = 1111 \times K$.

	A	*B*	*C*	*D*
	B	*C*	*D*	*A*
	C	*D*	*A*	*B*
+	*D*	*A*	*B*	*C*
	K	*K*	*K*	*K*

As noted in ***Strategy 1***, there are **21** different possible values for $K = A + B + C + D$.

The sums that can result are the different values for $1111K$: 1111×10, 1111×11, ..., 1111×30.

There are **21** different possible sums.

Follow-Up: *What is the average of all of the possible sums from the original question?* [22220]

4D. x and y are both integers. $x^2 + y^3 = 0$. If $0 < x < 100$, what is the greatest possible value for x?

Strategy: *Solve a simpler related problem.*

If $x^2 + y^3 = 0$,

then $x^2 = -y^3$

and so $x \times x = -y \times -y \times -y$.

Since x^2 must be positive, y must be negative.

To make this easier to think about, let's say that $p = -y$.

We now have: $x \times x = p \times p \times p$, where x and p are positive integers.

Method 1: *Build a table, and find a pattern.*

We begin by listing square numbers and cube numbers, with an aim of finding a value that is both a square and a cube.

	Square	Cube	$x \times x = p \times p \times p$
1	1	1	1 × 1 = 1 × 1 × 1 = 1
2	4	8	
3	9	27	
4	16	64	8 × 8 = 4 × 4 × 4 = 64
5	25	125	
6	36	216	
7	49	343	
8	64	256	

We have found **2** possible values for x: 1, and 8.

Since 1 and 8 are the first two cube numbers, we can reason that this might be significant for potential values for x.

In particular, we note that:

(1 × 1 × 1) × (1 × 1 × 1) = (1 × 1) × (1 × 1) × (1 × 1)

(2 × 2 × 2) × (2 × 2 × 2) = (2 × 2) × (2 × 2) × (2 × 2)

Another potential solution would be:

(3 × 3 × 3) × (3 × 3 × 3) = (3 × 3) × (3 × 3) × (3 × 3)

Here, x = 3 × 3 × 3 = 27, and p = 3 × 3 = 9.

Yet another potential solution would be:

(4 × 4 × 4) × (4 × 4 × 4) = (4 × 4) × (4 × 4) × (4 × 4)

Here, x = 4 × 4 × 4 = 64, and p = 4 × 4 = 16.

While x = 5 × 5 × 5 = 125 is a valid solution, we want the greatest value for x such that **0** < x < **100.**

The greatest possible value for x is **4 × 4 × 4 = 64.**

Method 2: *Reason algebraically.*

We need to find positive integer values for x and p so that $x \times x = p \times p \times p$ is a true statement.

$$x \times x = p \times p \times p$$
$$= p \times \sqrt{p} \times p \times \sqrt{p}$$
$$x = p \times \sqrt{p}$$
$$= \sqrt{p} \times \sqrt{p} \times \sqrt{p}.$$

x must be a cube number.

Listing cube numbers between **0** and **100**, we have:

	1	2	3	4	5
Cube	1	8	27	64	125 (too big)

The greatest possible value for x is **64.**

Follow-Up: *x and y are both integers. $x^2 + y^5 = 0$. If $0 < x < 100$, what is the greatest possible value for x?* [32]

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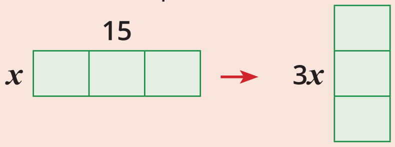
MATHS OLYMPIAD

OLYMPIAD 4

4E. How many 5-digit palindromes are multiples of 15?

Strategy: *Eliminate all but one possibility, and make an organised list.*

We begin by noting that multiples of **15** must also be multiples of **5**.



Since **2 × 5 = 10**, a multiple of **5** must be either:

- a multiple of **10**, or
- a multiple of **10**, plus **5**.

A multiple of **10** will have **0** in the ones place.

A number with **0** in the ones place that is a 5-digit palindrome would look like this:

0*XYX*0

where *X* and *Y* represent digits.

This is, at most, a 4-digit number.

There are no palindromes with **0** in the ones place.

A multiple of **10**, plus **5**, will have **5** in the ones place.

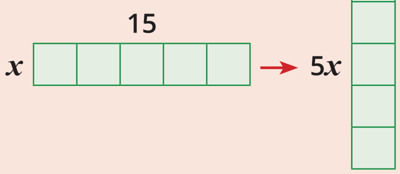
A number with **5** in the ones place that is a 5-digit palindrome would look like this:

5*XYX*5

where *X* and *Y* represent digits.

We want to find the number of multiples of **15** that are arranged in this configuration.

Multiples of **15** must also be multiples of **3**.



The 5-digit number ***ABCDE*** represents the value

$$10000A + 1000B + 100C + 10D + E$$

$$= 9999A + 999B + 99C + 9D + A + B + C + D + E$$

$$= 3(3333A + 333B + 33C + 3D) + A + B + C + D + E.$$

$3(3333A + 333B + 33C + 3D)$ is a multiple of **3**, so if $A + B + C + D + E$ is a multiple of **3**, then ***ABCDE*** would also be a multiple of **3**.

We need to find numbers configured **5*XYX*5**, where the digit sum is a multiple of **3**.

Method 1: *List palindromes with different values for X.*

We can use a table to list values for **50*Y*05**, **51*Y*15**, and so on, where the digit sum is a multiple of **3**.

50*Y*05	51*Y*15	52*Y*25	53*Y*35	54*Y*45	55*Y*55	56*Y*65	57*Y*75	58*Y*85	59*Y*95
50205	51015	52125	53235	54045	55155	56265	57075	58185	59295
50505	51315	52425	53535	54345	55455	56565	57375	58485	59595
50805	51615	52725	53835	54645	55755	56865	57675	58785	59895
	51915			54945			57975		

There are **33** 5-digit palindromes that are multiples of **15**.

Method 2: *List palindromes with digit sums that are different multiples of 3.*

We can use a table to list values configured **5*XYX*5** where the digit sum is **12**, **15, 18** and so on.

There are **33** such values.

12	15	18	21	24	27	30	33	36
50205	50505	50805	51915	53835	54945	56865	57975	59895
51015	51315	51615	52725	54645	55755	57675	58785	
	52125	52425	53535	55455	56565	58485	59595	
		53235	54345	56265	57375	59295		
		54045	55155	57075	58185			

Follow-Up: *How many palindromic multiples of 15 have 4 digits?* [3] *6 digits?* [33] *7 digits?* [333]
Is there a pattern? [Yes; the count is always represented by a string of 3s]