APSMO
2025 OLYMPIADS

IMPORTANT

APSMO

2025 OLYMPIADS

MATHS OLYMPIAD

Organisation and Procedures

For full details, see the Members' Area

To ensure the integrity of the competition, the Olympiads must be administered under examination conditions.

DO	DO NOT
• Supervise students at all times • Seat students apart • Maintain silence • Provide blank working paper • Give time warnings when 3 minutes remain, and again when 1 minute remains • Collect, mark and retain the papers	• Print the Olympiad papers prior to the Olympiad Date • Read the questions aloud to the students • Interpret the questions for students • Permit any discussion or movement around the room • Permit the use of calculators or other electronic devices

- Olympiad papers are scored by the PICO using the *Solutions and Answers* sheet provided.
- Results should be submitted in the Members' Area within 7 days of the Olympiad.
- Original student answer sheets should be retained by the PICO until the end of the year.
- *Solutions and Answers sheets* are not to be handed out to students. They are a teaching resource for use in class ***after*** completion of the Olympiad paper.

Timing of the Olympiad

- The *Total Time Allowed* for the Olympiad is **30 minutes**.

Absent Student Policy

A student who is legitimately absent on the Olympiad date, may sit the Olympiad under examination conditions on their first day back at school (if that date is within 2 weeks of the original Olympiad date). If these conditions cannot be met, the student must be marked as absent on the submitted results.

The Absent Student Policy is available in the **Contest Administration** section of the Members' Area.

APSMO

2025 : DIVISION S

WEDNESDAY 30 JULY 2025

OLYMPIAD 3

Total Time Allowed: ***30 Minutes.*** *Calculators NOT Permitted.*

Write your answers in the boxes on the back.

← ***Keep your answers hidden by folding backwards on this line.***

3A. What is the least positive integer n, such that 50% of 40% of n is also a positive integer?

3B. $10^a = \underbrace{10 \times 10 \times \ldots \times 10}_{a \text{ times}}$.

m and n are both positive integers.

The value of the expression $(1.5 \times 10^m) \div (1.35 \times 10^n)$ is between 100 and 1000.

What is the value of $m - n$?

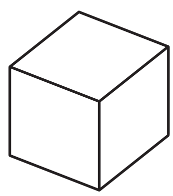
3C. How many distinct fractions $\frac{a}{b}$ can be formed where:

- Both the numerator and denominator are chosen from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, and
- $\frac{1}{2} < \frac{a}{b} < 1$?

Do not count equivalent fractions such as $\frac{2}{3}$ and $\frac{4}{6}$ as distinct. Consider just one of them.

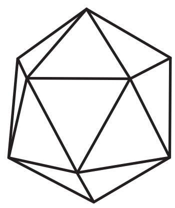
3D. A regular hexahedron (or cube) is a polyhedron which has 6 faces, 8 vertices, and 12 edges.

In the diagram, not all of the faces, vertices, and edges of the cube are visible.



A regular icosahedron is a polyhedron which has 20 faces, v vertices, and e edges.

Find the whole number value $v + e$.



3E. In how many ways can the letters in the word *FUZZY* be rearranged into a different string of letters, so that the two *Z*s are <u>**not**</u> next to one another?

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MATHS OLYMPIAD

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OLYMPIAD

3

3A.

Student Name:

3B.

Fold here. Keep your answers hidden.

3C.

3D.

3E.

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OLYMPIAD 3

Solutions and Answers

(Items in parentheses are not required)

For teacher use only. Not for Distribution.

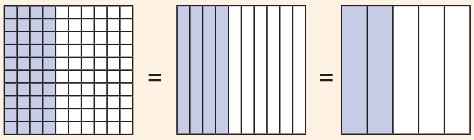
3A: 5	**3B:** 2	**3C:** 15	**3D:** 42	**3E:** 36

3A. What is the least positive integer n, such that 50% of 40% of n is also a positive integer?

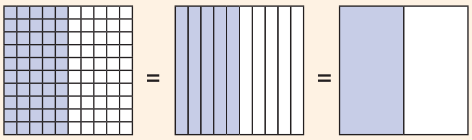
Strategy 1: *Convert to a more convenient form.*

We can begin by converting **40%** and **50%** into fractions.

$40\% = \frac{40}{100} = \frac{4}{10} = \frac{2}{5}.$



$50\% = \frac{50}{100} = \frac{5}{10} = \frac{1}{2}.$



Using a bar to represent the value of n:

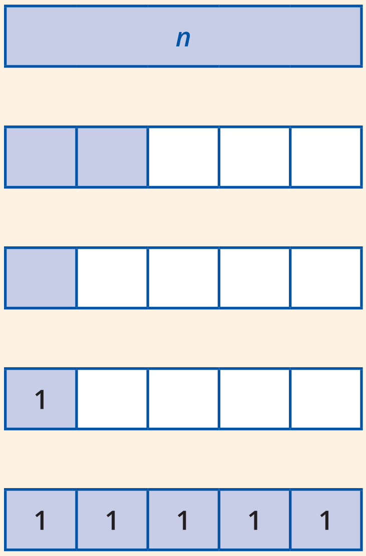
40% of n is equal to $\frac{2}{5}$ of n.

We want to find **50%**, or $\frac{1}{2}$, of **40%** of n.

This value needs to be a positive integer.

The least positive integer is **1**.

Therefore, the least positive integer n is **5**.



Strategy 2: *Reason algebraically.*

Since the least positive integer is **1**, we can form the equation: $50\% \times 40\% \times n = 1$

Recognising that **50%** of **40%** is **20%**: $20\% \times n = 1$

Method 1: 20% is equivalent to $\frac{1}{5}$.

$\frac{1}{5} \times \frac{n}{1} = 1$

$\frac{n}{5} = 1$

Multiplying both sides by **5**: $n = 5$

Method 2: 20% is equivalent to $\frac{20}{100}$.

$\frac{20n}{100} = 1$

Multiplying both sides by **100**: $20n = 100$

Dividing both sides by **20**: $n = 5$

Let's check: **40%** of **5** is **2**, and **50%** of **2** is **1**.

Since **1** is a positive integer, we can see that **5** is the least positive integer that satisfies this condition.

Follow-Up: *What whole number is 50% of 40% of 2025?* [405]

3B. The value of the expression $(1.5 \times 10^m) \div (1.35 \times 10^n)$ is between 100 and 1000. What is the value of $m - n$?

Strategy 1: *Guess, check, and refine.*

Suppose $m = 4$. The value of (1.5×10^m) would then be

$$1.5 \times \underbrace{10 \times 10 \times 10 \times 10}_{4 \text{ times}}.$$

Since we have a decimal place value system, multiplying by **10** would shift a number up one place value.

Multiplying by **10** four times causes 1.5 to be shifted up 4 place values.

$1.5 \times 10 \times 10 \times 10 \times 10 = 15000$.

10000s	1000s	100s	10s	1s	10ths	100ths
				1 .	5	
			1	5 .		
		1	5	0 .		
	1	5	0	0 .		
1	5	0	0	0 .		

We want a value for n such that $15000 \div (1.35 \times 10^n)$ is between **100** and **1000**.

- $15000 \div 10 = 1500$, which is too big.
- $15000 \div 10^2 = 150$, which is in the correct range.
- $15000 \div 10^3 = 15$, which is too small.

Let's guess that $n = 2$.

The value of the expression would then be $15000 \div 135$.

To get a sense of the value $15000 \div 135$, we can see that:

- $135 \times 100 = 13500$, and $13500 < 15000$.
- $135 \times 1000 = 135000$, and $135000 > 15000$.

The value $15000 \div 135 = (1.5 \times 10^4) \div (1.35 \times 10^2)$ must therefore be between **100** and **1000**.

Since the expression is in the correct range when $m = 4$ and $n = 2$, the value of $m - n$ is $4 - 2 = $ **2**.

Strategy 2: *Solve a simpler related problem.*

This problem may be more readily solved by making use of an equivalent number sentence.

The value of $m - n$ is **2**.

We know that	$(1.5 \times 10^m) \div (1.35 \times 10^n) > 100$.
Multiplying both sides by 1.35×10^n:	$1.5 \times 10^m > 1.35 \times 10^n \times 10 \times 10$
	$1.5 \times 10^m > 1.35 \times 10^{n+2}$.
Since $1.5 > 1.35$, this inequality is true if	$10^m = 10^{n+2}$.
Therefore,	$m = n + 2$.
Subtracting n from both sides:	$m - n = 2$.

Strategy 3: *Work with an estimated value.*

Recognising that

$$1 < \frac{1.5}{1.35} < 2,$$

Let's say that

$$\frac{1.5}{1.35} = 1.**.$$

We now have:

$$\frac{1.5 \times 10^m}{1.35 \times 10^n} = 1.** \times \frac{10^m}{10^n}.$$

Since we want the expression to have a value between **100** and **1000**, $10^m \div 10^n$ must equal **100**.

$$10^m \div 10^n = 100$$

$$10^m = 10^2 \times 10^n$$

$$\underbrace{10 \times \ldots \times 10}_{m \text{ times}} = 10 \times 10 \times \underbrace{10 \times \ldots \times 10}_{n \text{ times}}$$

Therefore, we see that $m - n = $ **2**.

Follow-Up: *Given that $(1.5 \times 10^m) \times (1.35 \times 10^n) = 2025$, what is the value of $m + n$?* [3]

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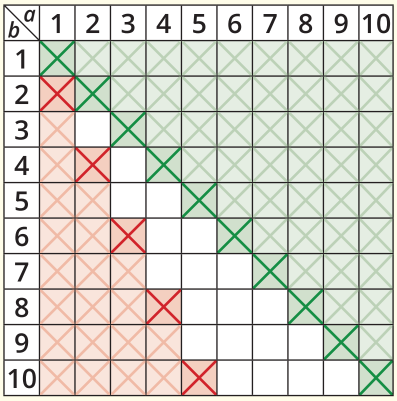
3C. How many distinct fractions $\frac{a}{b}$ can be formed where a and b are between 1 and 10, and $\frac{1}{2} < \frac{a}{b} < 1$?

Strategy 1: *Build a table.*

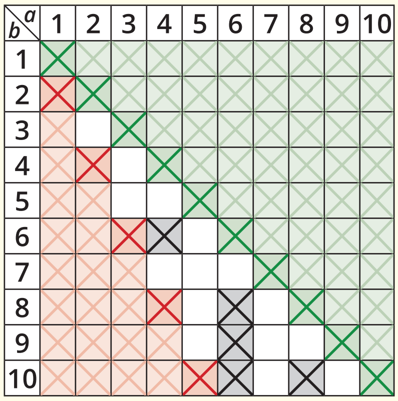
We can draw a table to visualise all of the fractions $\frac{a}{b}$ with a and b chosen from **1** to **10**.

We begin by eliminating fractions which are:

- Greater than or equal to 1, or
- Less than or equal to $\frac{1}{2}$.



Next, to identify distinct fractions, we would remove equivalent fractions where the numerator and denominator share a common factor.



There are **15** distinct fractions $\frac{a}{b}$ that are:

- between $\frac{1}{2}$ and **1**, where
- both a and b are between **1** and **10**.

Strategy 2: *Make an organised list.*

For each value for ***b***, we can begin with the value of ***a*** that would result in a fraction that is equivalent to $\frac{1}{2}$.

We then continually increase the numerator ***a*** by **1**, until the fraction is equivalent to **1**.

Finally, we would identify fractions that are expressed in lowest terms.

There are **15** distinct fractions that fit the requirements.

b	Fractions
1	1/1
2	1/2, 2/2
3	**2/3**, 3/3
4	2/4, **3/4**, 4/4
5	**3/5**, **4/5**, 5/5
6	3/6, 4/6, **5/6**, 6/6
7	**4/7**, **5/7**, **6/7**, 7/7
8	4/8, **5/8**, 6/8, **7/8**, 8/8
9	**5/9**, 6/9, **7/9**, **8/9**, 9/9
10	5/10, 6/10, **7/10**, 8/10, **9/10**, 10/10

Alternatively, for each value for ***a***, we can consider possible values for ***b*** that result in fractions between $\frac{1}{2}$ and **1**.

Since we are increasing the denominator, we will be listing fractions that decrease in value, from **1** to $\frac{1}{2}$.

We can then identify the **15** fractions that are expressed in lowest terms.

a	Fractions
1	1/1, 1/2
2	2/2, **2/3**, 2/4
3	3/3, **3/4**, **3/5**, 3/6
4	4/4, **4/5**, 4/6, **4/7**, 4/8
5	5/5, **5/6**, **5/7**, **5/8**, **5/9**, 5/10
6	6/6, **6/7**, 6/8, 6/9, 6/10
7	7/7, **7/8**, **7/9**, **7/10**
8	8/8, **8/9**, 8/10
9	9/9, **9/10**
10	10/10

Follow-Up: *Using all of the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9 exactly once, build two fractions whose sum is 1. Multi-digit numbers will necessarily be used here.* [Answers will vary; an example is 29/58 + 67/134.]

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OLYMPIAD 3

3D. A regular icosahedron is a polyhedron which has 20 faces, *v* vertices, and *e* edges.

Find the whole number value *v* + *e*.

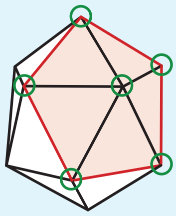
Strategy 1: *Divide a complex shape.*

A regular polyhedron is a three-dimensional shape where:
- All of the faces are identical, regular polygons, and
- The same number of faces meet at each vertex.

A regular icosahedron has five faces meeting at each vertex, as shown in the diagram.

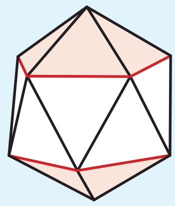
These faces form a pentagonal pyramid.

Each pentagonal pyramid has 6 vertices: 5 on its base, and 1 at the apex.

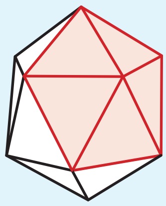


Rotating the icosahedron, we can see that there is a pentagonal pyramid on either side of a band of triangles.

In total, the icosahedron has 2 × 6 = 12 vertices.

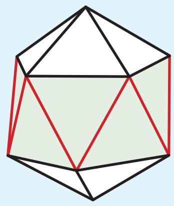


Each pentagonal pyramid has 10 edges:
- 5 edges around the base, and
- 5 between the base and the apex.



With 5 edges around the base of each pentagonal pyramid, there are 5 + 5 = 10 triangles in the band around the middle.

There are 10 edges that join each of these triangles to the next.



In total, the icosahedron has 10 + 10 + 10 = 30 edges.

There are 12 vertices and 30 edges, so the value ***v*** **+** ***e*** = 12 + 30 = 42.

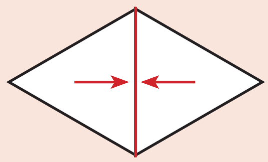
Strategy 2: *Divide a complex shape (Alternative approach).*

Each face of the icosahedron is triangular, and each triangular face has 3 edges.

With **20** faces, there would be **20** × **3** = **60** edges.

When the faces are formed into an icosahedron, each edge is shared between 2 faces.

In total, there are **60** ÷ 2 = 30 edges.

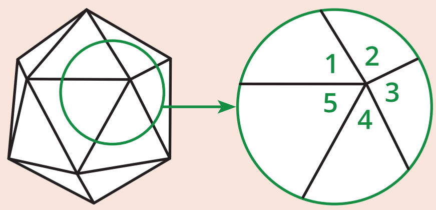


Each triangular face has **3** vertices.

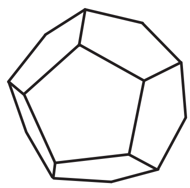
With **20** faces, there would be **20** × **3** = **60** vertices.

When the faces are formed into an icosahedron, each vertex is shared between 5 faces.

In total, there are **60** ÷ 5 = 12 vertices.



With 12 vertices and 30 edges, the value ***v*** **+** ***e*** = 12 + 30 = 42.



Follow-Up: *A regular dodecahedron is a polyhedron which has 12 faces. Find the number of vertices and the number of edges.* [Vertices = 20, Edges = 30]

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OLYMPIAD **3**

3E. In how many ways can the letters in the word *FUZZY* be rearranged into a different string of letters, so that the two *Z*s are <u>not</u> next to one another?

Strategy 1: *Make an organised list.*

We can begin by arranging the letters ***FUY***.

If the first letter is ***F***, then there are two ways to arrange the ***U*** and ***Y***.

We can apply the same reasoning for arrangements starting with ***U*** or ***Y***.

In total, there are **6** ways to arrange ***FUY***.

FUY
FYU
UFY
UYF
YFU
YUF

We now need to place the two *Z*s so that they are not next to each other.

This means that they must be separated by at least one letter.

Using placeholder boxes for the letters ***FUY***, we can see that there are **6** ways to arrange the two *Z*s.

Z □ Z □ □
Z □ □ Z □
Z □ □ □ Z
□ Z □ Z □
□ Z □ □ Z
□ □ Z □ Z

For each arrangement of the letters ***FUY***, there are **6** ways to arrange the two *Z*s.

There are **6** × **6** = **36** ways to arrange ***FUZZY***, so that the two *Z*s are <u>not</u> next to one another.

Strategy 2: *Make an organised list (Alternative approach).*

Suppose we begin by finding the number of ways to rearrange ***FUZZY***, regardless of whether the *Z*s are together or not.

To do this, it would help to think of the two *Z*s as being different letters.

Let one be *Z*, the other *Ƶ*; the letters we are rearranging are ***FUZƵY***.

There are **5** options for the first letter in the string. [5] [ ] [ ] [ ] [ ]

With one letter used in the first position, there are **4** options for the second letter. [5] [4] [ ] [ ] [ ]

Continuing this reasoning, there are **3**, **2**, and **1** option for the remaining three letters. [5] [4] [3] [2] [1]

There are **5** × **4** × **3** × **2** × **1** = **120** ways to arrange the letters ***FUZƵY***.

This list of arrangements actually double-counts every string, since (for example) ***FUZƵY*** and ***FUƵZY*** would be identical after we convert all of the *Ƶ*s back into *Z*s.

Therefore, there are **120** ÷ **2** = **60** ways to arrange the letters ***FUZZY***.

We now need to find how many of these arrangements have the *Z*s together.

To do this, we can pretend to fuse the two *Z*s together, to form the 4-letter string ***FUZZY***.

There are **4** × **3** × **2** × **1** = **24** ways to arrange the characters ***FUZZY***.

This means that there are **24** ways to arrange ***FUZZY***, so that the two *Z*s <u>are</u> together.

[4] [3] [2] [1]

Therefore, there are **60** – **24** = **36** ways to arrange ***FUZZY***, so that the two *Z*s are <u>not</u> together.

FOLLOW-UP: *In how many ways can the letters in the word PIZZAZZ be arranged so that no two Zs are next to each other?* [6]