APSMO

2025 OLYMPIADS

Important

The information contained in this file is ONLY for the use of registered participants of the 2025 Australasian Problem Solving Mathematical Olympiads.

APSMO
2025 OLYMPIADS

MATHS OLYMPIAD

Organisation and Procedures

For full details, see the Members' Area

To ensure the integrity of the competition, the Olympiads must be administered under examination conditions.

DO	DO NOT
• Supervise students at all times • Seat students apart • Maintain silence • Provide blank working paper • Give time warnings when 3 minutes remain, and again when 1 minute remains • Collect, mark and retain the papers	• Print the Olympiad papers prior to the Olympiad Date • Read the questions aloud to the students • Interpret the questions for students • Permit any discussion or movement around the room • Permit the use of calculators or other electronic devices

- Olympiad papers are scored by the PICO using the *Solutions and Answers* sheet provided.
- Results should be submitted in the Members' Area within 7 days of the Olympiad.
- Original student answer sheets should be retained by the PICO until the end of the year.
- *Solutions and Answers sheets* are not to be handed out to students. They are a teaching resource for use in class ***after*** completion of the Olympiad paper.

Timing of the Olympiad

- The *Total Time Allowed* for the Olympiad is **30 minutes**.

Absent Student Policy

A student who is legitimately absent on the Olympiad date, may sit the Olympiad under examination conditions on their first day back at school (if that date is within 2 weeks of the original Olympiad date). If these conditions cannot be met, the student must be marked as absent on the submitted results.

The Absent Student Policy is available in the **Contest Administration** section of the Members' Area.

APSMO

2025 : DIVISION S

WEDNESDAY 11 JUNE 2025

OLYMPIAD 2

Total Time Allowed: ***30 Minutes.*** *Calculators NOT Permitted.*

Write your answers in the boxes on the back.

Keep your answers hidden by folding backwards on this line.

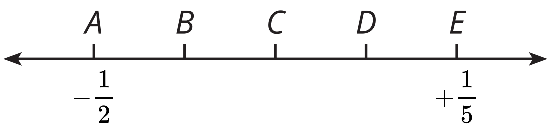
2A. A lattice point is a point whose coordinates are both integers.

How many lattice points are contained either inside or on the right triangle with vertices (0, 0), (6, 0), and (6, 2) ?

2B. The points A, B, C, D, and E are equally spaced in the given order on the number line shown.

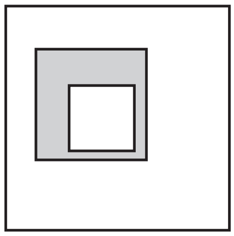
Point A is at $-\frac{1}{2}$.

Point E is at $+\frac{1}{5}$.



Find the position of point D, as a fraction in lowest terms.

2C. The diagonals of the 3 squares shown in the diagram have lengths 3 cm, 5 cm, and 10 cm.



A point is randomly selected within the largest square.

The probability that the randomly selected point lies in the shaded portion, inside the middle square but not in the smallest square, is K%.

Find K.

2D. 2^n means that 2 is multiplied by itself n times.

For example, 2^4 means $2 \times 2 \times 2 \times 2 = 16$.

Given that $2025 = 3^4 \times 5^2$, find the number of factors of 2025.

2E. In the cryptarithm shown, different letters represent different digits, and there is no letter that represents 0.

$$\begin{array}{r} A\ B\ C \\ +\ D\ E\ F \\ \hline G\ H\ J \end{array}$$

Find the greatest whole number value that could be GHJ.

APSMO

2025 : DIVISION S

WEDNESDAY 11 JUNE 2025

OLYMPIAD 2

Student Name:

Fold here. Keep your answers hidden.

2A.

2B.

2C.

2D.

2E.

APSMO

2025 : DIVISION S

WEDNESDAY 11 JUNE 2025

OLYMPIAD 2

MATHS OLYMPIAD

Solutions and Answers

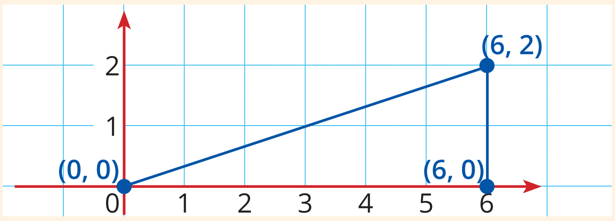
(Items in parentheses are not required)

For teacher use only. Not for Distribution.

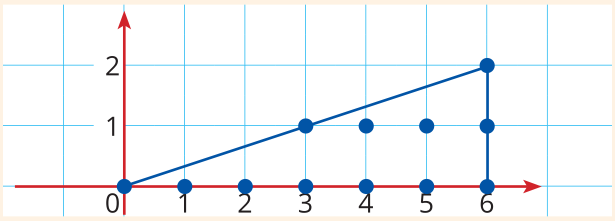
2A: 12	**2B:** $\frac{1}{40}$	**2C:** 16 (%)	**2D:** 15	**2E:** 981

2A. How many lattice points are either inside or on the right triangle with vertices (0, 0), (6, 0), and (6, 2) ?

Strategy 1: *Draw a diagram.*



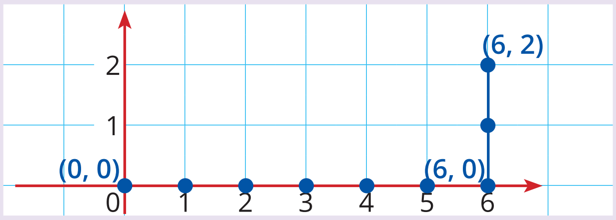
We can draw a Cartesian plane, and construct a triangle with vertices **(0, 0)**, **(6, 0)**, and **(6, 2)**.



Lattice points can then be drawn directly on the diagram.

There are **12** lattice points either inside or on the right triangle with vertices **(0, 0)**, **(6, 0)**, and **(6, 2)**.

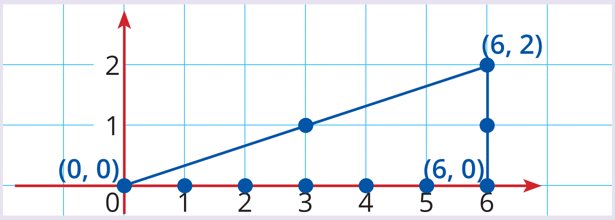
Strategy 2: *Draw a diagram, and build a table.*



There are **7** lattice points on the side of the triangle between points **(0, 0)** and **(6, 0)**.

There are another **2** lattice points on the side of the triangle between points **(6, 0)** and **(6, 2)**.

2							(6, 2)
1							(6, 1)
0	(0, 0)	(1, 0)	(2, 0)	(3, 0)	(4, 0)	(5, 0)	(6, 0)
y/x	0	1	2	3	4	5	6



The line through the origin and **(6, 2)** is $y = \frac{1}{3}x$.

Points with integer co-ordinates on this line are **(0, 0)**, **(3, 1)**, **(6, 2)**, and so on.

2							(6, 2)
1				(3, 1)			(6, 1)
0	(0, 0)	(1, 0)	(2, 0)	(3, 0)	(4, 0)	(5, 0)	(6, 0)
y/x	0	1	2	3	4	5	6

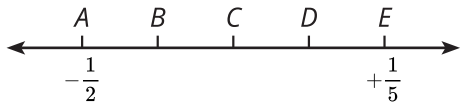
We can now fill in the lattice points that exist on horizontal or vertical lines, between points that we have already identified.

There are **12** lattice points that satisfy these conditions.

2							(6, 2)
1				(3, 1)	(4, 1)	(5, 1)	(6, 1)
0	(0, 0)	(1, 0)	(2, 0)	(3, 0)	(4, 0)	(5, 0)	(6, 0)
y/x	0	1	2	3	4	5	6

Follow-Up: *Rectangle ABCD has a diagonal between A(0, 0) and C(20, 25). How many lattice points lie inside, but not on, ABCD?* [456]

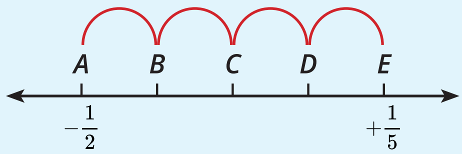
2B. Find the position of point *D*, as a fraction in lowest terms.



Strategy 1: *Draw a diagram.*

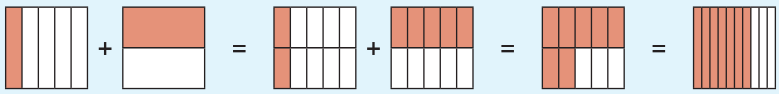
On the number line, there are **5** equally spaced points.

These **5** points are separated by **4** equally sized spaces.



The distance between ***A*** and ***E***

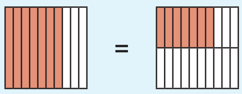
is $+\frac{1}{5} - (-\frac{1}{2}) = +\frac{1}{5} + \frac{1}{2} = \frac{2}{10} + \frac{5}{10} = \frac{7}{10}$.



$+\frac{1}{5} + \frac{1}{2} = \frac{2}{10} + \frac{5}{10} = \frac{7}{10} = \frac{7}{10}$

Point ***C*** is halfway between ***A*** and ***E***.

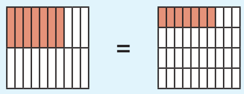
Half of $\frac{7}{10}$ is $\frac{1}{2} \times \frac{7}{10} = \frac{7}{20}$.



$\frac{1}{2} \times \frac{7}{10} = \frac{7}{20}$

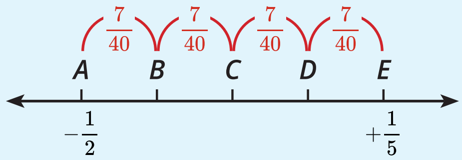
Point ***D*** is halfway between ***C*** and ***E***.

Half of $\frac{7}{20}$ is $\frac{1}{2} \times \frac{7}{20} = \frac{7}{40}$.



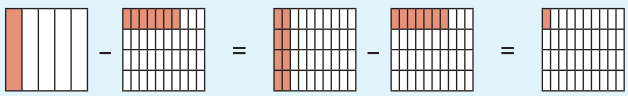
$\frac{1}{2} \times \frac{7}{20} = \frac{7}{40}$

Each of the **4** spaces is $\frac{7}{40}$ units long.



The position of ***D*** is therefore

$+\frac{1}{5} - \frac{7}{40} = \frac{8}{40} - \frac{7}{40} = \frac{1}{40}$.



$+\frac{1}{5} - \frac{7}{40} = \frac{8}{40} - \frac{7}{40} = \frac{1}{40}$

Strategy 2: *Find the mean (average).*

Point ***C*** is halfway between ***A*** and ***E***.

Since ***A*** is at $-\frac{1}{2}$, and ***E*** is at $+\frac{1}{5}$,

C will be at position $\frac{1}{2} \times (-\frac{1}{2} + +\frac{1}{5})$

$= \frac{1}{2} \times (-\frac{5}{10} + +\frac{2}{10})$

$= \frac{1}{2} \times -\frac{3}{10}$

$= -\frac{3}{20}$.

Point ***D*** is halfway between ***C*** and ***E***.

Since ***C*** is at $-\frac{3}{20}$, and ***E*** is at $+\frac{1}{5}$,

D will be at position $\frac{1}{2} \times (-\frac{3}{20} + +\frac{1}{5})$

$= \frac{1}{2} \times (-\frac{3}{20} + +\frac{4}{20})$

$= \frac{1}{2} \times \frac{1}{20}$

$= \frac{1}{40}$.

The position of ***D*** is $\frac{1}{40}$.

Follow-Up: *Which number on a number line is one-fifth of the way from –20 to 2025?* [389]

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2025 : DIVISION S

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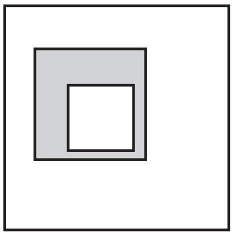
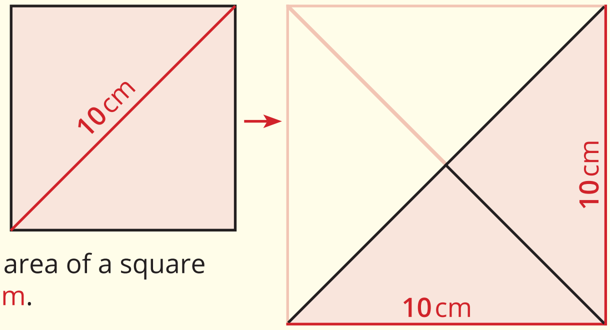
OLYMPIAD 2

2C. Find, as a percentage, the probability that a randomly selected point lies in the shaded portion.



Strategy 1: *Divide a complex shape.*

If we cut the largest square along its 10 cm diagonal, we can rearrange it to form a right-angled triangle with base 10 cm and height 10 cm.



This triangle is half the area of a square measuring 10 cm × 10 cm.

The square with a 10 cm diagonal has an area of 100 cm² ÷ 2 = 50 cm².

Using a similar method, we find that the middle-sized square has area 5 cm × 5 cm ÷ 2 = 12.5 cm².

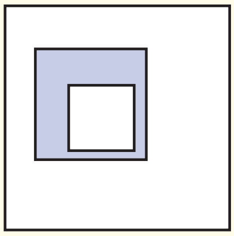
The smallest square has area 3 cm × 3 cm ÷ 2 = 4.5 cm².

The shaded portion has an area of 12.5 cm² – 4.5 cm² = 8 cm².

The total area is 50 cm².

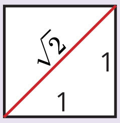
The probability of selecting a point in the shaded area is $\frac{8\text{cm}^2}{50\text{cm}^2} = \frac{16}{100}$.

As a percentage, the probability of selecting a point in the shaded area is **16%**.



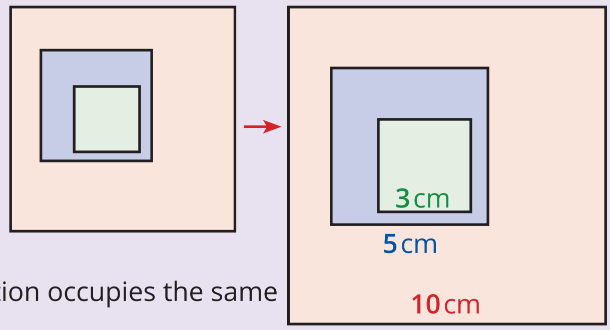
Strategy 2: *Convert to a more convenient form.*

The diagonal of a square is always the same multiple ($\sqrt{2}$) of its side length.



This can be determined in a number of different ways, including Pythagoras' Theorem, or by using the area model from Strategy 1.

If we multiply all of the side lengths of all of the squares by $\sqrt{2}$, the proportions of the resulting diagram remain the same.



In both the original diagram and the new diagram, the shaded portion occupies the same percentage of the area.

In the new diagram, the areas of the squares are 3 cm × 3 cm = 9 cm², 5 cm × 5 cm = 25 cm², and 10 cm × 10 cm = 100 cm².

The shaded portion has an area of 25 cm² – 9 cm² = 16 cm².

The total area is 100 cm².

The probability of selecting a point in the shaded area is $\frac{16\text{cm}^2}{100\text{cm}^2} = \frac{16}{100}$ = **16%**.

Follow-Up: *Three concentric circles, centered at O, have radii A cm, B cm, and 10 cm, where A and B are whole numbers and A < B < 10. The middle ring is shaded. The probability that a randomly selected point that lies in the circle of radius 10 also lies in the shaded ring, is 20%. Find A and B.* [*A* = 4 and *B* = 6]

2D. Find the number of factors of 2025.

Strategy 1: *Build a table.*

We know that **2025** = $\mathbf{3^4 \times 5^2}$.

Therefore, both $\mathbf{3^4}$ and $\mathbf{5^2}$ are factors of **2025**.

Since both **3** and **5** are prime, $\mathbf{3^4}$ and $\mathbf{5^2}$ are relatively prime - they do have any factors in common, except for **1**.

Factors of 3^4 are:

	Expansion	Value
3^0	1	1
3^1	3	3
3^2	3 × 3	9
3^3	3 × 3 × 3	27
3^4	3 × 3 × 3 × 3	81

Factors of 5^2 are:

	Expansion	Value
5^0	1	1
5^1	5	5
5^2	5 × 5	25

All of the factors of **2025** can now be expressed in terms of factors of $\mathbf{3^4}$ and $\mathbf{5^2}$.

×	3^0	3^1	3^2	3^3	3^4
5^0	1	3	3 × 3 = 9	3 × 3 × 3 = 27	3 × 3 × 3 × 3 = 81
5^1	5	3 × 5 = 15	3 × 3 × 5 = 45	3 × 3 × 3 × 5 = 135	3 × 3 × 3 × 3 × 5 = 405
5^2	5 × 5 = 25	3 × 5 × 5 = 75	3 × 3 × 5 × 5 = 225	3 × 3 × 3 × 5 × 5 = 675	3 × 3 × 3 × 3 × 5 × 5 = 2025

2025 has **15** factors.

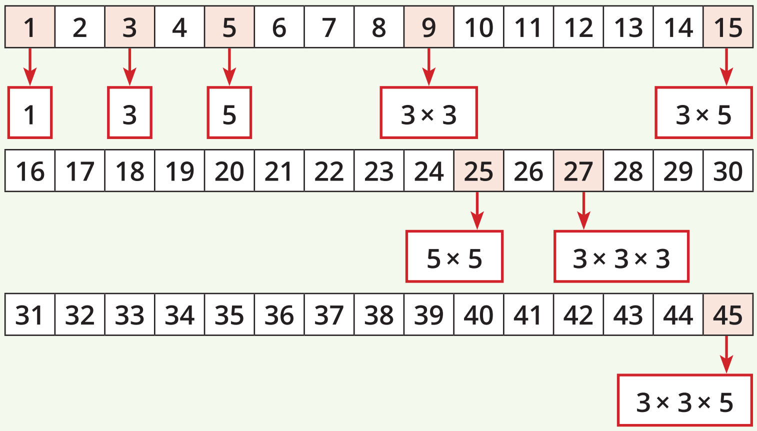
Strategy 2: *Make an organised list, and draw a diagram.*

3 × 3 × 3 × 3 × 5 × 5 = **(3 × 3 × 5) × (3 × 3 × 5)**.

We can see that **2025** is a square number.

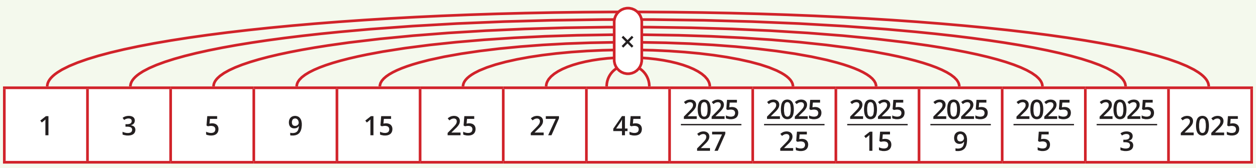
Aside from **3 × 3 × 5 = 45,** every pair of factors must include one value that is less than **45**.

We can identify factors that are less than **45** by inspection. They are numbers where the prime factorisation can be expressed as some portion of **3 × 3 × 3 × 3 × 5 × 5**.



We can now use a diagram to find the number of factors of **2025**.

Note that we do not need to know what those factors are, as long as we know they are unique.



2025 has **15** factors.

Follow-Up: *What is the least integer greater than 2025 that has exactly 15 positive integer factors?* [2500]

APSMO

2025 : DIVISION S

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OLYMPIAD 2

MATHS OLYMPIAD

2E. Find the greatest whole number value that could be *GHJ*.

Strategy: *Eliminate All But One Possibility.*

We know that different letters represent different digits, and there is no letter that represents **0**.

$$\begin{array}{r} A\ B\ C \\ +\ D\ E\ F \\ \hline G\ H\ J \end{array}$$

Let's begin by assuming the greatest possible value for ***GHJ***, which is **987**.

1	2	3	4	5	6	7	8	9
						J	*H*	*G*

With this assumption, either:

- *C* + *F* = 7, or
- *C* + *F* = 17.

$$\begin{array}{r} A\ B\ C \\ +\ D\ E\ F \\ \hline 9\ 8\ 7 \end{array}$$

Since **9** and **8** are both taken, there is no combination of digits that gives ***C* + *F* = 17**.

If ***C* + *F* = 7**, ***C*** and ***F*** could be **1** and **6**, **2** and **5**, or **3** and **4.**

1	2	3	4	5	6	7	8	9
(*C*)					(*F*)	*J*	*H*	*G*

There is no redistribution to the tens place, and ***B* + *E*** = **18** is impossible, so ***B* + *E* = 8**.

$$\begin{array}{r} A\ B\ C \\ +\ D\ E\ F \\ \hline 9\ 8\ 7 \end{array}$$

So, (***B*** **+** ***E***) **+** (***C*** **+** ***F***) **= 8 + 7 = 15.**

Since **1 + 2 + 3 + 4 + 5 + 6 = 21**, we have ***A* + *D* = 21 – 15 = 6.**

We want ***A* + *D* = 9**, so this does not work. Therefore, ***GHJ* ≠ 987**.

1	2	3	4	5	6	7	8	9
(*C*)		(*B*)		(*E*)	(*F*)	*J*	*H*	*G*

To maximise the value of ***GHJ***, we will endeavour to maintain ***G*** = **9**, and ***H*** = **8**.

From the above reasoning, we know that:

B* + *E = **17** and ***B* + *E*** = **18** are both impossible.

Therefore, ***B* + *E* = 7** (with redistribution from the ones place), or ***B* + *E* = 8.**

$$\begin{array}{r} {}^{1}\ \ \ \\ A\ B\ C \\ +\ D\ E\ F \\ \hline 9\ 8\ J \end{array} \qquad \begin{array}{r} A\ B\ C \\ +\ D\ E\ F \\ \hline 9\ 8\ J \end{array}$$

Either way, there is no redistribution from the tens place. We can therefore be sure that ***A* + *D*** = **9.**

Digits **1 - 7** are available for use.

1	2	3	4	5	6	7	8	9
							H	*G*

1 + 2 + 3 + 4 + 5 + 6 + 7 = 28, so we know that ***A* + *B* + *C* + *D* + *E* + *F* + *J* = 28.**

Suppose ***B* + *E* = 7**, with redistribution from the ones place.

$$\begin{array}{r} {}^{1}\ \ \ \\ A\ B\ C \\ +\ D\ E\ F \\ \hline 9\ 8\ J \end{array}$$

Then:

- ***C* + *F* ≥ 10.**
- ***C* + *F* = *J* + 10.**
- ***C* + *F* + *J* = 28 – 9 – 7 = 12.**

If ***C* + *F* = 10**, then ***J* = 0**, which is not an available digit.

If ***C* + *F* = 11**, then ***J* = 1.**

As shown at the right, this solution is possible.

$$\begin{array}{r} {}^{1}\ \ \ \\ 2\ 3\ 5 \\ +\ 7\ 4\ 6 \\ \hline 9\ 8\ 1 \end{array}$$

Suppose ***B* + *E* = 8**, with no redistribution from the ones place.

$$\begin{array}{r} A\ B\ C \\ +\ D\ E\ F \\ \hline 9\ 8\ J \end{array}$$

Then:

- ***C* + *F* < 10.**
- ***C* + *F* = *J*.**
- ***C* + *F* + *J* = 28 – 9 – 8 = 11.**

Since ***C* + *F* = *J*, and *C* + *F* + *J*** = **11**, we have ***J* + *J* = 11.**

J is a digit, so this is impossible.

The greatest whole number value that could be ***GHJ*** *is* **981**.

Follow-Up: *Suppose it is possible to use the digit 0, but no leading digit can be 0.*

(a) Find the least whole number that could be GHJ. [356]

(b) Find the least multiple of 10 that could be GHJ. [450]