APSMO

2025 OLYMPIADS

Important

APSMO

2025 OLYMPIADS

Organisation and Procedures

For full details, see the Members' Area

To ensure the integrity of the competition, the Olympiads must be administered under examination conditions.

DO

- Supervise students at all times
- Seat students apart
- Maintain silence
- Provide blank working paper
- Give time warnings when 3 minutes remain, and again when 1 minute remains
- Collect, mark and retain the papers

DO NOT

- Print the Olympiad papers prior to the Olympiad Date
- Read the questions aloud to the students
- Interpret the questions for students
- Permit any discussion or movement around the room
- Permit the use of calculators or other electronic devices

- Olympiad papers are scored by the PICO using the *Solutions and Answers* sheet provided.
- Results should be submitted in the Members' Area within 7 days of the Olympiad.
- Original student answer sheets should be retained by the PICO until the end of the year.
- *Solutions and Answers sheets* are not to be handed out to students. They are a teaching resource for use in class ***after*** completion of the Olympiad paper.

Timing of the Olympiad

- The *Total Time Allowed* for the Olympiad is **30 minutes**.

Absent Student Policy

A student who is legitimately absent on the Olympiad date, may sit the Olympiad under examination conditions on their first day back at school (if that date is within 2 weeks of the original Olympiad date). If these conditions cannot be met, the student must be marked as absent on the submitted results.

The Absent Student Policy is available in the **Contest Administration** section of the Members' Area.

APSMO

2025 : DIVISION S

WEDNESDAY 7 MAY 2025

OLYMPIAD 1

Total Time Allowed: ***30 Minutes.*** *Calculators NOT Permitted.*

Write your answers in the boxes on the back.

Keep your answers hidden by folding backwards on this line.

1A. The value of $\sqrt{9}$ is 3, because 3 is the positive square root of 9.

What integer is equal to $\sqrt{2025} - \sqrt{1600} - \sqrt{25}$?

1B. Consider the following sequence of whole numbers:

1, 2, 4, 7, 11, 16, ...

The positive difference between each pair of consecutive terms increases by one each time.

Find the 20th term in this sequence.

1C. The 1st century CE encompassed the years 1 through to 100.

What is the second year in the 21st century CE that is exactly divisible by 43?

1D. x and y are two positive integers.

$2x + xy + y = 32$.

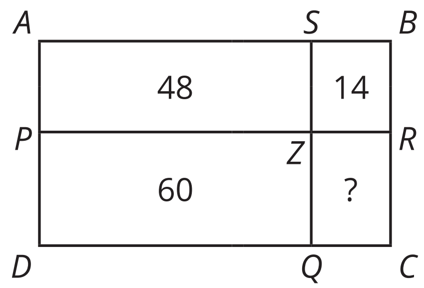
Find the value of $x + y$.

1E. Rectangle *ABCD* is partitioned into smaller rectangles by line segments *PR* and *QS*.

PR and *QS* meet at interior point *Z*.

The areas of three of the four regions are shown, in square units.

Find the exact area of rectangle *ZRCQ*, in square units.



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2025 : DIVISION S

WEDNESDAY 7 MAY 2025

MATHS OLYMPIAD

OLYMPIAD 1

Student Name:

Fold here. Keep your answers hidden.

1A.

1B.

1C.

1D.

1E.

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2025 : DIVISION S

WEDNESDAY 7 MAY 2025

OLYMPIAD 1

MATHS OLYMPIAD

Solutions and Answers

(Items in parentheses are not required)

For teacher use only. Not for Distribution.

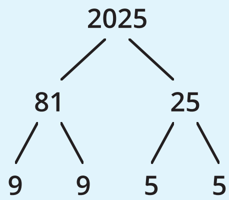
1A: 0	**1B:** 191	**1C:** 2064	**1D:** 16	**1E:** 17.5 or $\frac{35}{2}$ (u²)

1A. What integer is equal to $\sqrt{2025} - \sqrt{1600} - \sqrt{25}$?

Strategy 1: *Convert to a more convenient form.*

To find factors of **2025**, we might notice that:

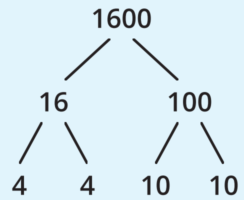
100 = 4 × 25
2000 = 20 × 100
= 20 × 4 × 25
= 80 × 25
2025 = 81 × 25
= 9 × 9 × 5 × 5
= 45 × 45.



Therefore, $\sqrt{2025}$ = 45.

To find factors of **1600**, we might notice that:

1600 = 16 × 100
= 4 × 4 × 10 × 10
= 40 × 40.



Therefore, $\sqrt{1600}$ = 40.

Finally, we know that $\sqrt{25}$ = 5.

$\sqrt{2025} - \sqrt{1600} - \sqrt{25} = 45 - 40 - 5 =$ **0**.

Strategy 2: *Build a table.*

To find the value of $\sqrt{2025}$, we could build a table to list the squares of different integers, until we find the integer whose square is **2025**.

Integer	1	2	3	...										
Square	1	4	9											

Since **2025** is quite a large value, it might be worth skipping some integers in our search.

We might, for example, choose to list the squares of multiples of **10**.

Through this, we might notice that **40² = 1600.**

Integer	1	2	3	...	10	...	20	...	30	...	40	...	50	
Square	1	4	9		100		400		900		1600		2500	

We can see that the value of $\sqrt{2025}$ is between **40** and **50**.

Upon testing **45**, we find that **45² = 2025.**

Integer	1	2	3	...	10	...	20	...	30	...	40	45	50	
Square	1	4	9		100		400		900		1600	2025	2500	

We can see that $\sqrt{2025} - \sqrt{1600} - \sqrt{25} = 45 - 40 - 5 =$ **0**.

Follow-Up: *What is the value of* $\sqrt{1369} - \sqrt{1089} - \sqrt{9}$? [1. Note that 1369 and 1089 both occur between 30² and 40². With 9 in the ones place, the ones digit of the square root must be 3 or 7. 37 – 33 – 3 = 1.]

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WEDNESDAY 7 MAY 2025

OLYMPIAD

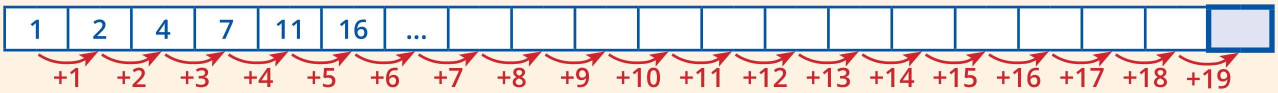
1

MATHS OLYMPIAD

1B. Find the 20th term in the sequence 1, 2, 4, 7, 11, 16, ...

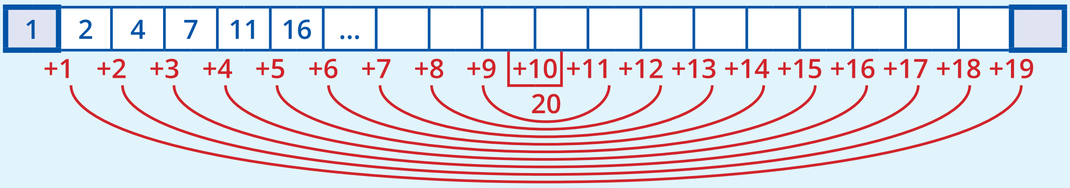
Strategy: *Find a pattern.*

To find the twentieth term, we need to add **19** values that continue to increase by one.



Method 1:
Find "friends of 20".

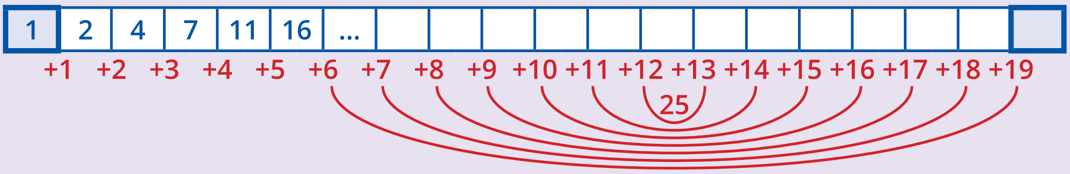
To add 1 + 2 + 3 + ... + 19, we can pair values that together result in a sum of 20.



The twentieth term is equal to **1** + (9 × 20) + 10 = **191**.

Method 2:
Find "friends of 25".

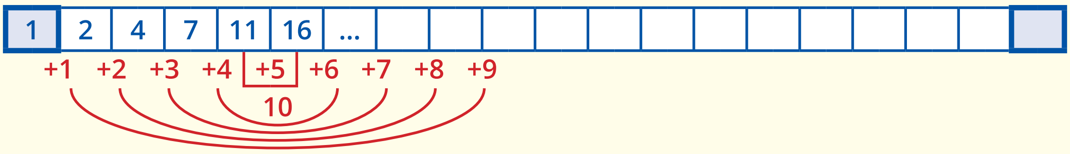
We can also begin with 16 and add 6 + 7 + 8 + ... + 19.



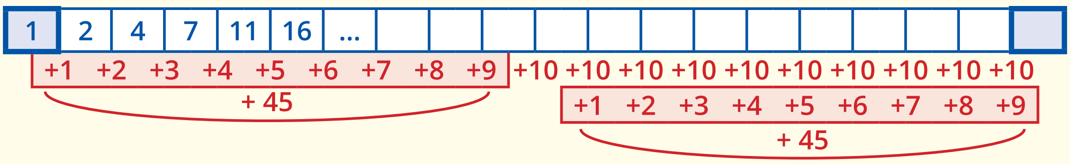
The twentieth term is equal to **16** + (7 × 25) = **191**.

Method 3:
Find the sum of 1 + 2 + ... + 9.

We begin by finding that 1 + 2 + ... + 9 = (4 × 10) + 5 = 45**.**



Then, by recognising that 11 = 10 + 1, 12 = 10 + 2, and so on, we can use place value partitioning to find the value of 1 + 2 + 3 + ... + 19.



The twentieth term is equal to **1** + 45 + (10 × 10) + 45 = **191**.

Follow-Up: *A sequence begins 2, 3, 5, 9, 17, ... The positive differences between each two consecutive terms continues to double. What is the 10th term of the sequence?* [513]

1C. What is the second year in the 21st century CE that is exactly divisible by 43?

The **1st century CE** encompassed the years **1 – 100**.

The **2nd century CE** would then be **101 – 200**.

We can see that the century is defined by its final year.

Using this idea, we can work out the range of years for the **21st century CE**.

Century	First Year	Final Year
1 CE	1	100
2 CE	101	200
10 CE	901	1000
20 CE	1901	2000
21 CE	2001	2100

Strategy 1: *Convert to a more convenient form.*

To find values that are exactly divisible by **43**, we can begin by listing convenient multiples of **43**.

Using multiples of **10 × 43 = 430**, we find that the range of values lies between **40 × 43 = 1720** and **50 × 43 = 2150**.

This range can be further refined to **47 × 43 = 2021** and **48 × 43 = 2064**.

These are the only two years in the **21st century CE** that are exactly divisible by **43**.

The second year in the **21st century CE** that is exactly divisible by **43**, is **2064**.

1 × 43	= 43
10 × 43	= 430
20 × 43	= 860
40 × 43	= 1720
50 × 43	= 2150
49 × 43	= 2150 – 43 = 2107
48 × 43	= 2107 – 43 = 2064
47 × 43	= 2064 – 43 = 2021
46 × 43	= 2021 – 43 = 1978

Strategy 2: *Use a written algorithm.*

We know that the last year in the **21st century CE** is **2100**.

Dividing **2100** by **43**:

```
       4 8
43 ) 2 1 0 0
     1 7 2
     -----
       3 8 0
       3 4 4
       -----
         3 6
```

2100 ÷ 43 = 48, with a remainder of **36**.

Therefore, we know that **43 × 48 < 2100**.

The value of **43 × 48** can be found in many different ways.

```
        4 8
    ×   4 3
    -------
      1 4 4
    1 9 2 0
    -------
    2 0 6 4
```

	40	8
40	1600	320
3	120	24

```
  1600
   320
   120
+   24
------
  2064
```

The question is asking for the second year in the **21st century CE** that is exactly divisible by **43**.

We can see that:

- The year **2064 – 43 = 2021** is also divisible by **43**.
- There can be no other years in the **21st century CE** that are divisible by **43**.

The first year in the **21st century CE** that is exactly divisible by **43**, is **2021**.

The second year in the **21st century CE** that is exactly divisible by **43**, is **2064**.

Follow-Up: *How many positive integers between 4301 and 42999 are divisible by 43?* [899]

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OLYMPIAD 1

1D. Find the value of $x + y$, if $2x + xy + y = 32$, and x and y are both positive integers.

Strategy 1: *Build a table, and find a pattern.*

Since x and y are both positive integers, we can build a table with values for x and y, placing the value of $2x + xy + y$ at each intersection.

y \ x	1	2	3	4	5
1	4	7	10	13	16
2	6	10	14	18	22
3	8	13	18	23	28
4	10	16	22	28	34
5	12	19	26	33	40

Through doing so, we can see arithmetic sequences in each row and column.

By inspection, the column for $x = 1$ appears to include every even number greater than 2.

Therefore, 32 must occur in the column associated with $x = 1$.

The question indicates that there is exactly one value for $x + y$ for which $2x + xy + y = 32$, so setting $x = 1$ must lead to the answer.

Substituting $x = 1$:

$$\begin{aligned} 2(1) + (1)y + y &= 32 \\ 2y &= 30 \\ y &= 15 \\ x + y &= 1 + 15 \\ &= 16 \end{aligned}$$

y \ x	1	2	3	4	5	6	7	8	9	10	11
1	4	7	10	13	16	19	22	25	28	31	34
2	6	10	14	18	22	26	30	34			
3	8	13	18	23	28	33					
4	10	16	22	28	34						
5	12	19	26	33	40						
6	14	22	30								
7	16	25	34								
8	18	28									
9	20	31									
10	22	34									
11	24										
12	26										
13	28										
14	30										
15	32										

Alternatively, we can use the table to situate values for $2x + xy + y$ that are close to 32.

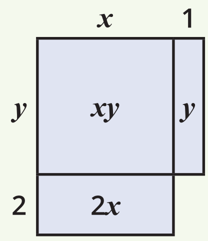
The arithmetic sequences can facilitate a quicker calculation of $2x + xy + y$ in each case.

Since $2x + xy + y = 32$ when $x = 1$ and $y = 15$, the value of $x + y$ is $1 + 15 = 16$.

Strategy 2: *Draw a diagram.*

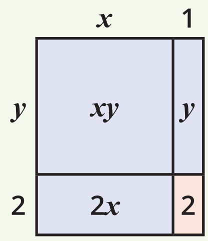
We can represent $2x + xy + y$ diagramatically, using an area model.

This entire area is 32 square units.



If we add an extra 2 square units, we can turn the area into a rectangle.

The area of the entire rectangle is 34 square units.



The area of the rectangle is $(x + 1) \times (y + 2)$.

Since the entire area is 34 square units, and x and y are both positive integers, we know that both $(x + 1)$ and $(y + 2)$ must be factors of 34.

We can see that there is only one rectangle where both x and y are positive integers.

Therefore, the value of $x + y$ is $1 + 15 = 16$.

$x + 1$	$y + 2$	x	y
1	34	0	32
2	17	1	15
17	2	16	0
34	1	33	-1

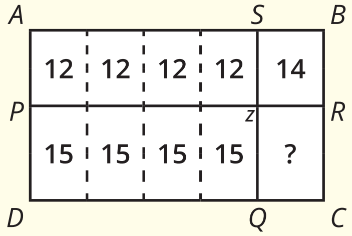
Follow-Up: *Suppose p and q are primes such that pq + 6p + q = 27. Find p and q.* [p = 2, q = 5]

MATHS OLYMPIAD

1E. Find the exact area of rectangle *ZRCQ*.

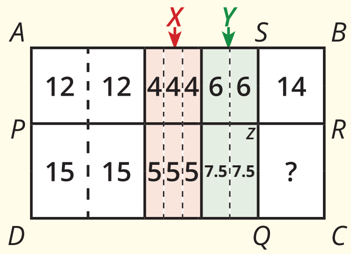
Strategy 1: *Divide a complex shape.*

If we divide rectangle ***ASQD*** in quarters, there would be four rectangles with an area of **12** units^2, and four with an area of **15** units^2.

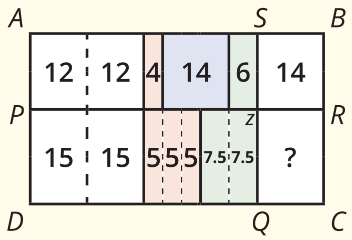


Suppose we select two of these rectangles, ***X*** and ***Y***.

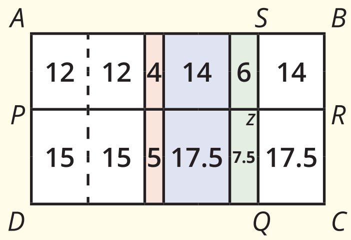
We can divide rectangle ***X*** into thirds, and ***Y*** in half.



We can combine three of the rectangles in the top row to make a single rectangle that, like ***SBRZ***, has area **14** units^2.



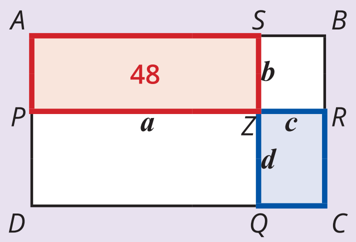
Combining the corresponding rectangles in the bottom row, we have a rectangle that, like ***ZRCQ***, has area **17.5** units^2.



Strategy 2: *Reason algebraically.*

Let a, b, c, and d represent the lengths of *PZ*, *SZ*, *ZR* and *ZQ*.

$ab = 48$, $bc = 14$, and $ad = 60$.



Our aim is to find the value cd.

Finding c in terms of a:

$$\frac{bc}{ab} = \frac{14}{48}$$
$$\frac{c}{a} = \frac{7}{24}$$
$$c = \frac{7}{24}a$$

Finding d in terms of b:

$$\frac{ad}{ab} = \frac{60}{48}$$
$$\frac{d}{b} = \frac{5}{4}$$
$$d = \frac{5}{4}b$$

ZRCQ is $\frac{7}{24}$ as wide as *ASZP*, and $\frac{5}{4}$ as high.

Therefore the area of *ZRCQ* is equal to

$\frac{7}{24} \times \frac{5}{4} \times$ Area(*ASZP*)

$= \frac{7}{24} \times \frac{5}{4} \times 48$

$= \frac{7}{24} \times 60$

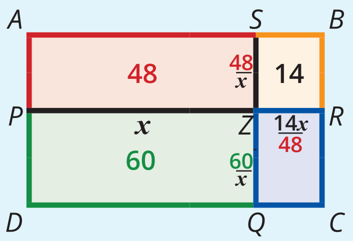
$= 7 \times 2.5$

= 17.5 units^2.

Strategy 3: *Reason algebraically (Alternative approach).*

Let x represent the length of *PZ.*

Other lengths can then be expressed in terms of x.



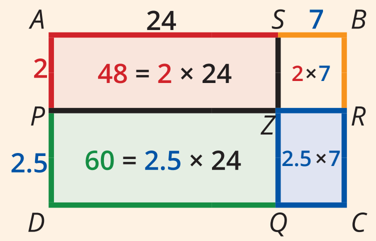
We have $ZR = \frac{14x}{48}$ and $ZQ = \frac{60}{x}$.

The area of *ZRCQ* is equal to $\frac{14x}{48} \times \frac{60}{x} = \frac{35}{2}$ units^2.

Strategy 4: *Use a convenient value.*

Suppose the length of *AP* is 2 units.

Other lengths can then be deduced.



The area of *ZRCQ* is $2.5 \times 7 =$ **17.5** units^2.

Follow-Up: *Consider the original diagram. Suppose that instead of the numbers being the areas of the smaller rectangles, they are the perimeters of the smaller rectangles. What is the perimeter of ZRCQ?* [26]