2025 Maths Games Senior - Years 7 & 8
Resource Kit 3
Teaching Problem Solving

Problem Solving Strategies

This resource kit focuses on the following problem solving strategies:

1. Solve a Simpler Related Problem

Many hard problems are actually simpler problems that have been extended to larger numbers.

Patterns can sometimes be identified by trying the problem with smaller numbers.

2. Eliminate All But One Possibility

Deciding what a quantity is not, can narrow the field to a very small number of possibilities.

These can then be tested against the conditions of the original problem.

It follows on from strategies introduced in the preparation resource kit and resource kits 1 and 2:

Guess, Check and Refine

Draw a Diagram

Find a Pattern

Build a Table

Work Backwards

Make an Organised List

Resource Kit 3 focuses on:

Solve a Simpler Related Problem

Eliminate All But One Possibility

Set Yellow

Example problems for which full worked solutions are included.

Set Green

Problems that are designed to be similar to Set Yellow, but with fewer difficult elements.

Set Orange

Problems that are similar in mathematical structure to the corresponding Yellow problems.

Further questions and solution methods can be found in the APSMO resource book "Building Confidence in Maths Problem Solving", available from www.apsmo.edu.au.

How to use these problems

At the start of the lesson, present the problem and ask the students to think about it. Encourage students to try to solve it in any way they like. When the students have had enough time to consider their solutions, ask them to describe or present their methods, taking particular note of different ways of arriving at the same solution.

Each question includes at least one solution method that the majority of students should be able to follow. By participating in lessons that demonstrate achievable problem solving techniques, students may gain increased confidence in their own ability to address unfamiliar problems.

Finally, the consideration of different solution methods is fundamental to the students' development as effective and sophisticated problem solvers. Even when students have solved a problem to their own satisfaction, it is important to expose them to other methods and encourage them to judge whether or not the other methods are more efficient.

Preparation Kit

Guess, Check and Refine

This involves making a reasonable guess of the answer, and checking it against the conditions of the problem. An incorrect guess may provide more information that may lead to the answer.

Draw a Diagram

A diagram may reveal information that may not be obvious just by reading the problem.

It is also useful for keeping track of where the student is up to in a multi-step problem.

Resource Kit 1

Find a Pattern

A frequently used problem solving strategy is that of recognising and extending a pattern.

Students can often simplify a difficult problem by identifying a pattern in the problem situation.

Build a Table

A table displays information so that it is easily located and understood.

A table is an excellent way to record data so the student doesn't have to repeat their efforts.

Resource Kit 2

Work Backwards

If a problem describes a procedure and then specifies the final result, this method usually makes the problem much easier to solve.

Make an Organised List

Listing every possibility in an organised way is an important tool.

How students organise the data often reveals additional information.

Resource Kit 3

Solve a Simpler Related Problem

Many hard problems are actually simpler problems that have been extended to larger numbers.

Patterns can sometimes be identified by trying the problem with smaller numbers.

Eliminate All But One Possibility

Deciding what a quantity is not, can narrow the field to a very small number of possibilities.

These can then be tested against the conditions of the original problem.

Resource Kit 4

Convert to a More Convenient Form

There are times when changing some of the conditions of a problem makes a solution clearer or more convenient.

Divide a Complex Shape

Sometimes it is possible to divide an unusual shape into two or more common shapes that are easier to work with.

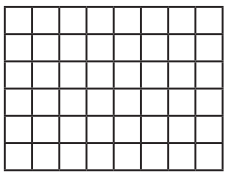
Set Yellow

3.1) Evaluate 2023 × 2022 – 2022 × 2021.

3.2) There are ten students in the book club.
For an end of year celebration, each student brought one book each for everyone else in the club.
How many books were there all together?

3.3) Three machines can make ninety doughnuts in two minutes.
How many doughnuts will four machines make in three minutes?

3.4) The 6 × 8 rectangular array shown is composed entirely of identical small squares.
How many squares can be traced along the lines in this diagram?



Set Yellow

3.5) Adam, Bree, Chloe, Duncan, Elena and Fahim are holding hands in a circle, facing the centre.

Adam is opposite Bree.

Duncan is holding Bree's left hand.

Elena is holding hands with both Adam and Fahim.

Who is holding Chloe's left hand?

3.6) Each person has some information about a secret number.

- Amelia knows that the sum of the digits is either 9 or 18.
- Blake knows that it is a 3-digit number where the three digits are different and in increasing order.
- Chelsea knows that the difference between the last and middle digits does not equal the difference between the middle and first digits.
- Daniel knows that neither 1 nor 9 is used anywhere as a digit.

What is the secret number?

3.7) Ray, Bryn and Max each brought a plate of food for a party.
One brought nachos, one brought quiche and one brought sushi.

Only one of the following statements is true:

1. Ray brought nachos.
2. Ray did not bring quiche.
3. Max did not bring quiche.
4. Max did not bring sushi.

What did Bryn bring?

3.8) P and Q are both prime numbers.

- $P \times Q = 2021$.
- $P - Q = 4$.

Find the value of $P + Q$.

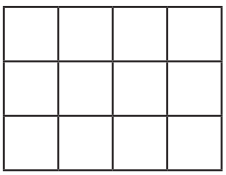
Set Green

3.1) Evaluate 203 × 202 – 202 × 201.

3.2) There are five students in the book club.
For an end of year celebration, each student brought one book each for everyone else in the club.
How many books were there all together?

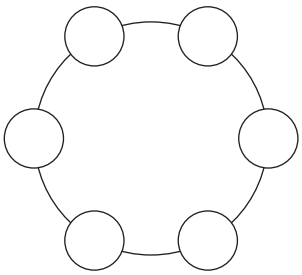
3.3) One machine can make thirty doughnuts in two minutes.
How many doughnuts will four machines make in three minutes?

3.4) The 3 × 4 rectangular array shown is composed entirely of identical small squares.
How many squares can be traced along the lines in this diagram?



Set Green

3.5) Adam, Bree, Chloe, Duncan, Elena and Fahim are holding hands in a circle, facing the centre.



Adam is opposite Bree.

Duncan is holding Bree's left hand.

Elena is holding hands with both Adam and Fahim.

Who is holding Chloe's left hand?

3.6) Each person has some information about a secret number.

- Amelia knows that the sum of the digits is 18.
- Blake knows that it is a 3-digit number where the three digits are different and in increasing order.
- Chelsea knows that the middle digit is 4 more than the first digit.

What is the secret number?

3.7) Ray, Bryn and Max each brought a plate of food for a party.
One brought nachos, one brought quiche and one brought sushi.

Only one of the following statements is true:

1. Ray did not bring quiche.
2. Max did not bring quiche.
3. Max did not bring sushi.

What did Bryn bring?

3.8) P and Q are both prime numbers.

- $P \times Q = 221$.
- $P - Q = 4$.

Find the value of $P + Q$.

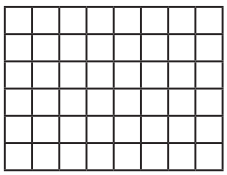
Set Orange

3.1) Evaluate 65432 × 65432 – 65432 × 65421 – 65432.

3.2) When two people shake hands with one another, that counts as one "handshake".
Every person in a room shakes hands with each other person in the room exactly once.
There are a total of 15 "handshakes".
How many people are in the room?

3.3) If 4 people can paint 2 fences in 5 hours, how many hours will it take for 8 people to paint 8 fences?
Assume all painters work at the same rate for the full time, and all of the fences are the same size.

3.4) The 6 × 8 rectangular array shown is composed entirely of identical small squares.
How many rectangles with integer length sides are nested in the 6 × 8 rectangle?



Set Orange

3.5) Amy, Dawn and Soumiya sit at a round table facing the centre.
Each girl wears a ring on her finger.
Each ring is a different colour. One ring is red, one is green, and one is blue.
The girl with the green ring is to the right of Amy.
The girl with the red ring is to the left of Soumiya.
State the colour of each girl's ring.

3.6) Janine's number has three digits.
One digit is a prime number.
Another digit is a square number.
The other digit is neither prime nor square.
Janine's number is NOT divisible by 3.
What is the greatest possible value of Janine's number?

3.7) Ray, Bryn and Max each brought a plate of food for a party.
One brought nachos, one brought quiche and one brought sushi.

Exactly three of the following statements are true:

1. Ray brought nachos.
2. Ray did not bring quiche.
3. Max did not bring quiche.
4. Max did not bring sushi.

What did Ray bring?

3.8) Find the sum of all prime numbers less than 100, which contain the digit 5.

Maths Games – Example Problem 3.1

Example Problem 3.1 - Green

Evaluate $203 \times 202 - 202 \times 201$.

Example Problem 3.1 - Yellow

Evaluate $2023 \times 2022 - 2022 \times 2021$.

Example Problem 3.1 - Orange

Evaluate $65432 \times 65432 - 65432 \times 65421 - 65432$.

Maths Games Example Solution 3.1 - Yellow

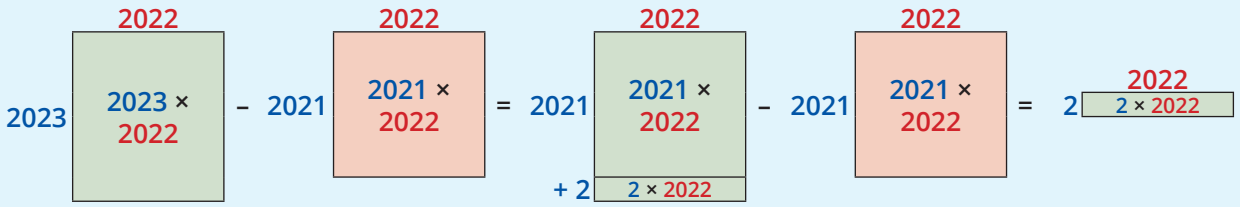
Evaluate 2023 × 2022 – 2022 × 2021.

Strategy 1: Solve a Simpler Related Problem

By recognising that 2022 × 2021 = 2021 × 2022, we can solve this problem in a number of different ways.

Method 1: Draw a Diagram.

We can use an area model to represent the quantities 2023 × 2022 and 2021 × 2022.



2023 × 2022 – 2021 × 2022 = 2 × 2022 = **4044**.

Method 2: Use the Distributive Law.

2023 × 2022 – 2021 × 2022

= 2022(2023 – 2021)

= 2022(2)

= **4044**.

Method 3: Consider the Expression Algebraically.

We can think of 2023 × 2022 – 2021 × 2022

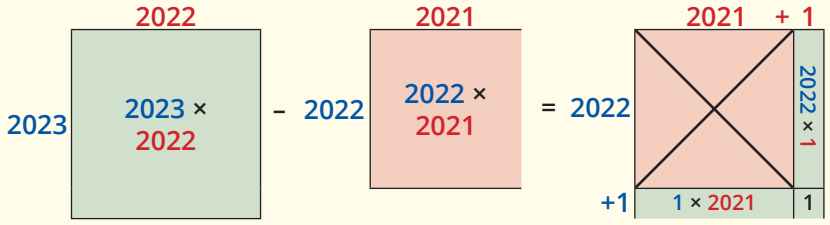
in the same way that we might think about

2023x – 2021x, where x = 2022.

By collecting like terms, we have 2023x – 2021x = 2x

and since x = 2022, 2x = **4044**.

Strategy 2: Draw a Diagram



After subtracting the 2022 × 2021 rectangle, what remains is 2021 + 2022 + 1 = **4044**.

Strategy 3: Perform the Calculation

2023	2022	4090506
× 2022	× 2021	– 4086462
4046	2022	4044
40460	40440	
00	00	
+ 4046000	+ 4044000	
4090506	4086462	

The value is **4044**.

Answers

3.1 - Green: 404

3.1 - Orange: 654320

3.1 - Yellow: 4044

Maths Games – Example Problem 3.2

Example Problem 3.2 - Green

There are five students in the book club.
For an end of year celebration, each student brought one book each for everyone else in the club.
How many books were there all together?

Example Problem 3.2 - Yellow

There are ten students in the book club.
For an end of year celebration, each student brought one book each for everyone else in the club.
How many books were there all together?

Example Problem 3.2 - Orange

When two people shake hands with one another, that counts as one "handshake".
Every person in a room shakes hands with each other person in the room exactly once.
There are a total of 15 "handshakes".
How many people are in the room?

Maths Games Example Solution 3.2 - Yellow

There are ten students in the book club.

For an end of year celebration, each student brought one book each for everyone else in the club.

How many books were there all together?

Strategy 1: Solve a Simpler Related Problem

Let's pretend there are only **2** students: Adam and Bella.

Adam would give 1 book to Bella.

Bella would give 1 book to Adam.

In total, there are 1 + 1 = 2 books.

Let's pretend there are **3** students: Adam, Bella and Cam.

Adam would give 1 book to Bella, and 1 book to Cam.

Bella would give 1 book to Adam, and 1 book to Cam.

Cam would give 1 book to Adam, and 1 book to Bella.

In total, there are 2 + 2 + 2 = 6 books.

Is there a pattern?

Adam gives books to everyone but himself.

So, if there are 5 students, then Adam would bring 5 – 1 = 4 books.

Each student does the same thing, so with 5 students, there would be 5 × 4 = 20 books.

With 10 students, each student would bring 10 – 1 = 9 books.

There would be 10 × 9 = **90 books**.

Strategy 2: Build a Table

There are 10 students in the book club.

Let's call them Adam, Bella, Cam, Danny, Evie, Frances, George, Henry, Imogen and Jake.

Adam would give 1 book to each of the other students in the club.

Adam brings a book for	~~A~~	B	C	D	E	F	G	H	I	J

Bella would also give 1 book to everyone except herself.

Adam brings a book for	~~A~~	B	C	D	E	F	G	H	I	J
Bella brings a book for	A	~~B~~	C	D	E	F	G	H	I	J

The same happens for Cam, Danny, and everyone else.

Our table shows that there would have been 10 × 10 = 100 books if all of the students brought a book for everyone, *including* themselves.

Since none of the students brought books for themselves, the number of books is reduced by 10.

In total, there were 100 – 10 = **90** books.

Adam brings a book for	~~A~~	B	C	D	E	F	G	H	I	J
Bella brings a book for	A	~~B~~	C	D	E	F	G	H	I	J
Cam brings a book for	A	B	~~C~~	D	E	F	G	H	I	J
Danny brings a book for	A	B	C	~~D~~	E	F	G	H	I	J
Evie brings a book for	A	B	C	D	~~E~~	F	G	H	I	J
Frances brings a book for	A	B	C	D	E	~~F~~	G	H	I	J
George brings a book for	A	B	C	D	E	F	~~G~~	H	I	J
Henry brings a book for	A	B	C	D	E	F	G	~~H~~	I	J
Imogen brings a book for	A	B	C	D	E	F	G	H	~~I~~	J
Jake brings a book for	A	B	C	D	E	F	G	H	I	~~J~~

Answers

3.2 - Green: 20

3.2 - Orange: 6

3.2 - Yellow: 90

Maths Games – Example Problem 3.3

Example Problem 3.3 - Green

One machine can make thirty doughnuts in two minutes.
How many doughnuts will four machines make in three minutes?

Example Problem 3.3 - Yellow

Three machines can make ninety doughnuts in two minutes.
How many doughnuts will four machines make in three minutes?

Example Problem 3.3 - Orange

If 4 people can paint 2 fences in 5 hours, how many hours will it take for 8 people to paint 8 fences?
Assume all painters work at the same rate for the full time, and all of the fences are the same size.

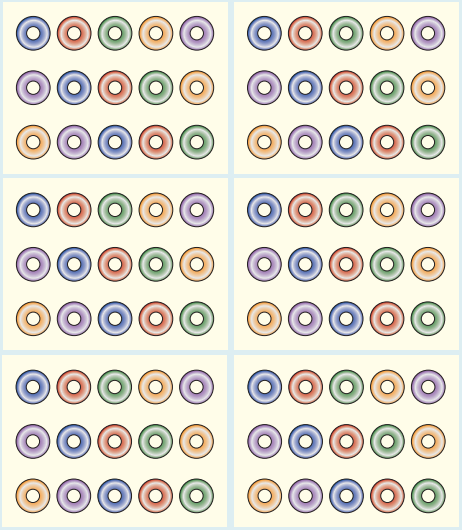
Maths Games Example Solution 3.3 - Yellow

Three machines can make ninety doughnuts in two minutes.

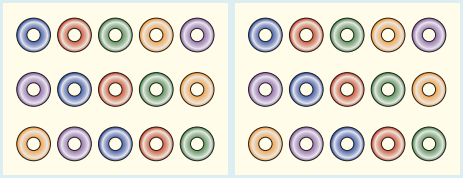
How many doughnuts will four machines make in three minutes?

Strategy: Solve a Simpler Related Problem

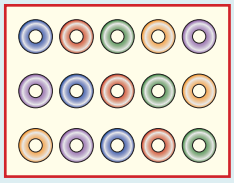
We know that 3 machines will make 90 doughnuts in 2 minutes.



So, in 2 minutes, 1 of the machines will make 90 ÷ 3 = 30 doughnuts.

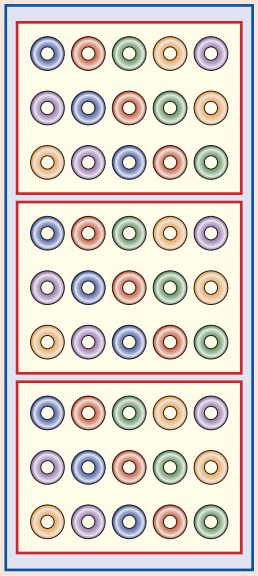


In 1 minute, 1 of the machines will make 30 ÷ 2 = 15 doughnuts.

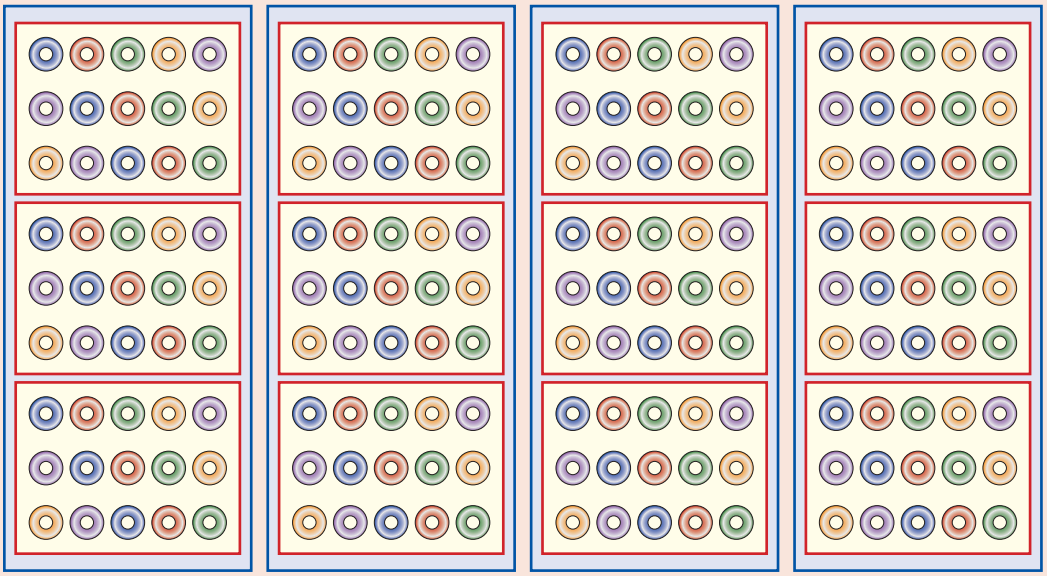


We now know that it takes 1 minute for 1 machine to make 15 doughnuts.

In 3 minutes, 1 machine will make 3 × 15 = 45 doughnuts.



If we run 4 of the machines for 3 minutes, together they will make 4 × 45 = 180 doughnuts.



In 3 minutes, 4 machines will make **180** doughnuts.

Answers

3.3 - Green: 180

3.3 - Orange: 10 (hours)

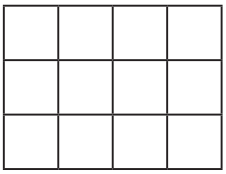
3.3 - Yellow: 180

Maths Games – Example Problem 3.4

Example Problem 3.4 - Green

The 3 × 4 rectangular array shown is composed entirely of identical small squares.

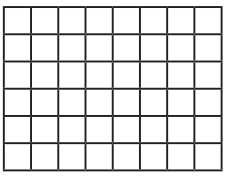
How many squares can be traced along the lines in this diagram?



Example Problem 3.4 - Yellow

The 6 × 8 rectangular array shown is composed entirely of identical small squares.

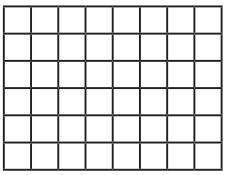
How many squares can be traced along the lines in this diagram?



Example Problem 3.4 - Orange

The 6 × 8 rectangular array shown is composed entirely of identical small squares.

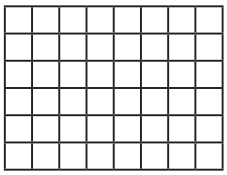
How many rectangles with integer length sides are nested in the 6 × 8 rectangle?



Maths Games Example Solution 3.4 - Yellow

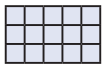
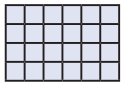
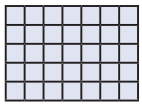
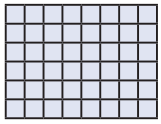
The 6 × 8 rectangular array shown is composed entirely of identical small squares.

How many squares can be traced along the lines in this diagram?



Strategy 1: Solve a Simpler Related Problem

By working with a smaller grid, we can identify patterns that can help us to count the numbers of squares.

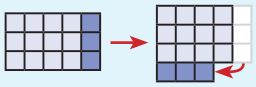
Grid size	1 × 1	2 × 2	3 × 3	4 × 4	5 × 5	6 × 6	Total
	3						3
	8	3					11
	15	8	3				26
	24	15	8	3			50
	35	24	15	8	3		85
	48	35	24	15	8	3	133

The following patterns then emerge:

- Each new row adds a new column to the left. The value in the new column is the number of 1 × 1 squares, which is the size of the grid.

  (**Strategy 2** demonstrates why the pattern makes sense.)

- Each value in the table is **1** less than a perfect square. We can see that this is a logical result by rearranging the grid, as follows:



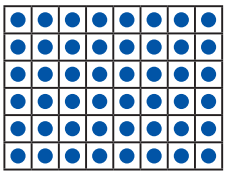
133 squares can be traced along the lines in this diagram.

Strategy 2: Count in an Organised Way

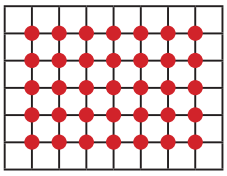
We can count all of the squares systematically, by size, beginning with 1 × 1 squares, then 2 × 2 squares, 3 × 3 squares, and so on.

In the following diagrams, each dot represents the centre of a specified square that measures $n \times n$.

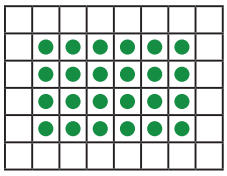
There are $8 \times 6 = 48$ 1 × 1 squares.



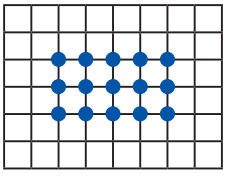
There are $7 \times 5 = 35$ 2 × 2 squares.



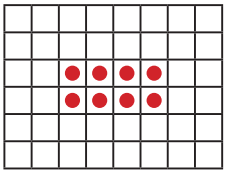
There are $6 \times 4 = 24$ 3 × 3 squares.



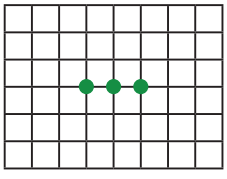
There are $5 \times 3 = 15$ 4 × 4 squares.



There are $4 \times 2 = 8$ 5 × 5 squares.



There are $3 \times 1 = 3$ 6 × 6 squares.



There are, in total, 48 + 35 + 24 + 15 + 8 + 3 = **133** squares that can be traced along the lines in this diagram.

Answers

3.4 - Green: 20

3.4 - Orange: 756

3.4 - Yellow: 133

Maths Games – Example Problem 3.5

Example Problem 3.5 - Green

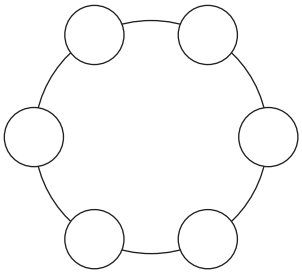
Adam, Bree, Chloe, Duncan, Elena and Fahim are holding hands in a circle, facing the centre.

Adam is opposite Bree.

Duncan is holding Bree's left hand.

Elena is holding hands with both Adam and Fahim.

Who is holding Chloe's left hand?



Example Problem 3.5 - Yellow

Adam, Bree, Chloe, Duncan, Elena and Fahim are holding hands in a circle, facing the centre.

Adam is opposite Bree.

Duncan is holding Bree's left hand.

Elena is holding hands with both Adam and Fahim.

Who is holding Chloe's left hand?

Example Problem 3.5 - Orange

Amy, Dawn and Soumiya sit at a round table facing the centre.

Each girl wears a ring on her finger.

Each ring is a different colour. One ring is red, one is green, and one is blue.

The girl with the green ring is to the right of Amy.

The girl with the red ring is to the left of Soumiya.

State the colour of each girl's ring.

Maths Games Example Solution 3.5 - Yellow

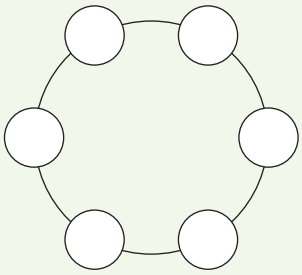
Adam, Bree, Chloe, Duncan, Elena and Fahim are holding hands in a circle, facing the centre.

Adam is opposite Bree. Duncan is holding Bree's left hand. Elena is holding hands with both Adam and Fahim.

Who is holding Chloe's left hand?

Strategy 1: Eliminate All But One Possibility

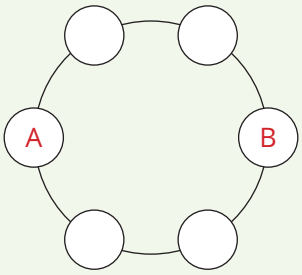
Let's draw a diagram showing how the children are standing.



Adam is opposite Bree.

Let's choose a position for Adam.

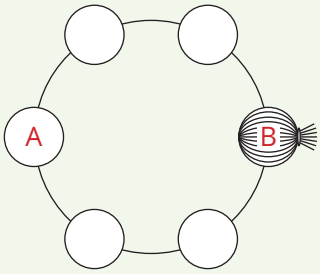
We will put Bree directly opposite.



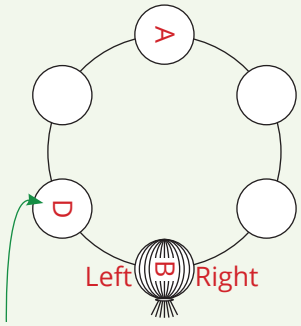
Duncan is holding Bree's left hand - but which side is Bree's left?

The children are facing the centre of the circle.

Let's draw Bree with a ponytail, so that we can imagine her facing the centre.



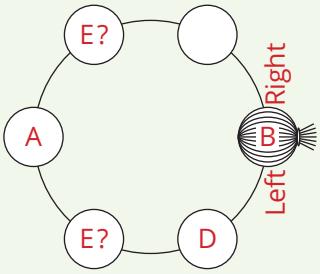
It is easier to find Bree's left hand if we rotate the picture so that we are in Bree's position.



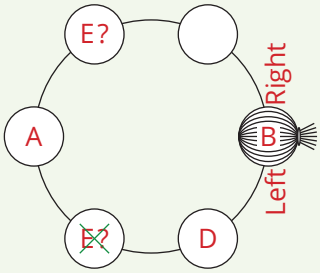
Now we can place Duncan, who is holding Bree's left hand.

Elena is holding hands with both Adam and Fahim.

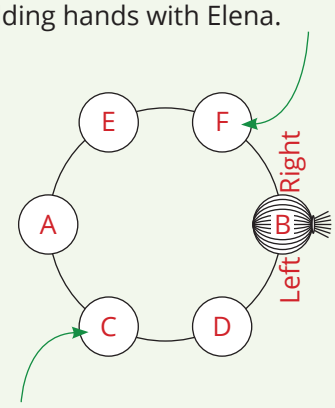
So Elena must be in one of the positions next to Adam.



Since Fahim is on Elena's other side, Elena can't be next to Duncan.

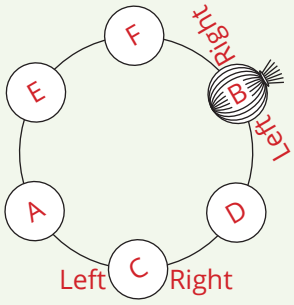


Now we can place Fahim, who is holding hands with Elena.



Chloe must be in the last remaining position.

To find Chloe's left hand, let's rotate the picture so we are in Chloe's position.



So **Adam** is holding Chloe's left hand.

Answers

3.5 - Green: Adam

3.5 - Yellow: Adam

3.5 - Orange: Amy: red, Dawn: blue, Soumiya: green

Maths Games – Example Problem 3.6

Example Problem 3.6 - Green

Each person has some information about a secret number.

- Amelia knows that the sum of the digits is 18.
- Blake knows that it is a 3-digit number where the three digits are different and in increasing order.
- Chelsea knows that the middle digit is 4 more than the first digit.

What is the secret number?

Example Problem 3.6 - Yellow

Each person has some information about a secret number.

- Amelia knows that the sum of the digits is either 9 or 18.
- Blake knows that it is a 3-digit number where the three digits are different and in increasing order.
- Chelsea knows that the difference between the last and middle digits does not equal the difference between the middle and first digits.
- Daniel knows that neither 1 nor 9 is used anywhere as a digit.

What is the secret number?

Example Problem 3.6 - Orange

Janine’s number has three digits.

One digit is a prime number.

Another digit is a square number.

The other digit is neither prime nor square.

Janine's number is NOT divisible by 3.

What is the greatest possible value of Janine’s number?

Maths Games Example Solution 3.6 - Yellow

Each person has some information about a secret number.

- Amelia knows that the sum of the digits is either 9 or 18.
- Blake knows that it is a 3-digit number where the three digits are different and in increasing order.
- Chelsea knows that the difference between the last and middle digits does not equal the difference between the middle and first digits.
- Daniel knows that neither 1 nor 9 is used anywhere as a digit.

What is the secret number?

Strategy 1: Eliminate All But One Possibility

Together, Amelia and Blake know that the number has increasing digits, and the digit sum is **9** or **18**.

	H	T	O	H	T	O	H	T	O	H	T	O	H	T	O
Digit sum is 9	1	2	6	2	3	4									
	1	3	5												
Digit sum is 18	1	8	9	2	7	9	3	6	9	4	5	9	5	6	7
							3	7	8	4	6	8			

Chelsea knows that the difference between the last and middle digits does not equal the difference between the middle and first digits.

Finally, Daniel knows that neither **1** nor **9** is used anywhere as a digit.

H	T	O	H	T	O	H	T	O	H	T	O	H	T	O
~~1~~	~~2~~	~~6~~	~~2~~	~~3~~	~~4~~									
~~1~~	~~3~~	~~5~~												
~~1~~	~~8~~	~~9~~	~~2~~	~~7~~	~~9~~	~~3~~	~~6~~	~~9~~	~~4~~	~~5~~	~~9~~	~~5~~	~~6~~	~~7~~
						3	7	8	~~4~~	~~6~~	~~8~~			

The secret number must be **378**.

Strategy 2: Eliminate All But One Possibility (Alternative Approach)

Together, Blake and Daniel know that the number is between **234** and **678** inclusive, with increasing digits.

H	T	O	H	T	O	H	T	O	H	T	O	H	T	O
2	3	4	3	4	5	4	5	6	5	6	7	6	7	8
2	3	5	3	4	6	4	5	7	5	6	8			
2	3	6	3	4	7	4	5	8	5	7	8			
2	3	7	3	4	8	4	6	7						
2	3	8	3	5	6	4	6	8						
2	4	5	3	5	7	4	7	8						
2	4	6	3	5	8									
2	4	7	3	6	7									
2	4	8	3	6	8									
2	5	6	3	7	8									
2	5	7												
2	5	8												
2	6	7												
2	6	8												
2	7	8												

Amelia knows that the sum of the digits is either **9** or **18**.

H	T	O	H	T	O	H	T	O	H	T	O	H	T	O
~~2~~	~~3~~	~~4~~	~~3~~	~~4~~	~~5~~	~~4~~	~~5~~	~~6~~	~~5~~	~~6~~	~~7~~	~~6~~	~~7~~	~~8~~
~~2~~	~~3~~	~~5~~	~~3~~	~~4~~	~~6~~	~~4~~	~~5~~	~~7~~	~~5~~	~~6~~	~~8~~			
~~2~~	~~3~~	~~6~~	~~3~~	~~4~~	~~7~~	~~4~~	~~5~~	~~8~~	~~5~~	~~7~~	~~8~~			
~~2~~	~~3~~	~~7~~	~~3~~	~~4~~	~~8~~	~~4~~	~~6~~	~~7~~						
~~2~~	~~3~~	~~8~~	~~3~~	~~5~~	~~6~~	4	6	8						
~~2~~	~~4~~	~~5~~	~~3~~	~~5~~	~~7~~	~~4~~	~~7~~	~~8~~						
~~2~~	~~4~~	~~6~~	~~3~~	~~5~~	~~8~~									
~~2~~	~~4~~	~~7~~	~~3~~	~~6~~	~~7~~									
~~2~~	~~4~~	~~8~~	~~3~~	~~6~~	~~8~~									
~~2~~	~~5~~	~~6~~	3	7	8									
~~2~~	~~5~~	~~7~~												
~~2~~	~~5~~	~~8~~												
~~2~~	~~6~~	~~7~~												
~~2~~	~~6~~	~~8~~												
~~2~~	~~7~~	~~8~~												

Finally, Chelsea knows that the difference between the last and middle digits does not equal the difference between the middle and first digits.

This information eliminates all but **378**.

Answers

3.6 - Green: 378

3.6 - Orange: 985

3.6 - Yellow: 378

Maths Games – Example Problem 3.7

Example Problem 3.7 - Green

Ray, Bryn and Max each brought a plate of food for a party.
One brought nachos, one brought quiche and one brought sushi.

Only one of the following statements is true:

1. Ray did not bring quiche.
2. Max did not bring quiche.
3. Max did not bring sushi.

What did Bryn bring?

Example Problem 3.7 - Yellow

Ray, Bryn and Max each brought a plate of food for a party.
One brought nachos, one brought quiche and one brought sushi.

Only one of the following statements is true:

1. Ray brought nachos.
2. Ray did not bring quiche.
3. Max did not bring quiche.
4. Max did not bring sushi.

What did Bryn bring?

Example Problem 3.7 - Orange

Ray, Bryn and Max each brought a plate of food for a party.
One brought nachos, one brought quiche and one brought sushi.

Exactly three of the following statements are true:

1. Ray brought nachos.
2. Ray did not bring quiche.
3. Max did not bring quiche.
4. Max did not bring sushi.

What did Ray bring?

Maths Games Example Solution 3.7 - Yellow

Ray, Bryn and Max each brought a plate of food for a party.
One brought nachos, one brought quiche and one brought sushi.

Only one of the following statements is true:

1. Ray brought nachos.
2. Ray did not bring quiche.
3. Max did not bring quiche.
4. Max did not bring sushi.

What did Bryn bring?

Strategy: Eliminate All But One Possibility

Guess: Statement 1 is true. All of the other statements are false.

"Ray brought nachos" is true.

Therefore, Ray did not bring quiche or sushi, and nobody except Ray brought nachos.

	Ray	Bryn	Max
Nachos	✓	✗	✗
Quiche	✗		
Sushi	✗		

"Ray did not bring quiche" is false.

This means that Ray must have brought quiche.

This is a contradiction, so our guess is incorrect.

	Ray	Bryn	Max
Nachos	✓	✗	✗
Quiche	?		
Sushi	✗		

Guess: Statement 2 is true. All of the other statements are false.

"Ray did not bring quiche" is true.

	Ray	Bryn	Max
Nachos			
Quiche	✗		
Sushi			

"Ray brought nachos" is false.

This means that Ray must have brought sushi, and nobody except Ray brought sushi.

	Ray	Bryn	Max
Nachos	✗		
Quiche	✗		
Sushi	✓	✗	✗

"Ray did not bring sushi" is false.

This means that Max must have brought sushi.

This is a contradiction, so our guess is incorrect.

	Ray	Bryn	Max
Nachos	✗		
Quiche	✗		
Sushi	✓	✗	?

Guess: Statement 3 is true. All of the other statements are false.

"Max did not bring quiche" is true.

	Ray	Bryn	Max
Nachos			
Quiche			✗
Sushi			

"Ray brought nachos" is false.

	Ray	Bryn	Max
Nachos	✗		
Quiche			✗
Sushi			

"Ray did not bring quiche" is false.

This means that Ray must have brought quiche, and nobody except Ray brought quiche.

	Ray	Bryn	Max
Nachos	✗		
Quiche	✓	✗	✗
Sushi	✗		

"Max did not bring sushi" is false.

This means that Max must have brought sushi, and nobody except Max brought sushi.

	Ray	Bryn	Max
Nachos	✗		✗
Quiche	✓	✗	✗
Sushi	✗	✗	✓

We have considered every statement.

In each case, we were able to make it work, with no contradictions.

Therefore, Ray brought quiche, Max brought sushi, and so Bryn must have brought **nachos**.

Answers

3.7 - Green: Nachos

3.7 - Orange: Nachos

3.7 - Yellow: Nachos

Maths Games – Example Problem 3.8

Example Problem 3.8 - Green

P and *Q* are both prime numbers.

- $P \times Q = 221$.
- $P - Q = 4$.

Find the value of $P + Q$.

Example Problem 3.8 - Yellow

P and *Q* are both prime numbers.

- $P \times Q = 2021$.
- $P - Q = 4$.

Find the value of $P + Q$.

Example Problem 3.8 - Orange

Find the sum of all prime numbers less than 100, which contain the digit 5.

Maths Games Example Solution 3.8 - Yellow

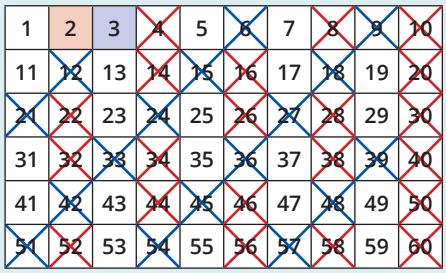
P and Q are both prime numbers.

- $P \times Q = 2021$.
- $P - Q = 4$.

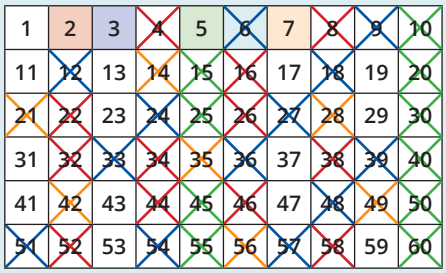
Find the value of $P + Q$.

Strategy 1: Eliminate All But One Possibility

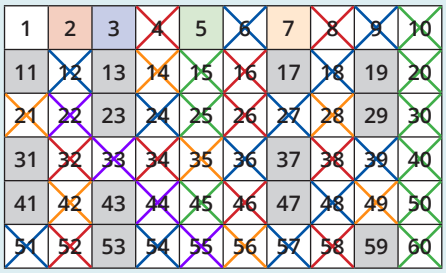
To find prime numbers, we can use the "Eratosthenes Sieve" method of eliminating multiples.



We begin by eliminating multiples of 2, then 3,



then multiples of 5 and 7,



and so on.

By continuing with this process, we eliminate all of the composite numbers.

The numbers that remain must be prime.

We now need to find prime numbers that differ by **4**, and multiply them to see if the result is **2021**.

Primes that differ by 4	3 × 7	7 × 11	13 × 17	19 × 23	37 × 41	43 × 47	
Product	21	77	221	437	1517	2021	

Since **43 × 47 = 2021**, the sum of the two primes is **43 + 47 = 90**.

Strategy 2: Guess, Check and Refine

Since $P - Q = 4$, we have $P = Q + 4$.

Let's consider different values for Q to see what the value $P \times Q$ might look like.

Q	10	20	30	40	50
P	14	24	34	44	54
$P \times Q$	140	480	1020	1760	2700

Since $P \times Q = 2021$, we now know that the value of Q will be between **40** and **50**.

Let's now consider values for Q that lie within this range.

We'll work out the multiplication if both P and Q are prime.

Q	41	42	43	44	45	46	47	48	49
P	45	46	47	48	49	50	51	52	53
$P \times Q$			2021						

Therefore $Q = 43$, $P = 47$, and the value of $P + Q$ is **47 + 43 = 90**.

Answers

3.8 - Green: 30

3.8 - Orange: 117

3.8 - Yellow: 90

Answers

Set Green

3.1	404
3.2	20
3.3	180
3.4	20
3.5	Adam
3.6	378
3.7	Nachos
3.8	30

Set Yellow

3.1	4044
3.2	90
3.3	180
3.4	133
3.5	Adam
3.6	378
3.7	Nachos
3.8	90

Set Orange

3.1	654320
3.2	6
3.3	10 (hours)
3.4	756
3.5	Amy: red, Dawn: blue, Soumiya: green
3.6	985
3.7	Nachos
3.8	117