MATHS GAMES

APSMO

2025 MATHS GAMES

Important

The information contained in this file is ONLY for the use of registered participants of the 2025 APSMO Maths Games.

Organisation and Procedures

For full details, see the Members' Area

- Maths Games papers are to be conducted under test conditions.

DO	DO NOT
• Supervise students at all times. • Maintain silence. • Provide blank working paper. • Collect, mark and retain the papers.	• Print the papers prior to the scheduled date. • Read the questions aloud to the students. • Interpret the questions for students. • Permit any discussion or movement around the room. • Permit the use of calculators or other electronic devices.

- Papers should be scored by the PICO using the *Solutions and Answers* sheet provided.
- Original student answer sheets should be retained by the PICO until the end of the year.

Absent Students

- A student who is legitimately absent on the date of the Maths Games paper, may sit the paper on their return to school.
- If an absent student does not sit the paper on their return to school they should be marked as 'absent'.
- *Note: This policy differs from the Maths Olympiads Absent Student Policy which has additional requirements*.

APSMO

MATHS GAMES

WEDNESDAY 10 SEPTEMBER 2025

MATHS GAMES
SENIOR

Suggested Time: ***30 Minutes.*** *Calculators NOT Permitted.*

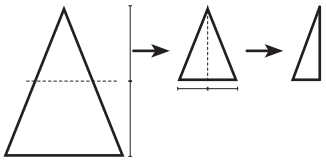
Write your answers in the boxes on the back.

Keep your answers hidden by folding backwards on this line.

4A. An isosceles triangle has two equal sides.

It is first cut horizontally, resulting in a triangle with half of the original height.

It is then cut in half vertically, as shown.

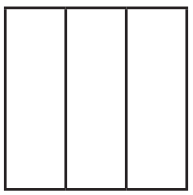


What fraction of the area of the original triangle remains?

4B. The square at the right is divided into three identical rectangles, as shown.

The perimeter of each of the three rectangles is 24 cm.

What is the perimeter of the square, in centimetres?

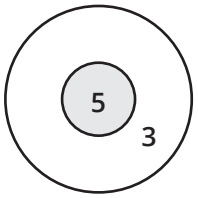


4C. Hitting the inner circle scores 5 points.

Hitting the outer circle scores 3 points.

I scored a total of 26 points after 7 attempts.

How many times did I miss the target completely?



4D. You are counting from 1 to 100, and writing the numbers in the table as shown.

What is the letter (*A*, *B*, *C* or *D*) at the top of the column that contains the number 100?

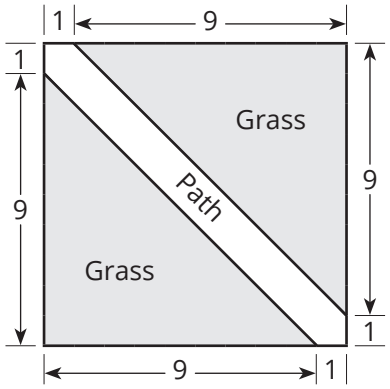
A	*B*	*C*	*D*
1	2	3	4
7	6	5	
	8	9	10
...	12	11	

4E. Each of the sides of a grassy square park is 10 metres long.

The council is building a straight path connecting two diagonally opposite corners, as shown in the diagram.

The long edges of the path start and end 1 metre from the corners of the original square of grass.

What is the area of the path in square metres?



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MATHS GAMES

WEDNESDAY 10 SEPTEMBER 2025

MATHS GAMES
SENIOR

4A.

Student Name:

4B.

Fold here. Keep your answers hidden.

4C.

4D.

4E.

APSMO

MATHS GAMES

WEDNESDAY 10 SEPTEMBER 2025

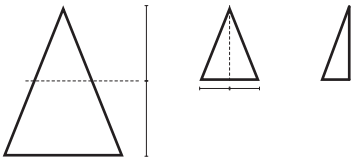
MATHS GAMES
SENIOR

Solutions and Answers

(Items in parentheses are not required)

4A: ⅛	**4B:** 36 (cm)	**4C:** 1	**4D:** *D*	**4E:** 19 (m^2)

4A. What fraction of the area of the original triangle remains?

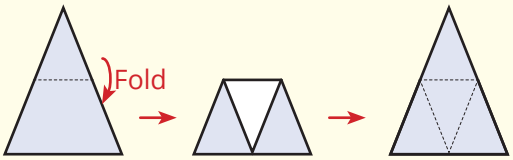


Strategy 1: **Divide a Complex Shape**

We can imagine folding the triangle in half horizontally.

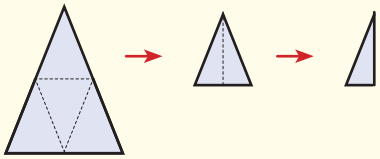
By doing so, we would end up with four identical triangles.

Each of these triangles is ¼ of the area of the original triangle.

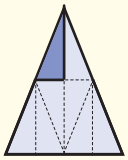


After halving the topmost triangle vertically, we would have two identical (mirror-image) triangles.

Each of these triangles is ½ of the area of the ¼-sized triangle.



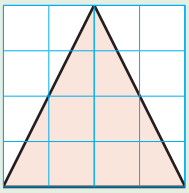
The remaining area is ½ × ¼ = ⅛ of the area of the original triangle.



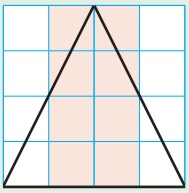
Strategy 2: **Convert to a More Convenient Form**

The question did not specify the dimensions of the isosceles triangle.

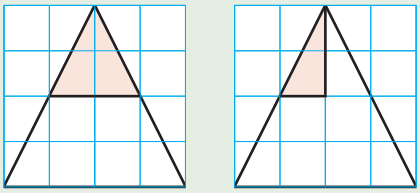
That being the case, we can try a specific isosceles triangle.



By rearranging the shaded regions within the grid, we can see that the area of our original triangle is 8 square units.

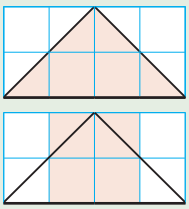


After halving horizontally and halving vertically, the area is **1** square unit.

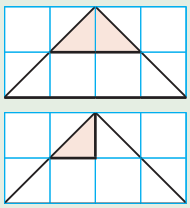


Suppose it was a different isosceles triangle.

The area of our original triangle is **4** square units.



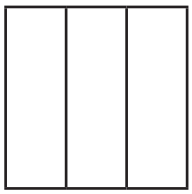
After halving horizontally and vertically, the area is ½ of a square unit.



In both cases, the area of the resulting triangle is ⅛ of the area of the original triangle.

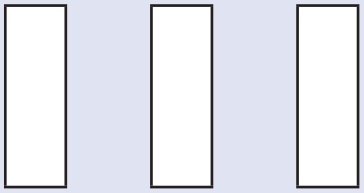
Follow-Up: *When the triangle is cut horizontally to create a triangle with half of the original height, a trapezium is also created. What fraction of the original triangle is the area of the trapezium?* [¾]

4B. What is the perimeter of the square, in centimetres?

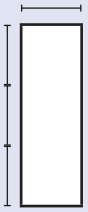


Strategy: **Divide a Complex Shape**

Each of the three identical rectangles has a perimeter of **24** cm.

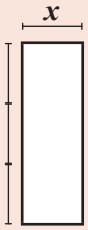


Since each rectangle is one-third of the square, the length of each vertical side is three times the length of each horizontal side.



Method 1: Reason algebraically.

Let x represent the length of the horizontal side.



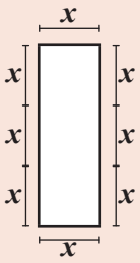
The perimeter of each of these rectangles is $8x$.

We now have:

$8x = 24$ cm

Dividing both sides by 8:

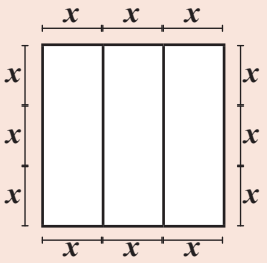
$x = 3$ cm



The perimeter of the square is equal to

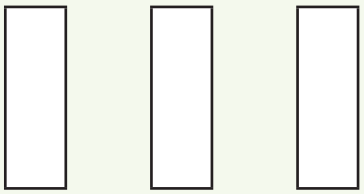
$12x = 12 \times 3$ cm

$= \mathbf{36}$ cm.



Method 2: Subtract the extra side lengths.

The total perimeter of the three rectangles is 3×24 cm = 72 cm.

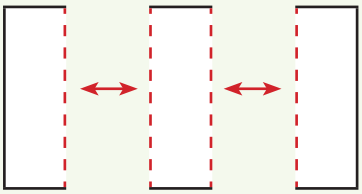


Included in this **72** cm length are the **4** sides of the square, plus 4 extra sides that only belong to the rectangles.

72 cm comprises $4 + 4 = 8$ side lengths of the square.

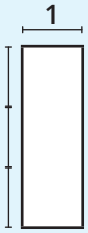
Therefore, each side of the square is $72 \div 8 = 9$ cm long.

The perimeter of the square is 4×9 cm = **36** cm.



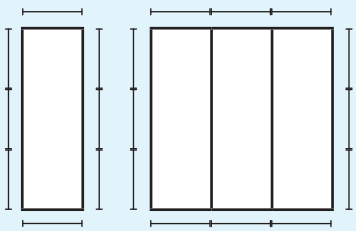
Method 3: Solve a simpler related problem.

Suppose the length of the horizontal side is **1** unit long.



The perimeter of the rectangle would then be **8** units long.

The perimeter of the original square would be **12** units long.

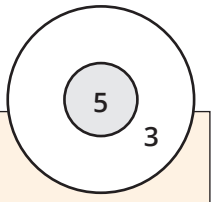


The perimeter of the square is $12 \div 8 = 1.5$ times the perimeter of the rectangle.

Since the rectangle has a perimeter of **24** cm, the perimeter of the square would be 1.5×24 cm = **36** cm.

Follow-Up: *Suppose the perimeter of each of the 3 rectangles was 5 cm. What is the perimeter of the square?* [7.5 cm]

4C. How many times did I miss the board completely?



Strategy 1: **Convert to a More Convenient Form, and Build a Table**

Every time I hit the target, I will score at least **3** points.

Hitting the inner circle scores an extra **5 – 3 = 2** points.

Let's begin by supposing that I hit the target on every one of the **7** attempts.

Attempts that hit the target	Base score	Extra score	Hits on the inner circle
7	7 × 3 = 21	26 – 21 = 5	5 ÷ 2 = 2.5

It is not possible to score **26** if I hit the target on each of the **7** attempts.

Suppose I hit the target on **6** attempts.

If so, I would score **6 × 3 + 4 × 2 = 26** if I hit the inner circle **4** times.

Attempts that hit the target	Base score	Extra score	Hits on the inner circle
7	7 × 3 = 21	26 – 21 = 5	5 ÷ 2 = 2.5
6	6 × 3 = 18	26 – 18 = 8	8 ÷ 2 = 4

Continuing on with this method, we can see that the next time we could score **26** is when we hit the target **4** times, and hit the inner circle **7** times.

Attempts that hit the target	Base score	Extra score	Hits on the inner circle
7	7 × 3 = 21	26 – 21 = 5	5 ÷ 2 = 2.5
6	6 × 3 = 18	26 – 18 = 8	8 ÷ 2 = 4
5	5 × 3 = 15	26 – 15 = 11	11 ÷ 2 = 5.5
4	4 × 3 = 12	26 – 12 = 14	14 ÷ 2 = 7

Since it is impossible to hit the inner circle without also hitting the target, this scenario doesn't make any sense.

I scored **26** when I hit the target **6** times, and missed the target completely on **7 – 6 = 1** of my attempts.

Strategy 2: **Guess, Check and Refine**

Let's begin by supposing that I hit the inner circle on every one of my **7** attempts.

If so, my total score would be **7 × 5 = 35**.

Scores 5	7
Scores 3	0
Scores 0	0
Total Score	35

If I miss the inner circle but hit the outer circle once, my score would be **6 × 5 + 1 × 3 = 33**.

My score is reduced by **35 – 33 = 2.**

Scores 5	6
Scores 3	1
Scores 0	0
Total Score	33

If I miss the target completely, just once, my total score would be **6 × 5 + 1 × 0 = 30**.

My score is reduced by **35 – 30 = 5.**

Scores 5	6
Scores 3	0
Scores 0	1
Total Score	30

To end up with a score of **26**, I need to reduce the score by **35 – 26 = 9.**

The only way to combine **2**s and **5**s to make **9** is **2 + 2 + 5 = 9**.

There must have been **1** time when I missed the target.

Scores 5	7	6	6	4
Scores 3	0	1	0	2
Scores 0	0	0	1	1
Total Score	35	33	30	26

Follow-Up: *In how many different ways could I score 26 if I had 10 attempts?*
[2: it could have been 1 × 5 + 7 × 3 + 2 × 0, or 4 × 5 + 2 × 3 + 4 × 0]

4D. What is the letter at the top of the column that contains the number 100?

A	B	C	D
1	2	3	4
7	6	5	
	8	9	10
...	12	11	

***Strategy 1:* Convert to a More Convenient Form**

We can reconfigure the table so that the numbers are used as the column headings.

1	2	3	4	5	6	7	8	9	10	11	12	...
A	*B*	*C*	*D*	*C*	*B*	*A*	*B*	*C*	*D*	*C*	*B*	...

Doing so helps us to identify a repeating pattern:

1	2	3	4	5	6	7	8	9	10	11	12	...
A	*B*	*C*	*D*	*C*	*B*	*A*	*B*	*C*	*D*	*C*	*B*	...

We can see that every **6**th number, starting from **6**, is going to be in column ***B***.

6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96
B	*B*	*B*	*B*	*B*	*B*	*B*	*B*	*B*	*B*	*B*	*B*	*B*	*B*	*B*	*B*

90 and **96** are both multiples of **6**, so they will both occur at the end of the pattern.

90	91	92	93	94	95	96	97	98	99	100
B	*A*	*B*	*C*	*D*	*C*	*B*	*A*	*B*	*C*	*D*

By extending the pattern to **100**, the letter at the top of the column that contains the number **100** is ***D***.

***Strategy 2:* Find a Pattern, and Reason Arithmetically**

As can be seen in ***Strategy 1***, the pattern repeats for every **6** numbers.

This pattern can be identified in a few different ways.

For example, we can shift the ***A*** and ***D*** numbers slightly to align with a particular row.

A	B	C	D
1	2	3	4
	6	5	
7	8	9	10
	12	11	

A	B	C	D
1	2	3	
	6	5	4
7	8	9	
	12	11	10

We want to find the column for the **100**th number.

If **100** was a multiple of **6**, then we would know that it would occur in Column ***B***.

A	B	C	D
1	2	3	
	6	5	4
7	8	9	
	12	11	10

Using a division algorithm to find **100 ÷ 6**:

$$\begin{array}{r} 16 \\ 6\overline{)100} \\ \underline{60} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

100 ÷ 6 = 16 r4.

Therefore, **100 – 4 = 96** must be a multiple of **6**.

Since **96** is a multiple of **6**, we can continue the pattern from **96** to find the column for **100**.

The column for **100** is ***D***.

A	B	C	D
91	92	93	
	96	95	94
97	98	99	
	102	101	100

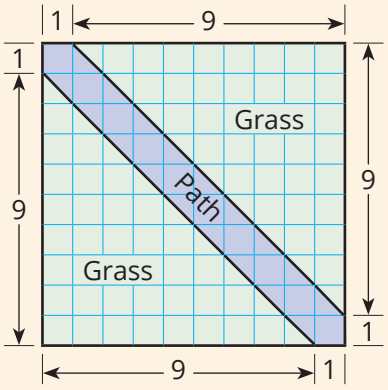
Follow-Up: *What is the letter at the top of the column containing 1000?* [*D*]

4E. What is the area of the path, in square metres?

Strategy 1: **Divide a Complex Shape**

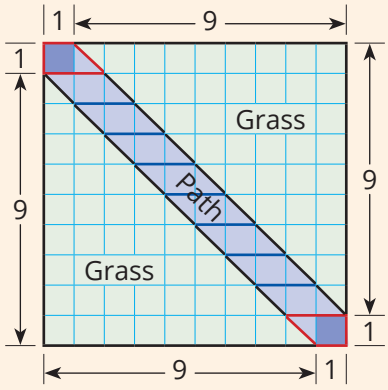
The park is a square, **10** m long and **10** m wide.

We can divide the park into **100** **1** m × **1** m square units.



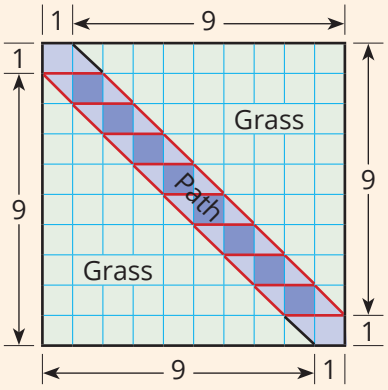
We can consider horizontal stripes across the path, where each stripe is **1** m thick.

The top and bottom stripes each have an area of 1.5 m².



The other **8** stripes each have an area of 2 m².

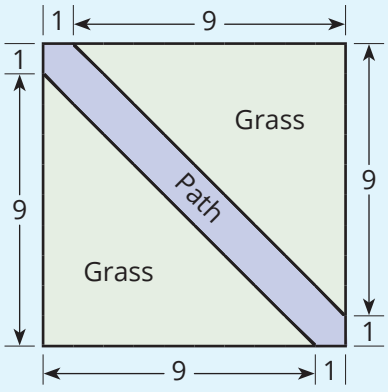
The area of the path is
2 × 1.5 m² + 8 × 2 m² = **19** m².



Strategy 2: **Divide a Complex Shape (Alternative Approach)**

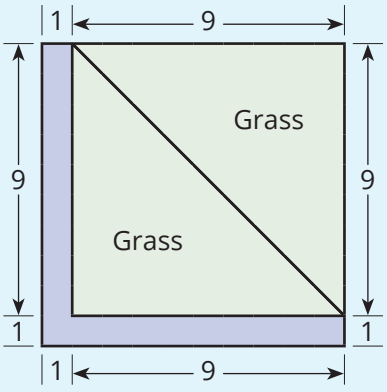
The original area of the grass area is
10 m × **10** m = **100** m².

After putting in the path, the grass area becomes two triangles, each **9** m long and **9** m wide.

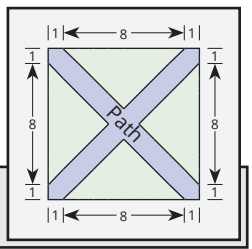


We can combine the two grass triangle areas into a square measuring **9** m × **9** m = **81** m².

The path must occupy the amount of area that is not included in the new grass area.



The area of the path is **100** m² – **81** m² = **19** m².



Follow-Up: *Suppose the council decided to build paths on both diagonals. What is the total area occupied by the two paths?* [36 m²]