MATHS GAMES

APSMO

2025 MATHS GAMES

Important

The information contained in this file is ONLY for the use of registered participants of the 2025 APSMO Maths Games.

Organisation and Procedures

For full details, see the Members' Area

- Maths Games papers are to be conducted under test conditions.

DO	DO NOT
• Supervise students at all times. • Maintain silence. • Provide blank working paper. • Collect, mark and retain the papers.	• Print the papers prior to the scheduled date. • Read the questions aloud to the students. • Interpret the questions for students. • Permit any discussion or movement around the room. • Permit the use of calculators or other electronic devices.

- Papers should be scored by the PICO using the *Solutions and Answers* sheet provided.
- Original student answer sheets should be retained by the PICO until the end of the year.

Absent Students

- A student who is legitimately absent on the date of the Maths Games paper, may sit the paper on their return to school.
- If an absent student does not sit the paper on their return to school they should be marked as 'absent'.
- *Note: This policy differs from the Maths Olympiads Absent Student Policy which has additional requirements*.

APSMO

WEDNESDAY 30 JULY 2025

MATHS GAMES

MATHS GAMES SENIOR

Suggested Time: ***30 Minutes.*** *Calculators NOT Permitted.*

Write your answers in the boxes on the back.

←

Keep your answers hidden by folding backwards on this line.

3A. What is the value of 123 × 26 + 76 × 26 + 199 × 74?

3B. Consider the two fractions:

$\frac{n}{n+1}$ and $\frac{n+1}{n+2}$, where n is a positive integer.

Which fraction has the larger value?

3C. Three commercial knitting machines take 5 hours to produce 75 pairs of socks.

How long will it take two of these machines to produce 100 pairs of socks?

3D. Alex, Bianca, Sofia and Taylor are 10, 11, 12, and 13 years old, but not necessarily in that order.

Exactly one of the following four statements is false:

1. Taylor is more than one year older than Bianca.
2. Sofia is one year older than Alex.
3. Bianca is two years older than Alex.
4. Sofia is two years older than Bianca.

Who is the youngest?

3E. 12 is a multiple of 4.

10 is a multiple of 5.

20 is a multiple of 4, and also a multiple of 5.

What fraction of two-digit numbers are multiples of 4, 5, or both?

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WEDNESDAY 30 JULY 2025

MATHS GAMES

SENIOR

3A.

3B.

3C.

3D.

3E.

Student Name:

Fold here. Keep your answers hidden.

APSMO
MATHS GAMES
WEDNESDAY 30 JULY 2025

MATHS GAMES
SENIOR

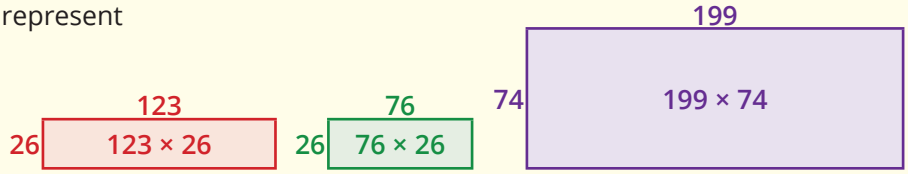
Solutions and Answers
(Items in parentheses are not required)

3A: 19900	**3B:** $\frac{n+1}{n+2}$	**3C:** 10 (hrs)	**3D:** Bianca	**3E:** $\frac{2}{5}$

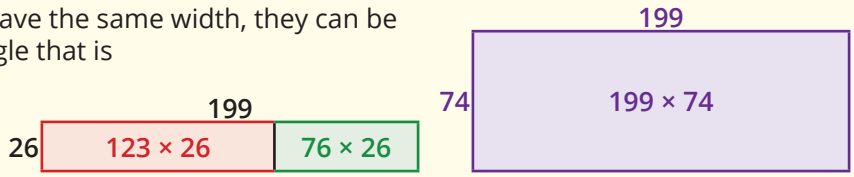
3A. What is the value of 123 × 26 + 76 × 26 + 199 × 74?

Strategy 1: **Solve a Simpler Related Problem**

We can use area diagrams to represent
- 123 × 26,
- 76 × 26, and
- 199 × 74.



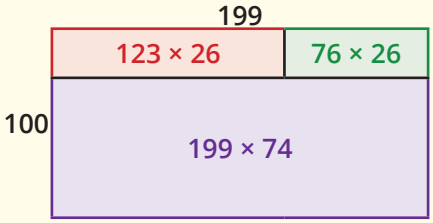
Since both 123 × 26 and 76 × 26 have the same width, they can be combined to form a single rectangle that is
- (123 + 76) = 199 units long, and
- 26 units wide.



Both 199 × 26 and 199 × 74 have the same length.

They can be combined to form a single rectangle that is
- 199 units long, and
- (26 + 74) = 100 units wide.

(123 + 76) × 26 + 199 × 74 = 199 × (26 + 74) = 199 × 100.



To find the value of 199 × 100, we can take advantage of our decimal place value system.

To multiply a number by 10, we would move all of the digits up by one place value.

To multiply a number by 100, we would move all of the digits up by two place values.

	10000s	1000s	100s	10s	1s
199 × 1:			1	9	9
199 × 10:		1	9	9	0
199 × 100:	1	9	9	0	0

123 × 26
+ 76 × 26
+ 199 × 74
= 199 × 100
= **19900**.

Strategy 2: **Use a Written Algorithm**

To find the value of 123 × 26 + 76 × 26 + 199 × 74, we can perform each multiplication separately.

123 × 26 + 76 × 26 + 199 × 74
= 3198 + 1976 + 14726
= **19900**.

1 2 3	7 6	1 9 9	1 2 2
× 2 6	× 2 6	× 7 4	3 1 9 8
7 3 8	4 5 6	7 9 6	1 9 7 6
2 4 6 0	1 5 2 0	1 3 9 3 0	+ 1 4 7 2 6
3 1 9 8	1 9 7 6	1 4 7 2 6	**1 9 9 0 0**

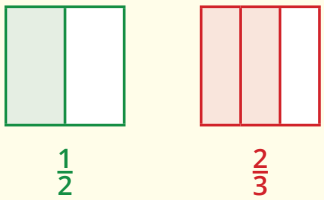
Follow-Up: *What is the value of 199 × 123 – 123 × 23 – 76 × 23?* [199 × (123 – 23) = 199 × 100 = 19900]

3B. Which fraction has the larger value: $\frac{n}{n+1}$, or $\frac{n+1}{n+2}$?

Strategy 1: **Solve a Simpler Related Problem, and Find a Pattern**

Suppose $n = 1$. The question would then be:

Which is larger: $\frac{1}{2}$, or $\frac{2}{3}$?

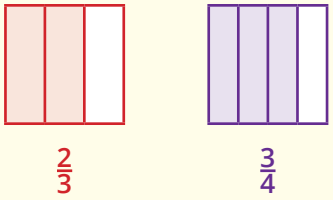


$\frac{1}{2}$ $\frac{2}{3}$

We can see that $\frac{2}{3} > \frac{1}{2}$.

To check this idea, we can try the question with $n = 2$.

Which is larger: $\frac{2}{3}$, or $\frac{3}{4}$?

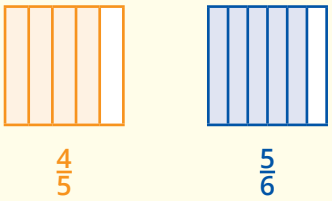


$\frac{2}{3}$ $\frac{3}{4}$

Here, we see that $\frac{3}{4} > \frac{2}{3}$.

Suppose $n = 4$.

Which is larger: $\frac{4}{5}$, or $\frac{5}{6}$?



$\frac{4}{5}$ $\frac{5}{6}$

We can see that $\frac{5}{6} > \frac{4}{5}$.

In each case we can see that, of the two fractions $\frac{n}{n+1}$ and $\frac{n+1}{n+2}$, $\frac{n+1}{n+2}$ has the larger value.

Strategy 2: **Solve a Simpler Related Problem (Alternative Approach)**

We can rewrite $\frac{n}{n+1}$ as follows:

$$\frac{n}{n+1} = \frac{n+1}{n+1} - \frac{1}{n+1}$$

$$= 1 - \frac{1}{n+1}.$$

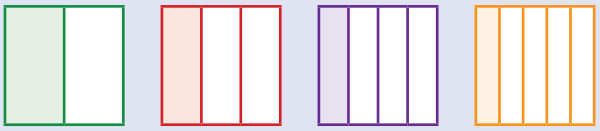
Likewise, we can rewrite $\frac{n+1}{n+2}$ as:

$$\frac{n+1}{n+2} = \frac{n+2}{n+2} - \frac{1}{n+2}$$

$$= 1 - \frac{1}{n+2}.$$

In each case, we are removing a unit fraction from the whole (1).

We can represent the quantity being removed diagrammatically:



$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$

As the denominator increases, the size of the fraction decreases.

Comparing $\frac{1}{n+1}$ and $\frac{1}{n+2}$, $\frac{1}{n+2}$ is smaller. The removal of $\frac{1}{n+2}$ reduces the value by a lesser amount.

$\frac{n+1}{n+2}$ is greater than $\frac{n}{n+1}$.

Strategy 3: **Draw a Diagram**

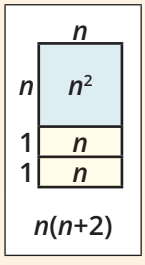
We can begin by creating equivalent fractions with the same denominator.

$$\frac{n}{n+1} = \frac{n(n+2)}{(n+1)(n+2)}, \text{ and}$$

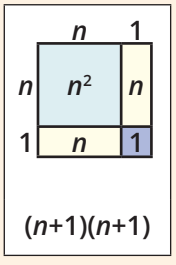
$$\frac{n+1}{n+2} = \frac{(n+1)(n+1)}{(n+1)(n+2)}.$$

To compare these fractions, we could just compare $n(n+2)$ and $(n+1)(n+1)$.

Using area diagrams to represent $n(n+2)$ and $(n+1)(n+1)$:



$n(n+2)$



$(n+1)(n+1)$

We can see that the area of $(n+1)(n+1)$ exceeds the area of $n(n+2)$ by 1 square unit.

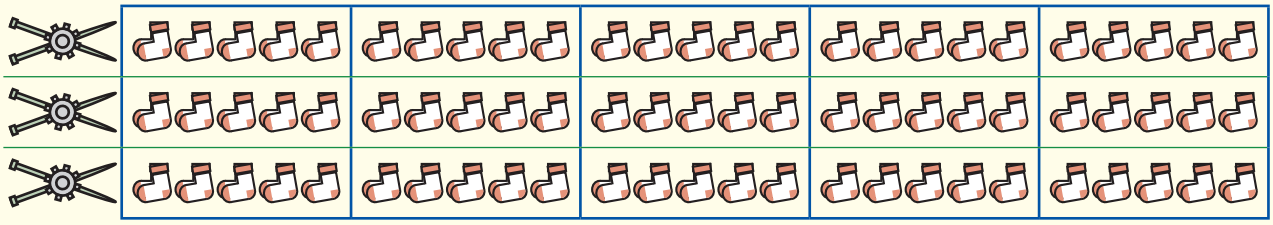
The larger value is $\frac{(n+1)(n+1)}{(n+1)(n+2)} = \frac{n+1}{n+2}$.

Follow-Up: *Suppose n is negative. Which is greater: n(n + 2) or (n + 1)(n + 1) ?* [(n + 1)(n + 1)]

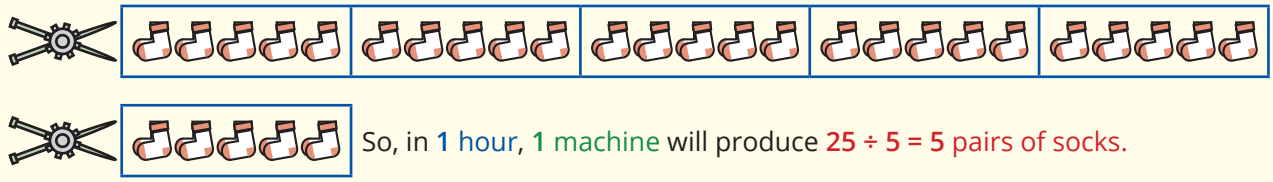
3C. How many hours will it take two of these machines to produce 100 pairs of socks?

Strategy 1: **Solve a Simpler Related Problem**

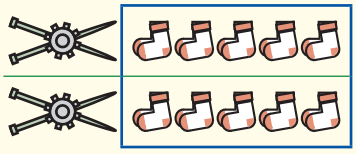
If 3 machines take 5 hours to produce 75 pairs of socks,



then 1 machine will take 5 hours to produce 75 ÷ 3 = 25 pairs of socks.



So, in 1 hour, 1 machine will produce 25 ÷ 5 = 5 pairs of socks.



In 1 hour, 2 machines will produce 2 × 5 = 10 pairs of socks.

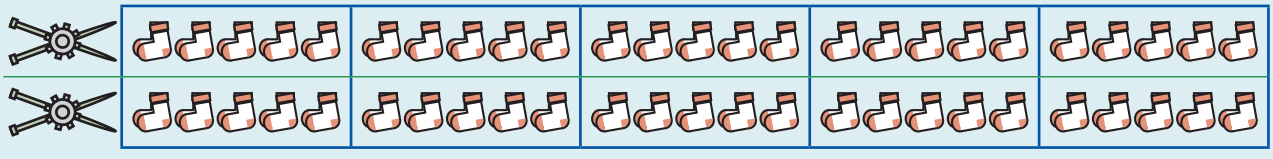
For those 2 machines to produce 100 pairs of socks,

it will take 100 ÷ 10 = **10 hours**.

Strategy 2: **Solve a Simpler Related Problem (Alternative Approach)**

From Strategy 1, 1 machine will take 5 hours to produce 75 ÷ 3 = 25 pairs of socks.

2 machines will take 5 hours to produce 2 × 25 = 50 pairs of socks.



If 2 machines take 5 hours to produce 50 pairs of socks, then we just need to run those 2 machines for another 5 hours to get the total pairs up to 100.

It will take 5 + 5 = **10 hours** for 2 machines to produce 100 pairs of socks.

Follow-Up: *How many machines would need to be operating to produce 80 pairs of socks in 3 hours?* [6. It is not possible to have a fractional machine.]

3D. Who is the youngest, if we know that exactly one of the given statements is false?

1. Taylor is more than one year older than Bianca.
2. Sofia is one year older than Alex.
3. Bianca is two years older than Alex.
4. Sofia is two years older than Bianca.

Strategy: **Eliminate All But One Possibility**

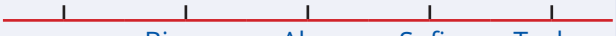
Scenario 1: Statement 1 is false; all other statements are true.

2. Sofia is one year older than Alex.	
3. Bianca is two years older than Alex.	
4. Sofia is two years older than Bianca.	→ This is impossible.

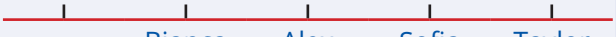
Scenario 2: Statement 2 is false; all other statements are true.

1. Taylor is more than one year older than Bianca.	
3. Bianca is two years older than Alex.	→ This is impossible; there would be at least 4 years between Alex and Taylor.

Scenario 3: Statement 3 is false; all other statements are true.

1. Taylor is more than one year older than Bianca.	
2. Sofia is one year older than Alex.	
4. Sofia is two years older than Bianca.	→ This statement works for the above scenario.
3. "Bianca is two years older than Alex" **is a FALSE statement**.	→ In the above scenario, Bianca is NOT two years older than Alex.

Scenario 4: Statement 4 is false; all other statements are true.

1. Taylor is more than one year older than Bianca.	
2. Sofia is one year older than Alex.	
3. Bianca is two years older than Alex.	→ This is impossible.

It must be the case that Statement 3 is false.

According to Scenario 3, where we assumed that Statement 3 is false, **Bianca** is the youngest.

Follow-Up: *Suppose Statement 1 was "Taylor is one year older than Bianca". One statement is false. Who is the youngest?* [Alex]

3E. What fraction of two-digit numbers are multiples of 4, 5, or both?

Strategy 1: **Eliminate All But One Possibility**

We are interested in two-digit numbers only. There are **9 × 10 = 90** two-digit numbers.

10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

To find multiples of **4** we would identify every **4**th number starting from **12**.

10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

To find multiples of **5** we would identify every **5**th number starting from **10**.

10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

To count the multiples of **4** and **5** in an organised way, we can (for example) see that there are **4** of these multiples from **10** to **19** inclusive.

10	11	12	13	14	15	16	17	18	19

Likewise, there are **4** of these multiples from **20** to **29** inclusive.

20	21	22	23	24	25	26	27	28	29

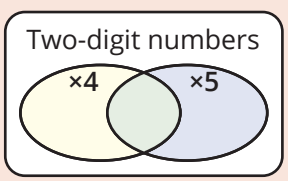
We can see that there are **4** of these multiples for every subsequent set of **10** numbers.

The fraction is therefore $\frac{4}{10} = \frac{2}{5}$.

Strategy 2: **Reason Logically**

There are **99** numbers less than **100**.

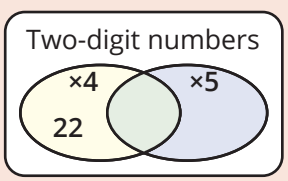
The nine numbers **1 – 9** have **1** digit only, so there are **99 – 9 = 90** two-digit numbers.



100 = 25 × 4, so **24** multiples of **4** are less than **100**.

Two of them (**4** and **8**) have **1** digit only.

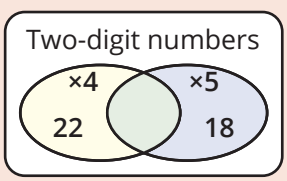
There are **24 – 2 = 22** two-digit multiples of **4**.



100 = 20 × 5, so **19** multiples of **5** are less than **100**.

One of them (**5**) has **1** digit only.

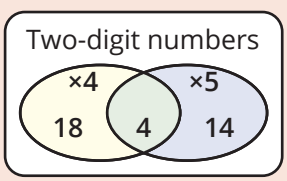
There are **19 – 1 = 18** two-digit multiples of **5**.



Multiples of **4** and **5** will also be multiples of **4 × 5 = 20**.

There are **4** such values: **20, 40, 60** and **80**.

Those **4** values belong in the overlap region.



There are **18 + 4 + 14 = 36** multiples of **4** and **5**, so as a fraction of all two-digit numbers this is $\frac{36}{90} = \frac{2}{5}$.

Follow-Up: *What fraction of two-digit numbers are multiples of 4, 5, and/or 6?* [41/90]