APSMO

MATHS GAMES

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2025 MATHS GAMES

IMPORTANT

The information contained in this file is ONLY for the use of registered participants of the 2025 APSMO Maths Games.

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2025 MATHS GAMES

ORGANISATION AND PROCEDURES

For full details, see the Members' Area

- Maths Games papers are to be conducted under test conditions.

DO	DO NOT
• Supervise students at all times. • Maintain silence. • Provide blank working paper. • Collect, mark and retain the papers.	• Print the papers prior to the scheduled date. • Read the questions aloud to the students. • Interpret the questions for students. • Permit any discussion or movement around the room. • Permit the use of calculators or other electronic devices.

- Papers should be scored by the PICO using the *Solutions and Answers* sheet provided.
- Original student answer sheets should be retained by the PICO until the end of the year.

ABSENT STUDENTS

- A student who is legitimately absent on the date of the Maths Games paper, may sit the paper on their return to school.
- If an absent student does not sit the paper on their return to school they should be marked as 'absent'.
- *Note: This policy differs from the Maths Olympiads Absent Student Policy which has additional requirements*.

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MATHS GAMES

WEDNESDAY 11 JUNE 2025

MATHS GAMES SENIOR

Suggested Time: ***30 Minutes.*** *Calculators NOT Permitted.*

Write your answers in the boxes on the back.

Keep your answers hidden by folding backwards on this line.

2A. In the school elections, there are three positions: Captain, Vice Captain, and Prefect. Students can only nominate for one position.

Two students have nominated for Captain.

Three students nominated for Vice Captain.

Two students nominated for Prefect.

How many different student leadership groups are possible?

2B. Amith, Benjamin, Cain, Daniel and Ethan line up at the canteen.

When the bell for the end of recess rang, only Benjamin and Daniel had not been served.

In how many different ways could the five students have stood in line?

2C. Sanvi needs to achieve a particular average (mean) result over 5 tests.

Her marks over the first four tests were 63, 75, 80, and 82.

Her fifth result was 90.

While she was happy with her progress, she knew that she didn't have the average result she needed.

What did she need to achieve on the fifth test, so that her average would have been 2 marks higher?

2D. At the music academy, there are 34 students who play the piano, and 26 students who play the guitar.

10 of the students play both piano and guitar.

Mr Marsh selected 20 students to attend a concert.

He chose half of the students who play only the piano, and one-fifth of the students who play both instruments.

What fraction of the students who play only guitar did Mr Marsh select?

Give your answer in lowest terms.

2E. The sum of the digits of the number 123 is 6.

How many 3-digit numbers have the sum of their digits equal to 6, including 123?

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MATHS GAMES

WEDNESDAY 11 JUNE 2025

MATHS GAMES SENIOR

2A.

Student Name:

2B.

Fold here. Keep your answers hidden.

2C.

2D.

2E.

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WEDNESDAY 11 JUNE 2025

MATHS GAMES SENIOR

Solutions and Answers

(Items in parentheses are not required)

2A: 12	2B: 12	2C: 100	2D: $\frac{3}{8}$	2E: 21

2A. How many different student leadership groups are possible?

Strategy 1: **Build a Table, and Make an Organised List**

Let's call the captain nominees *A* and *B*. Let's also name each of the vice captain nominees *P*, *Q* and *R*, and each of the prefect nominees *X* and *Y*.

We can begin by building a table that gives us all of the possible combinations for the captain and vice captain positions.

	Vice *P*	Vice *Q*	Vice *R*
Captain *A*	*A, P*	*A, Q*	*A, R*
Captain *B*	*B, P*	*B, Q*	*B, R*

This means there are **2** × **3** = **6** different ways to choose the captain and vice captain positions.

To add the options for the prefect position, we can build a new table, using the results of the first table for the first column.

There are **6** possible ways to select the captain and vice captain, and **2** possible prefects.

Therefore, there are **6** × **2** = **12** possible different student leadership groups.

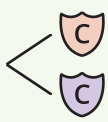
	Prefect *X*	Prefect *Y*
A, P	*A, P, X*	*A, P, Y*
A, Q	*A, Q, X*	*A, Q, Y*
A, R	*A, R, X*	*A, R, Y*
B, P	*B, P, X*	*B, P, Y*
B, Q	*B, Q, X*	*B, Q, Y*
B, R	*B, R, X*	*B, R, Y*

Strategy 2: **Draw a Diagram**

We can draw a tree diagram to list the choices for each position in the leadership team.

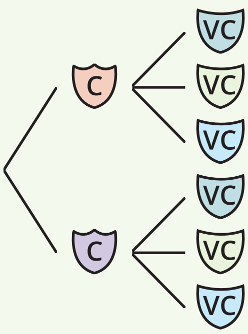
We can start by choosing the captain.

There are **2** nominees for captain.



Next we choose the vice captain.

Since there are **3** nominees, there are **3** possible vice captains for each choice of captain.

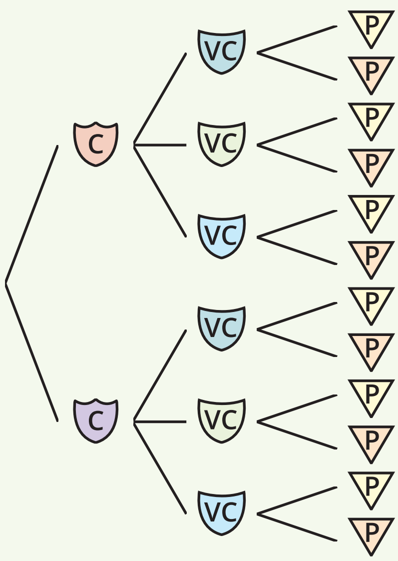


Finally, we choose the prefect from **2** nominees.

There are **2** possible prefects for each choice of captain and vice captain.

With our finished tree diagram, we can count all of the different combinations.

There are **12** possible different student leadership groups.



Follow-Up: *Suppose there were four nominations for captain, three for vice-captain, and five for prefect. How many different leadership groups are possible?* [60]

2B. In how many different ways could the five students have stood in line?

Strategy 1: **Make an Organised List**

Five students waited in line.

At the end of recess, Benjamin and Daniel had not been served, so they must have been at the end of the line:

			B	*D*

or

			D	*B*

Amith, Cain and Daniel occupied the first **3** positions in the line.

If Amith is first in line, there are **2** ways to arrange Cain and Daniel.

A	*C*	*D*		
A	*D*	*C*		

There are likewise **2** possible arrangements for Amith and Daniel if Cain was first in line,

C	*A*	*D*		
C	*D*	*A*		

and **2** possible arrangements for Amith and Cain if Daniel was first in line.

D	*A*	*C*		
D	*C*	*A*		

There are **6** arrangements for [*A*, *C*, *D*] and **2** arrangements for [*B*, *D*].

In total, there are **6 × 2 = 12** possible ways for the five students to have stood in line.

A	*C*	*D*	*B*	*D*	*A*	*C*	*D*	*D*	*B*
A	*D*	*C*	*B*	*D*	*A*	*D*	*C*	*D*	*B*
C	*A*	*D*	*B*	*D*	*C*	*A*	*D*	*D*	*B*
C	*D*	*A*	*B*	*D*	*C*	*D*	*A*	*D*	*B*
D	*A*	*C*	*B*	*D*	*D*	*A*	*C*	*D*	*B*
D	*C*	*A*	*B*	*D*	*D*	*C*	*A*	*D*	*B*

Strategy 2: **Count in an Organised Way**

Benjamin and Daniel were not served, so we know that the first in line must be Amith, Cain or Daniel.

There are **3** ways to choose the student who is at the front of the line.

3				

One of Amith, Cain and Daniel is standing at the front of the line; so there are **2** students remaining who could be second in line.

3	2			

After placing the student who is second in line, there is just **1** student remaining who was also served before the bell.

3	2	1		

Either Benjamin or Daniel would have to be fourth in line.

3	2	1	2	

There is just **1** student remaining to place.

3	2	1	2	1

There are **3 × 2 × 1 × 2 × 1 = 12** possible ways for the five students to have stood in line.

Follow-Up: *The next day, the same five students stood in line again at the canteen. In how many different ways could they stand ?* [120]

2C. What did Sanvi need to achieve on the fifth test, so that her average would have been 2 marks higher?

Strategy 1: **Work Backwards**

Sanvi's marks over five tests were **63**, **75**, **80**, **82**, and **90**.

Test 1	63
Test 2	75
Test 3	80
Test 4	82
Test 5	90

All together, Sanvi has achieved **63** + **75** + **80** + **82** + **90** = **390** marks over the five tests.

63	75	80	82	90
390				

Her average mark is **390** ÷ **5** = **78**.

78	78	78	78	78

Sanvi needed her average to be **2** marks higher: **78** + **2** = **80**.

This means that she needed to achieve **5** × **80** = **400** marks over the five tests.

80	80	80	80	80
400				

After the first **4** tests, Sanvi had **63** + **75** + **80** + **82** = **300** marks.

63	75	80	82	?
400				

To achieve an average of **80**, Sanvi needed **400** – **300** = **100** marks in the fifth test.

63	75	80	82	100
400				

Strategy 2: **Work Backwards (Alternative Approach)**

Sanvi needed to achieve an average that is **2** marks higher.

This result would have occurred if she had scored **2** marks more on every one of the five tests.

63	75	80	82	90
65	77	82	84	92

Adding **2** marks to each of the five tests has the same effect on the average as adding **5** × **2** = **10** marks to just one of the tests.

In this case, we would be adding the extra **10** marks to Sanvi's fifth test.

63	75	80	82	90 + 10

Sanvi needed **90** + **10** = **100** marks in the fifth test.

Follow-Up: *Suppose Sanvi did not need to count her worst mark. What would her average have been?* [81.75]

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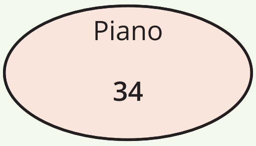
MATHS GAMES SENIOR

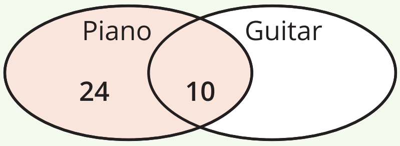
2D. What fraction of the students who play only guitar did Mr Marsh select?

Strategy 1: **Draw a Diagram, and Work Backwards**

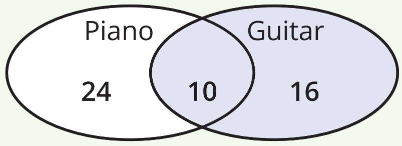
There are **34** students who play piano.



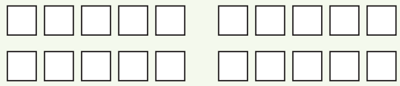
10 of those students play both piano and guitar.



There are another **26 – 10 = 16** students who only play guitar.

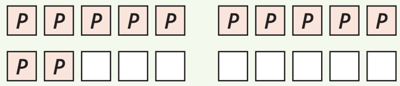


Mr Marsh has **20** concert tickets.



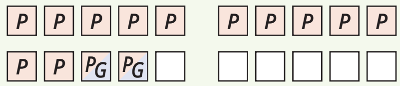
He selected half of the students who only play piano.

With **34 – 10 = 24** students who only play piano, Mr Marsh has selected **24 ÷ 2 = 12** students.



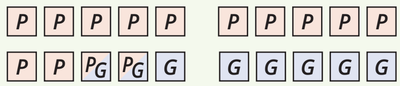
Mr Marsh selected one-fifth of the students who play both instruments.

With **10** students who play both instruments, Mr Marsh has selected another **10 ÷ 5 = 2** students.



There are **20 – 12 – 2 = 6** concert tickets remaining.

These were given to students who only play guitar.



Mr Marsh selected $\frac{6}{16} = \frac{3}{8}$ of the students who only play guitar.

Strategy 2: **Reason Logically**

If Mr Marsh wrote a list of the students who play piano, he would have **34** on the list.

There are **26** students on the list of guitar players.

Piano	Guitar
34	26

From each list, we can remove the **10** students who play both instruments.

Those students would be included on a separate list.

Piano	Both	Guitar
24	10	16

Mr Marsh selected **24 ÷ 2 = 12** piano-only students.

He also selected **10 ÷ 5 = 2** students who play both instruments.

Piano	Both	Guitar
12 / 24	2 / 10	/ 16

There are **20 – 12 – 2 = 6** concert tickets remaining, to be given to students who only play guitar.

Mr Marsh selected $\frac{6}{16} = \frac{3}{8}$ of the students who only play guitar.

Follow-Up: *One of the students was unwell and could not go. Mr Marsh selected a student at random to replace them. What is the probability that the selected student plays both piano and guitar ?* [8 / (12 + 8 + 10) = 4/15]

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MATHS GAMES SENIOR

2E. How many 3-digit numbers have the sum of their digits equal to 6, including 123?

Strategy 1: **Make an Organised List**

We can begin by considering 3-digit numbers with 1 in the hundreds place.

The sum of the tens place digit and the ones place digit would be **6 – 1 = 5**.

H	*T*	*O*
1	0	5
1	1	4
1	2	3
1	3	2
1	4	1
1	5	0

For 3-digit numbers with 2 in the hundreds place, the sum of the tens place digit and the ones place digit would be **6 – 2 = 4**.

H	*T*	*O*
2	0	4
2	1	3
2	2	2
2	3	1
2	4	0

For 3-digit numbers with 3 in the hundreds place, the sum of the tens place digit and the ones place digit would be **6 – 3 = 3**.

H	*T*	*O*
3	0	3
3	1	2
3	2	1
3	3	0

By listing the numbers in this way, we can observe a pattern.

H	*T*	*O*
4	0	2
4	1	1
4	2	0

H	*T*	*O*
5	0	1
5	1	0

H	*T*	*O*
6	0	0

	+1	+1	+1	+1	+1	
Hundreds digit	1	2	3	4	5	6
3-digit numbers with digit sum of 6	6	5	4	3	2	1
	–1	–1	–1	–1	–1	

There are **6 + 5 + 4 + 3 + 2 + 1 = 21** 3-digit numbers where the sum of the digits equals **6**.

Strategy 2: **Make an Organised List (Alternative Approach)**

We can begin by considering sets of 3 digits that add up to **6**.

To avoid double-counting, we will list each set of 3 digits in ascending order.

0	0	6
0	1	5
0	2	4
0	3	3
1	1	4
1	2	3
2	2	2

There are **3** sets of digits where the digits are all different (***A***, ***B***, ***C***).

0	0	6
0	1	5
0	2	4
0	3	3
1	1	4
1	2	3
2	2	2

Since there are **6** different ways to arrange ***A***, ***B***, ***C***, there are **6** different ways to arrange each of these **3** sets of digits.

A	*B*	*C*
A	*C*	*B*
B	*A*	*C*
B	*C*	*A*
C	*A*	*B*
C	*B*	*A*

There are **3** sets of digits where exactly two of the the digits are the same (***A***, ***A***, ***B***).

Notice that there are **3** different possible positions for digit ***B***.

0	0	6
0	1	5
0	2	4
0	3	3
1	1	4
1	2	3
2	2	2

There are **3** different ways to arrange each of these **3** sets of digits.

A	*A*	*B*
A	*B*	*A*
B	*A*	*A*

There is **1** set of digits where all of the digits are the same (**2**, **2**, **2**).

In total, there are **3 × 6 + 3 × 3 + 1 = 28** ways to create strings of 3 digits, where the sum of the digits is equal to **6**.

0	0	6
0	1	5
0	2	4
0	3	3
1	1	4
1	2	3
2	2	2

A	*A*	*B*
A	*B*	*A*
B	*A*	*A*

However, a string of 3 digits would not form a valid 3-digit number if the first digit is zero.

There are **7** such arrangements.

0	0	6
0	1	5
0	2	4
0	3	3
0	4	2
0	5	1
0	6	0

There are **28 – 7 = 21** 3-digit numbers where the sum of the digits equals **6**.

Follow-Up: *How many 3-digit numbers have the sum of their digits equal to 7?* [28]