# 2025 Maths Games Junior - Years 5 & 6
# Resource Kit 1
# Teaching Problem Solving

## Problem Solving Strategies

This resource kit follows on from the Preparation Kit and its emphasis on:

Guess, Check and Refine

Draw a Diagram

The problems are sourced from previous Junior (Division J) Maths Olympiads and Maths Games papers.

They introduce two new problem solving strategies:

### 1. Find a Pattern

One of the most frequently used problem solving strategies is that of recognising and extending a pattern.

Students can often simplify a difficult problem by identifying a pattern in it, and then applying that pattern to the problem situation.

### 2. Build a Table

A table displays information so that it is easily located and understood, and missing information becomes obvious.

If students are not given the data for a problem, and must generate it themselves, a table is an excellent way to record what they have done so they don't have to repeat their efforts.

A table can also be invaluable for detecting significant patterns.

### Resource Kit 1 focuses on:

Find a Pattern

Build a Table

#### Set Yellow

Problems with fully worked solutions.

#### Set Green

Problems similar to Set Yellow, but with fewer difficult elements.

#### Preparation Tasks

Tasks designed to encourage discussion and collaboration in problem solving.

#### Set Orange

More challenging problems to extend students.

Further questions and solution methods can be found in the APSMO resource book "Building Confidence in Maths Problem Solving", available from www.apsmo.edu.au.

## How to use these problems

At the start of the lesson, present the problem and ask the students to think about it. Encourage students to try to solve it in any way they like. When the students have had enough time to consider their solutions, ask them to describe or present their methods, taking particular note of different ways of arriving at the same solution.

Each question includes at least one solution method that the majority of students should be able to follow. By participating in lessons that demonstrate achievable problem solving techniques, students may gain increased confidence in their own ability to address unfamiliar problems.

Finally, the consideration of different solution methods is fundamental to the students' development as effective and sophisticated problem solvers. Even when students have solved a problem to their own satisfaction, it is important to expose them to other methods and encourage them to judge whether or not the other methods are more efficient.

## Preparation Kit

### Guess, Check and Refine

This involves making a reasonable guess of the answer, and checking it against the conditions of the problem. An incorrect guess may provide more information that may lead to the answer.

### Draw a Diagram

A diagram may reveal information that may not be obvious just by reading the problem.

It is also useful for keeping track of where the student is up to in a multi-step problem.

## Resource Kit 1

### Find a Pattern

A frequently used problem solving strategy is that of recognising and extending a pattern.

Students can often simplify a difficult problem by identifying a pattern in the problem situation.

### Build a Table

A table displays information so that it is easily located and understood.

A table is an excellent way to record data so the student doesn't have to repeat their efforts.

## Resource Kit 2

### Work Backwards

If a problem describes a procedure and then specifies the final result, this method usually makes the problem much easier to solve.

### Make an Organised List

Listing every possibility in an organised way is an important tool.

How students organise the data often reveals additional information.

## Resource Kit 3

### Solve a Simpler Related Problem

Many hard problems are actually simpler problems that have been extended to larger numbers.

Patterns can sometimes be identified by trying the problem with smaller numbers.

### Eliminate All But One Possibility

Deciding what a quantity is not, can narrow the field to a very small number of possibilities.

These can then be tested against the conditions of the original problem.

## Resource Kit 4

### Convert to a More Convenient Form

There are times when changing some of the conditions of a problem makes a solution clearer or more convenient.

### Divide a Complex Shape

Sometimes it is possible to divide an unusual shape into two or more common shapes that are easier to work with.

# Set Yellow

1.1) David is getting fit.

He does one push-up on June 1st, three on June 2nd, five on June 3rd, and so on, through the first 10 days of the month.

Each day, he does two more push-ups than the day before.

In all, how many push-ups does David do in the first 10 days of June?

1.2) Suppose the time is now 5 o'clock on a twelve-hour clock.

What time will this clock show 125 hours from now?

1.3) Lily and Billy ate all 30 jelly beans in the packet.

Lily ate 6 more jelly beans than Billy.

How many did Billy eat?

1.4) A small coffee cup has a capacity of 100 mL.

A large coffee cup has a capacity of 200 mL.

10 people ordered 1 coffee each.

They are served 1.4 litres of coffee.

How many of them ordered a small coffee?

# Maths Games Example Solution 1.1 - Yellow

David is getting fit. He does one push-up on June 1st, three on June 2nd, five on June 3rd, and so on, through the first 10 days of the month.

Each day, he does two more push-ups than the day before.

In all, how many push-ups does David do in the first 10 days of June?

## Strategy 1: Build a Table, and Find a Pattern

We want to know how many push-ups David does in total over the first ten days of June.

| Date in June | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Push-ups each day | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 |

### Method 1: Use a Written Algorithm

$$\begin{array}{rcccc} & & & & 1 \\ & & & & 3 \\ & & & & 5 \\ & & & & 7 \\ & & & & 9 \\ & & {\scriptstyle 5} & & \\ & & 1 & 1 & \\ & & 1 & 3 & \\ & & 1 & 5 & \\ & & 1 & 7 & \\ + & & 1 & 9 & \\ \hline & 1 & 0 & 0 & \end{array}$$

### Method 2: Use a Split Algorithm

If the numbers are split into tens and ones, we can see that:

$1 + 3 + 5 + 7 + 9 = 25$

so

$1 + 3 + 5 + 7 + 9 + 1 + 3 + 5 + 7 + 9 = 50$

Adding the five tens, we have

$5 \times 10 + 50 = 100$

### Method 3: Find friends of 20


By grouping numbers that add to **20**, we can see that the number of push-ups is equal to

$(1 + 19) + (3 + 17) + (5 + 15) + (7 + 13) + (9 + 11)$

$= 20 + 20 + 20 + 20 + 20$

$= 100$

### Method 4: Perform the calculation

$1 + 3 = 4$

$4 + 5 = 9$

$9 + 7 = 16$

$16 + 9 = 25$

$25 + 11 = 36$

$36 + 13 = 49$

$49 + 15 = 64$

$64 + 17 = 81$

$81 + 19 = 100$

Therefore David does **100** push-ups in 10 days.

## Strategy 2: Find a Pattern

From **Strategy 1 Method 4**, we can see that the results go like this: **4, 9, 16, 25, 36, 49, 64, 81 ...**

Can you see a pattern? They look a lot like square numbers! Why might such a pattern occur?

Let's see what happens if we represent the results as square numbers.

| Date in June | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Push-ups each day | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 |
| Total push-ups so far | o | o x<br>x x | o x o<br>x x o<br>o o o | o x o x<br>x x o x<br>o o o x<br>x x x x | o x o x o<br>x x o x o<br>o o o x o<br>x x x x o<br>o o o o o | ... | ... | ... | ... | o x o x o x o x o x<br>x x o x o x o x o x<br>o o o x o x o x o x<br>x x x x o x o x o x<br>o o o o o x o x o x<br>x x x x x x o x o x<br>o o o o o o o x o x<br>x x x x x x x o x x<br>o o o o o o o o o x<br>x x x x x x x x x x |
| | 1<br>$= 1^2$ | 1 + 3<br>= 4<br>$= 2^2$ | 4 + 5<br>= 9<br>$= 3^2$ | 9 + 7<br>= 16<br>$= 4^2$ | 16 + 9<br>= 25<br>$= 5^2$ | $= 6^2$ | $= 7^2$ | $= 8^2$ | $= 9^2$ | $= 10^2$ |

From the pattern, we can see that in the first **10** days, David does $10^2$ = **100** push-ups.

**Answer** **100**

# Maths Games Example Solution 1.2 - Yellow

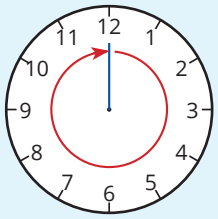
Suppose the time is now 5 o'clock on a twelve-hour clock.

What time will this clock show 125 hours from now?

## Strategy: Find a Pattern, and Build a Table or Draw a Diagram

125 hours is not a convenient amount to add. What is an easy number of hours to add?

How about 12 hours? This might be more convenient, because it takes 12 hours for the time on a 12-hour clock to complete a full cycle.




### Method 1: Count a 12 hour clock cycle from 12 o'clock to 12 o'clock.

We can build a table to keep track of how many hours have passed.

It may help to think of a 12 hour cycle from 12 o'clock to 12 o'clock.

7 hours after 5 o'clock, it will be 12 o'clock.

Every 12 hours after that, it will be 12 o'clock again.

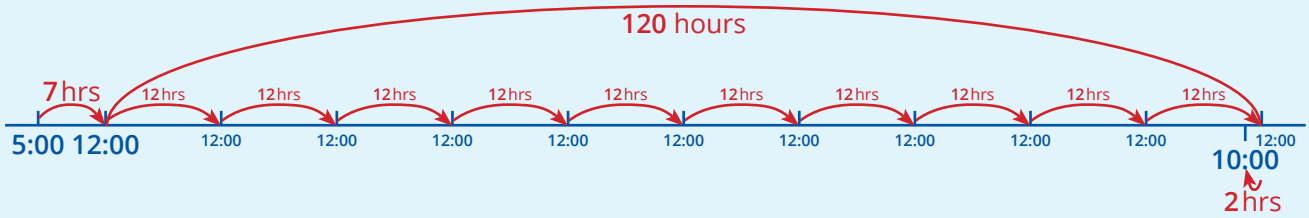
To get close to 125 hours in total, let's add 10 × 12 = 120 hours.

We've gone over by 127 – 125 = 2 hours.

2 hours before 12 o'clock, the time would have been 10 o'clock.

| Time now | Hours to add | Total hours | New time |
|---|---|---|---|
| 5:00 | +7 | 7 | 12:00 |
| 12:00 | +120 | 127 | 12:00 |
| 12:00 | –2 | 125 | 10:00 |

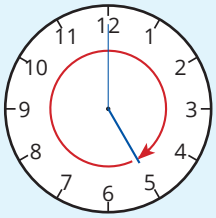
This method can also be represented as a time line.




We can see that, 125 hours after **5 o'clock**, the clock is going to show **10 o'clock**.

### Method 2: Count a 12 hour clock cycle from 5 o'clock to 5 o'clock.

If it's 5 o'clock now, in 12 hours' time the clock will show 5 o'clock again.

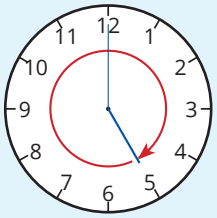


5:00

We want to find the time 125 hours from now.

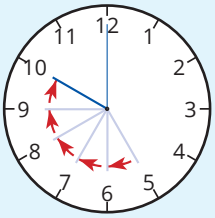
125 ÷ 12 = 10 r.5.

In 10 × 12 = 120 hours' time the clock will likewise show 5 o'clock.



5:00

5 hours after that, 120 + 5 = 125 hours will have passed.



10:00

125 hours after 5 o'clock, the clock will show **10 o'clock**.

**Answer** **10 o'clock**

# Maths Games Example Solution 1.3 - Yellow

Lily and Billy ate all 30 jelly beans in the packet.

Lily ate 6 more jelly beans than Billy. How many did Billy eat?

## Strategy 1: Build a Table

If Billy ate 10 jelly beans, then Lily ate 10 + 6 = 16 jelly beans.
Together, they would have eaten 10 + 16 = 26 jelly beans.

| Billy | Lily | Total |
|---|---|---|
| 10 | 16 | 26 |

If Billy ate 11 jelly beans, then Lily ate 11 + 6 = 17 jelly beans.
Together, they would have eaten 11 + 17 = 28 jelly beans.
If Billy ate 12 jelly beans, then Lily ate 12 + 6 = 18 jelly beans.
Together, they would have eaten 12 + 18 = 30 jelly beans.

| Billy | Lily | Total |
|---|---|---|
| 10 | 16 | 26 |
| 11 | 17 | 28 |
| 12 | 18 | 30 |

That matches the question, so Billy ate **12** jelly beans.

## Strategy 2: Build a Table, and Find a Pattern

Let's start by guessing in the middle, with Lily and Billy each eating 30 ÷ 2 = 15 jelly beans.

We know that Lily ate more jelly beans than Billy.
If Lily ate 16 jelly beans, then Billy must have eaten 30 – 16 = 14.
If so, then Lily would have eaten 16 – 14 = 2 jelly beans more than Billy.

| Lily | Billy | Difference |
|---|---|---|
| 15 | 15 | 0 |
| 16 | 14 | 2 |

When we guessed 1 more jellybean for Lily, the difference increased by 2.
Let's see if this pattern continues.

- If Lily ate 17 and Billy ate 13, then Lily had 17 – 13 = 4 more than Billy.
- If Lily ate 18 and Billy ate 12, then Lily had 18 – 12 = 6 more than Billy.

That matches the question, so Billy ate **12** jelly beans.

| Lily | Billy | Difference |
|---|---|---|
| 15 | 15 | 0 |
| 16 | 14 | 2 |
| 17 | 13 | 4 |
| 18 | 12 | 6 |

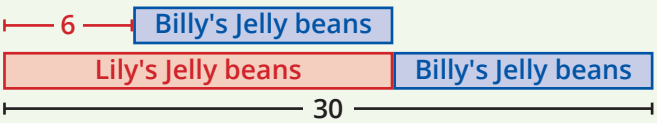
## Strategy 3: Draw a Diagram, or Use Algebra

We can use pronumerals (*B* and *L*) to represent the number of jelly beans eaten by Billy and Lily respectively.

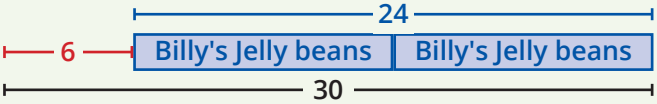
We can also use bars to represent the same idea.

Lily ate 6 more jelly beans than Billy.


All together, Lily and Billy ate 30 jelly beans.




30 – 6 = 24 is double the number of jelly beans eaten by Billy.




$L = B + 6$ - (1)
$L + B = 30$ - (2)

Substituting (1) into (2):
$B + 6 + B = 30$
$2B = 30 - 6$
$= 24$

$B = 24 \div 2$
$B = 12$

Billy ate 24 ÷ 2 = **12** jelly beans.

**Answer** 12

# Maths Games Example Solution 1.4 - Yellow

A small coffee cup has a capacity of 100 mL. A large coffee cup has a capacity of 200 mL.

10 people ordered 1 coffee each. They are served 1.4 litres of coffee. How many of them ordered a small coffee?

We know that **1.4 litres** of coffee was served.

However, all of the servings are measured in **millilitres**.

To make the quantities make sense, it is easier if we convert everything to use the same units. Let's choose to work in **millilitres**.

We know that **1 litre = 1000 mL**, so we can work it out from there.

| | | |
|---|---|---|
| 1 litre | = | 1000 mL |
| 2 litres | = | 2000 mL |
| 1.5 litres | = | 1500 mL |
| 1.4 litres | = | 1400 mL |

## Strategy 1: Build a Table, and Find a Pattern

Suppose all 10 people bought a small 100 mL coffee, and nobody bought a large 200 mL coffee.

In total, that's 10 × 100 mL = 1000 mL of coffee.

| | | | | | |
|---|---|---|---|---|---|
| Small coffees | 10 | | | | |
| Large coffees | 0 | | | | |
| mL of coffee | 1000 | | | | |

If 9 people bought a small 100 mL coffee, and 1 person bought a large 200 mL coffee, that's 9 × 100 mL + 1 × 200 mL = 1100 mL of coffee.

| | | | | | |
|---|---|---|---|---|---|
| Small coffees | 10 | 9 | | | |
| Large coffees | 0 | 1 | | | |
| mL of coffee | 1000 | 1100 | | | |

We can swap small coffees for large coffees, until the total amount of coffee served is 1400 mL.

Each swap increases the total amount of coffee by 100 mL.

(Why might this be the case?)

| | | | | | |
|---|---|---|---|---|---|
| Small coffees | 10 | 9 | 8 | 7 | 6 |
| Large coffees | 0 | 1 | 2 | 3 | 4 |
| mL of coffee | 1000 | 1100 | 1200 | 1300 | 1400 |
| | | +100 | +100 | +100 | +100 |

From our table, we can see that **6** people ordered a small coffee.

## Strategy 2: Draw a Diagram, and Find a Pattern

Suppose the entire 1400 mL was served in small 100 mL cups.

If so, there would be 1400 ÷ 100 = 14 small cups of coffee.

Every time someone orders a large coffee, we can imagine 2 small coffees being poured into 1 large 200 mL cup.

Every time 2 small coffees are poured into 1 large 200 mL cup, the number of cups goes down by 1, but the amount of coffee stays the same.

Is there a pattern here that might help us to work out what the coffee will look like, if we have 10 cups in total?

Let's continue to combine 2 small coffees until we end up with a total of 10 cups of coffee.

From our diagram, we can see that **6** small coffees were served.

**Answer** 6

# Set Green

1.1) David is getting fit.

He does one push-up on June 1st, two on June 2nd, three on June 3rd, and so on, through the first 10 days of the month.

Each day, he does one more push-up than the day before.

In all, how many push-ups does David do in the first 10 days of June?

1.2) Suppose the time is now 5 o'clock on a twelve-hour clock.

What time will this clock show 48 hours from now?

1.3) Lily and Billy ate all 20 jelly beans in the packet.

Lily ate 4 more jelly beans than Billy.

How many did Billy eat?

1.4) A small coffee cup has a capacity of 100 mL.

A large coffee cup has a capacity of 200 mL.

5 people ordered 1 coffee each.

They are served 600 mL of coffee.

How many of them ordered a small coffee?

# Preparation Task 1

A) Patrick and Stacey are working on this problem.

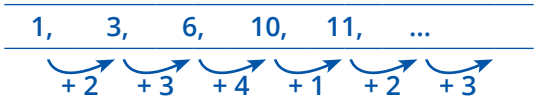
Stacey continues the sequence for a few more numbers.

She says, "I think I can see a pattern."

**Describe** the pattern that Stacey might be seeing.

**Billy is writing a sequence of numbers.**

**He starts with 1, then adds 2, 3, 4, 1, 2, 3, 4, 1 ... and so on, repeatedly adding 2, then 3, then 4, then 1, like this:**




**What is the 20th number in Billy's sequence?**

B) Stacey then says,

"Instead of 1 + 2 + 3 + 4, I can just add 10 in one step."

**Explain** how Stacey might use this observation to work out what the twentieth number might be.

C) Patrick says,

"I solved the problem another way.

Billy is adding twenty numbers.

In total, there are five 1s, five 2s, five 3s, and five 4s.

So I worked it out by adding all of the 1s first, then the 2s, and so on."

**Evaluate** Patrick's method. Would it work to add the numbers in a different order? Why or why not?

# Maths Games Example Solution - Preparation Task 1

Billy is writing a sequence of numbers.

He starts with 1, then adds 2, 3, 4, 1, 2, 3, 4, 1 ... and so on, repeatedly adding 2, then 3, then 4, then 1, like this:

| Billy's sequence: | 1, | 3, | 6, | 10, | 11, | ... |
|---|---|---|---|---|---|---|
| | +2 | +3 | +4 | +1 | +2 | +3 |

What is the 20th number in Billy's sequence?

## Strategy 1: Find a Pattern

Billy's sequence looks like this.

| Billy's Sequence: | 1, | 3, | 6, | 10, | 11, | ... |
|---|---|---|---|---|---|---|
| | +2 | +3 | +4 | +1 | +2 | +3 |

To find the **20**th number, we can just keep adding numbers according to Billy's pattern.

| 1, | 3, | 6, | 10, | 11, | 13, | 16, | 20, | 21, | 23, | 26, | 30, | 31, | 33, | 36, | 40, | 41, | 43, | 46, | 50 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| +2 | +3 | +4 | +1 | +2 | +3 | +4 | +1 | +2 | +3 | +4 | +1 | +2 | +3 | +4 | +1 | +2 | +3 | +4 | |

The 20th number in Billy's sequence is **50**.

## Strategy 2: Find a Pattern (Alternative Method)

Billy is creating his sequence by starting with 1 and then adding 2, then 3, then 4, then 1 ... and so on.

So the **20**th number in the sequence is the result of the following sum:

$$1 + 2 + 3 + 4 + 1 + 2 + 3 + 4 + 1 + 2 + 3 + 4 + 1 + 2 + 3 + 4 + 1 + 2 + 3 + 4$$

**Method 1: Consider groups of (1 + 2 + 3 + 4).**

Suppose we wrote the sum as:

$$1 + 2 + 3 + 4 \quad + 1 + 2 + 3 + 4 \quad + 1 + 2 + 3 + 4 \quad + 1 + 2 + 3 + 4 \quad + 1 + 2 + 3 + 4$$

Since **1 + 2 + 3 + 4 = 10**, we can see that the sum is $10 + 10 + 10 + 10 + 10 = \mathbf{50}$.

**Method 2: Rearrange to be a series of multiplications.**

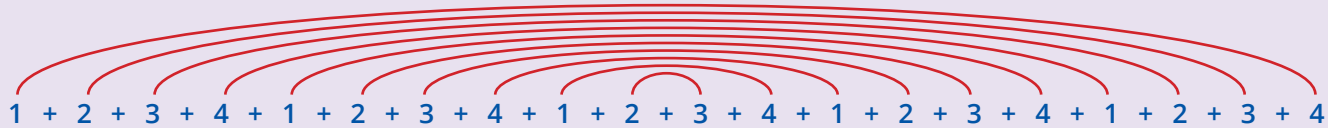
We could write the sum as:

$$1 + 1 + 1 + 1 + 1 \quad + 2 + 2 + 2 + 2 + 2 \quad + 3 + 3 + 3 + 3 + 3 \quad + 4 + 4 + 4 + 4 + 4$$

The sum is equal to $5 \times 1 + 5 \times 2 + 5 \times 3 + 5 \times 4 = 5 + 10 + 15 + 20 = \mathbf{50}$.

**Method 3: Find "Friends of Five".**

We can group the values two at a time to make "friends of five":




The sum is equal to $10 \times 5 = \mathbf{50}$.

**Answer** **50**

# Preparation Task 2

A) Tayah and Josie are working on this problem.

Josie drew the next border of grey tiles.

**Demonstrate** Josie's working by drawing the pattern of tiles as shown in the question, plus the next border of grey tiles.

**Hugo is using square tiles that are all the same size.**

**He begins with one white tile,**  **and surrounds it with a border of 8 grey tiles.**

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 8 | | 4 |
| 7 | 6 | 5 |

**He surrounds the grey tiles with a border of 16 white tiles.**

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
| 16 | | | | 6 |
| 15 | | | | 7 |
| 14 | | | | 8 |
| 13 | 12 | 11 | 10 | 9 |

**Hugo continues to alternate between grey and white borders.**

**How many tiles will Hugo need for the next white border?**

B) Josie says,

"I think there is a pattern in how the border grows."

**Complete the table** to see if Josie's idea makes sense.

| Border number | **1** (grey) | **2** (white) | **3** (grey) |
|---|---|---|---|
| Number of tiles | **8** | **16** | |

C) Tayah says,

"I noticed a different pattern.

The length of the side of the square is the next odd number."

| Border number | **1** (grey) | **2** (white) | **3** (grey) |
|---|---|---|---|
| Square side length | **3** | **5** | |

**Complete the table** with the side length of the next grey border.

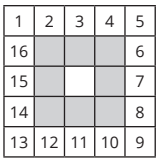
**How** might Tayah have calculated that the next white border has **32** tiles?

# Maths Games Example Solution - Preparation Task 2

Hugo is using square tiles that are all the same size.

He begins with one white tile,  and then surrounds it with a border of 8 grey tiles.


He surrounds the grey tiles with a border of 16 white tiles.

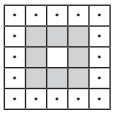
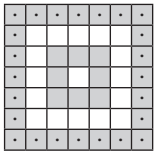
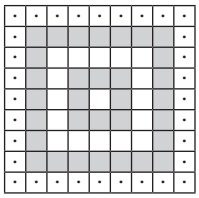



Hugo continues to alternate between grey and white borders.

How many tiles will Hugo need for the next white border?

## Strategy 1: Build a Table, and Draw a Diagram

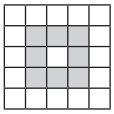
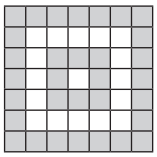
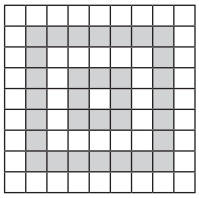
Let's use a table to record the number of tiles for each border.

| Tile pattern |  |  |  |  |
|---|---|---|---|---|
| Tiles in outside border | 8 | 16 | 24 | 32 |

Hugo will need **32** tiles for the next white border.

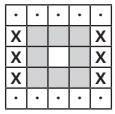
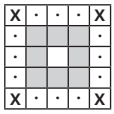
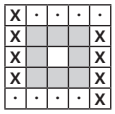
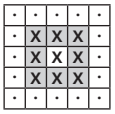
## Strategy 2: Build a Table, Draw a Diagram, and Find a Pattern

It may help to consider the side length for each pattern of squares.

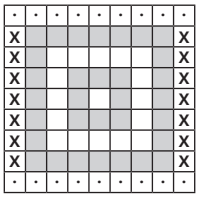
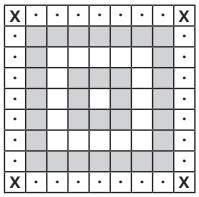
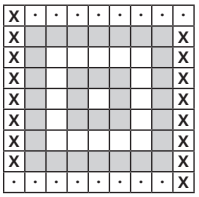
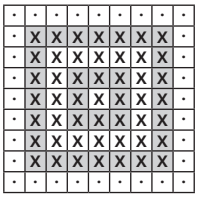
| Tile pattern |  |  |  |  |
|---|---|---|---|---|
| Side length of pattern | 3 | 5 | 7 | 9 |

Every time Hugo adds another border, the side length increases by **2**.

We can then use many different methods to count the tiles in the outside border.
For example, for a side length of **5**:

|  |  |  |  |
|---|---|---|---|
| 5 + 3 + 5 + 3 = 16 | 4 × 3 + 4 = 16 | 4 × 4 = 16 | (5 × 5) – (3 × 3) = 16 |

These patterns can then be applied to larger side lengths.
We want to find the number of tiles in the ouside border, for a side length of **9**.

|  |  |  |  |
|---|---|---|---|
| 9 + 7 + 9 + 7 = 32 | 4 × 7 + 4 = 32 | 4 × 8 = 32 | (9 × 9) – (7 × 7) = 32 |

Regardless of the method we choose to use, we find that Hugo needs **32** tiles for the next white border.

**Answer** 32

# Preparation Task 3

A) Aron and Amit are working on this problem.

Aron says, "Roger has the smallest number of books."

**Explain** how Aron worked this out.

> **Simon has one more book than Tilly.**
>
> **Tilly has four more books than Roger.**
>
> **If all three students shared their books equally, they would have eight books each.**
>
> **How many books does Tilly have?**

B) Aron decides to use a table to keep track of his work.

He starts by guessing that Roger has 1 book.

**Continue** Aron's table until you find a solution that matches the conditions in the question.

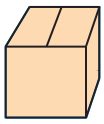
| Roger | Tilly | Simon | Total | Equal shares |
|---|---|---|---|---|
| 1 | 5 | 6 | 12 | 4 |
| 2 | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

C) Amit says,

"I started by figuring out how many books there are in total."

**Circle** the sentence that helped Amit to work out how many books there are in total.

D) Amit drew a box that was just big enough to hold all of Roger's books.



He drew the same box, plus four more books, for Tilly.

Using Amit's idea, draw Simon's books.

**Demonstrate** how Amit could use this diagram to work out how many books Tilly has.

# Maths Games Example Solution - Preparation Task 3

Simon has one more book than Tilly.

Tilly has four more books than Roger.

If all three students shared their books equally, they would have eight books each.

How many books does Tilly have?

## Strategy 1: Build a Table, and Find a Pattern

Let's build a table with possible numbers of books for Roger, Tilly and Simon.

We know that Tilly has **4** more than Roger, and Simon has **1** more than Tilly.

If Roger has **5** books, Tilly will have **9** books, and Simon will have **10** books, for a total of **24** books all together.

If those **24** books were shared equally, each student would have **8** books.

We can see that Tilly has **9** books.

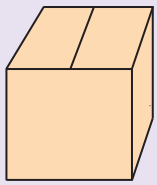
| Roger | Tilly | Simon | Total | Equal shares |
|---|---|---|---|---|
| 1 | 5 | 6 | 12 | 4 |
| 2 | 6 | 7 | 15 | 5 |
| 3 | 7 | 8 | 18 | 6 |
| 4 | 8 | 9 | 21 | 7 |
| 5 | 9 | 10 | 24 | 8 |

## Strategy 2: Work Backwards

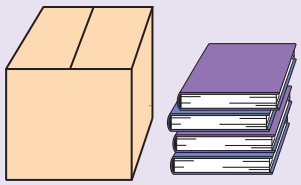
If Simon, Tilly and Roger shared their books equally, they would have **8** books each.

In total, they must have **8 × 3 = 24** books.

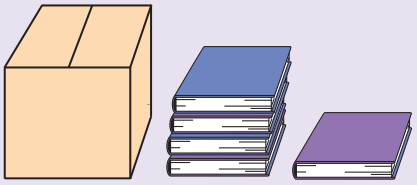
Let's suppose that Roger's books fit exactly in one box.



Tilly has **4** more books than Roger.



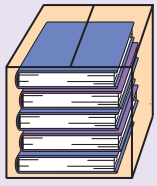
Simon has **1** more book than Tilly.



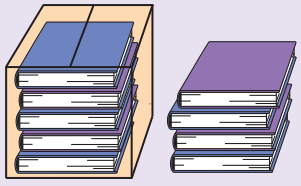
All together, Roger, Tilly and Simon have three boxes of books, plus another **4 + 5 = 9** books.

With **24** books in total, there must be **24 – 9 = 15** books in those three boxes.

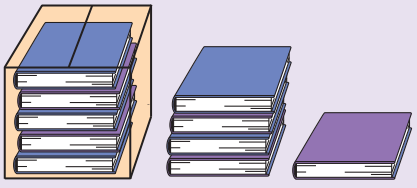
Roger's books



Tilly's books



Simon's books



Each box would then contain **15 ÷ 3 = 5** books.

Tilly must have **5 + 4 = 9** books.

**Answer** 9

# Preparation Task 4

A) Julia and Gareth are working on this problem.

Julia says,

"We can start with the first two sentences.

Let's write a list of all of the class sizes that it might be."

**Complete** Julia's list.

~~3~~, ~~6~~, ~~9~~, 12, 15,

**Mrs Waugh has between 10 and 35 students in her class.**

**She can put all of her students into groups of 3.**

**To put everyone into groups of 5, she would need to join one of the groups herself.**

**How many students are in her class?**

B) Gareth says,

"Okay. Now we need to add Mrs Waugh to the class."

He drew a table for Julia to fill in with her list, and then added a line for the class with Mrs Waugh.

**Complete** Gareth's table.

| Possible class sizes | 12 | 15 | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Class with Mrs Waugh | | | | | | | | |

C) Julia says,

"It looks like there's only one class size that works."

**Explain** how Julia can be sure that there is only one class size that works for the conditions of this problem.

# Maths Games Example Solution - Preparation Task 4

Mrs Waugh has between 10 and 35 students in her class.

She can put all of her students into groups of 3.

To put everyone into groups of 5, she would need to join one of the groups herself.

How many students are in her class?

## Strategy 1: Draw a Diagram, and Build a Table

Mrs Waugh has between **10** and **35** students.

She can put all of her students into groups of **3**.

Let's draw a diagram showing groups of **3** students.

We can use a table to keep track of the possible numbers of students in Mrs Waugh's class.

Then, we can work out how many people might be in the class if we count Mrs Waugh as well.

| Possible class size | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 |
|---|---|---|---|---|---|---|---|---|
| Class including Mrs Waugh | 13 | 16 | 19 | 22 | 25 | 28 | 31 | 34 |

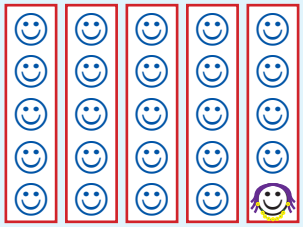
Mrs Waugh can put everyone into groups of **5** if she joins one of the groups herself.

Of all of the possible class sizes, the only one for which this works is the class of **24**.

When Mrs Waugh joins in, there would be **25** people.

This makes five groups of **5**. None of the other options results in a multiple of **5**.

So there must be **24** students in Mrs Waugh's class.



## Strategy 2: Build a Table

Let's draw a hundreds chart.

We'll stop short of **100** though, since Mrs Waugh's class only has between **10** and **35** students.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |

The number of students must be a multiple of **3**, since Mrs Waugh can put all of her students into groups of **3**.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | (3) | 4 | 5 | (6) | 7 | 8 | (9) | 10 |
| 11 | (12) | 13 | 14 | (15) | 16 | 17 | (18) | 19 | 20 |
| (21) | 22 | 23 | (24) | 25 | 26 | (27) | 28 | 29 | (30) |
| 31 | 32 | (33) | 34 | 35 | (36) | 37 | 38 | (39) | 40 |

If Mrs Waugh joins a group, the class can form groups of **5**.

This means that the number of students must be one less than a multiple of **5**.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | (3) | 4 | (5) | 6 | 7 | 8 | (9) | (10) |
| 11 | (12) | 13 | 14 | (15) | 16 | 17 | (18) | 19 | (20) |
| (21) | 22 | 23 | (24) | (25) | 26 | (27) | 28 | 29 | (30) |
| 31 | 32 | (33) | 34 | (35) | (36) | 37 | 38 | (39) | (40) |

We can see that **24** is both:

- a multiple of **3**, and
- one less than a multiple of **5**.

Mrs Waugh must have **24** students in her class.

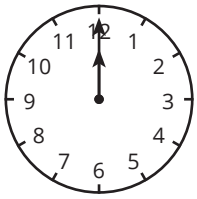
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | (3) | 4 | (5) | 6 | 7 | 8 | (9) | (10) |
| 11 | (12) | 13 | 14 | (15) | 16 | 17 | (18) | 19 | (20) |
| (21) | 22 | 23 | **(24)** | (25) | 26 | (27) | 28 | 29 | (30) |
| 31 | 32 | (33) | 34 | (35) | (36) | 37 | 38 | (39) | (40) |

**Answers** **24**

# Set Orange

1.1) The sum of the page numbers of Chapter 3 (of a certain book) is 374.
If there are 11 pages in Chapter 3, on what page does Chapter 3 begin?

1.2) At 12 o'clock, the hands of Akiko's watch are exactly on top of each other.
When the hands overlap again for the first time after 12 o'clock, between which two consecutive numbers are they pointing?




1.3) Judy has two more sisters than brothers.
Her brother, Mark, has twice as many sisters as brothers.
How many children are in their family?

1.4) Buses have 6 wheels.
Semi-trailers have 18 wheels.
At the truck and bus service station, there were 7 vehicles and a total of 78 wheels.
How many buses were there at the service station?

# Example Problem 1.1 - Summary

### Example Problem 1.1 - Green

David is getting fit.

He does one push-up on June 1st, two on June 2nd, three on June 3rd, and so on, through the first 10 days of the month.

Each day, he does one more push-up than the day before.

In all, how many push-ups does David do in the first 10 days of June?

### Example Problem 1.1 - Yellow

David is getting fit.

He does one push-up on June 1st, three on June 2nd, five on June 3rd, and so on, through the first 10 days of the month.

Each day, he does two more push-ups than the day before.

In all, how many push-ups does David do in the first 10 days of June?

### Example Problem 1.1 - Orange

The sum of the page numbers of Chapter 3 (of a certain book) is 374.

If there are 11 pages in Chapter 3, on what page does Chapter 3 begin?

# Example Problem 1.2 - Summary

### Example Problem 1.2 - Green

Suppose the time is now 5 o'clock on a twelve-hour clock.

What time will this clock show 48 hours from now?

### Example Problem 1.2 - Yellow

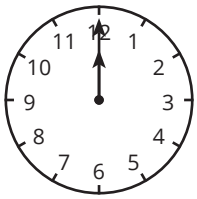
Suppose the time is now 5 o'clock on a twelve-hour clock.

What time will this clock show 125 hours from now?

### Example Problem 1.2 - Orange

At 12 o'clock, the hands of Akiko's watch are exactly on top of each other.

When the hands overlap again for the first time after 12 o'clock, between which two consecutive numbers are they pointing?



# Example Problem 1.3 - Summary

### Example Problem 1.3 - Green

Lily and Billy ate all 20 jelly beans in the packet.
Lily ate 4 more jelly beans than Billy.
How many did Billy eat?

### Example Problem 1.3 - Yellow

Lily and Billy ate all 30 jelly beans in the packet.
Lily ate 6 more jelly beans than Billy.
How many did Billy eat?

### Example Problem 1.3 - Orange

Judy has two more sisters than brothers.
Her brother, Mark, has twice as many sisters as brothers.
How many children are in their family?

# Example Problem 1.4 - Summary

### Example Problem 1.4 - Green

A small coffee cup has a capacity of 100 mL.
A large coffee cup has a capacity of 200 mL.
5 people ordered 1 coffee each.
They are served 600 mL of coffee.
How many of them ordered a small coffee?

### Example Problem 1.4 - Yellow

A small coffee cup has a capacity of 100 mL.
A large coffee cup has a capacity of 200 mL.
10 people ordered 1 coffee each.
They are served 1.4 litres of coffee.
How many of them ordered a small coffee?

### Example Problem 1.4 - Orange

Buses have 6 wheels.
Semi-trailers have 18 wheels.
At the truck and bus service station, there were 7 vehicles and a total of 78 wheels.
How many buses were there at the service station?

## Answers

| Set Yellow | |
|---|---|
| **1.1** | 100 |
| **1.2** | 10 o'clock |
| **1.3** | 12 |
| **1.4** | 6 |

| Set Green | |
|---|---|
| **1.1** | 55 |
| **1.2** | 5 o'clock |
| **1.3** | 8 |
| **1.4** | 4 |

| Preparation Tasks | |
|---|---|
| **1** | 50 |
| **2** | 32 |
| **3** | 9 |
| **4** | 24 |

| Set Orange | |
|---|---|
| **1.1** | 29 |
| **1.2** | 1 and 2 |
| **1.3** | 13 |
| **1.4** | 4 |