MATHS GAMES

APSMO

2025 MATHS GAMES

Important

The information contained in this file is ONLY for the use of registered participants of the 2025 APSMO Maths Games.

Organisation and Procedures

For full details, see the Members' Area

- Maths Games papers are to be conducted under test conditions.

DO	DO NOT
• Supervise students at all times. • Maintain silence. • Provide blank working paper. • Collect, mark and retain the papers.	• Print the papers prior to the scheduled date. • Read the questions aloud to the students. • Interpret the questions for students. • Permit any discussion or movement around the room. • Permit the use of calculators or other electronic devices.

- Papers should be scored by the PICO using the *Solutions and Answers* sheet provided.
- Original student answer sheets should be retained by the PICO until the end of the year.

Absent Students

- A student who is legitimately absent on the date of the Maths Games paper, may sit the paper on their return to school.
- If an absent student does not sit the paper on their return to school they should be marked as 'absent'.
- *Note: This policy differs from the Maths Olympiads Absent Student Policy which has additional requirements*.

APSMO

WEDNESDAY 7 MAY 2025

MATHS GAMES

MATHS GAMES JUNIOR

Suggested Time: ***30 Minutes.*** *Calculators NOT Permitted.*

Write your answers in the boxes on the back.

←

Keep your answers hidden by folding backwards on this line.

1A. Scarlett and Keren are sharing a box of twenty collectable cards.

Scarlett takes eight more cards than Keren.

How many cards does Keren get?

1B. A clown performs tricks for children's birthday parties.

He has a trick where the answer is the child's age.

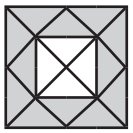
For this trick, he begins by saying: "Think of your age. Double it. Add 20. Halve your answer. Now subtract ..."

What should be subtracted, so that the answer is the child's age?

1C. Joey is building a pattern with identically sized triangular tiles.

He begins by arranging four white tiles into a square.

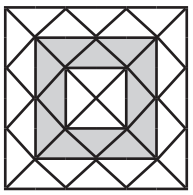
He surrounds this square with a border of 12 grey tiles.



He surrounds the grey tiles with another border of 20 white tiles.

Joey continues to alternate between grey and white borders.

How many tiles will Joey need for the next grey border?



1D. Water bottles come in two sizes, small (150 mL) and large (250 mL).

I have ten bottles, which can hold a total of 2.2 litres of water.

How many small bottles do I have?

1E. Chocolate bars come in boxes of 6 and 10.

If Sara wanted to buy exactly 22 chocolate bars, there is only one way she could do it: she would have to buy two boxes of 6, and one box of 10.

In how many different ways can Sara buy exactly 56 chocolate bars?

APSMO

WEDNESDAY 7 MAY 2025

MATHS GAMES

JUNIOR

1A.

1B.

1C.

1D.

1E.

Student Name:

Fold here. Keep your answers hidden.

APSMO
WEDNESDAY 7 MAY 2025

MATHS GAMES
JUNIOR

Solutions and Answers
(Items in parentheses are not required)

1A: 6	**1B:** 10	**1C:** 28	**1D:** 3	**1E:** 2

1A. How many cards does Keren get?

Strategy 1: **Build a Table, and Find a Pattern**

There are **20** cards in the box.

Suppose Scarlett and Keren took **20 ÷ 2 = 10** cards each.

Scarlett would then have **10 – 10 = 0** cards more than Keren.

Scarlett	10				
Keren	10				
Difference	0				

If Scarlett takes **11** cards, then Keren would take **20 – 11 = 9**.

Scarlett would then have **11 – 9 = 2** cards more than Keren.

We can see that increasing Scarlett's cards by **1** increases the difference by **2**.

Following the pattern, we find that when Scarlett takes **8** more cards than Keren, Keren will have **6** cards.

		+1	+1	+1	+1
Scarlett	10	11	12	13	14
Keren	10	9	8	7	6
Difference	0	2	4	6	8
		+2	+2	+2	+2

Strategy 2: **Build a Table, and Find a Pattern (Alternative Approach)**

Suppose Scarlett took **10** cards.

She got **8** more than Keren, so Keren got **10 – 8 = 2** cards.

In total, they distributed **10 + 2 = 12** cards.

Scarlett	10				
Keren	2				
Total	12				

If Scarlett took **11** cards, then Keren took **11 – 8 = 3**.

Between them, they would have **11 + 3 = 14** cards.

We can see that increasing Scarlett's cards by **1** increases the total by **2**.

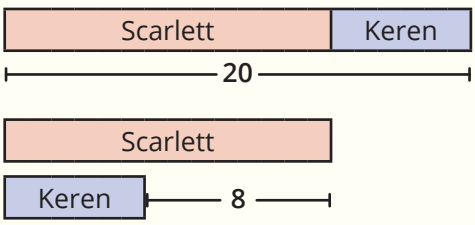
Following the pattern, we find that when there are **20** cards in total, Keren will have **6** cards.

		+1	+1	+1	+1
Scarlett	10	11	12	13	14
Keren	2	3	4	5	6
Total	12	14	16	18	20
		+2	+2	+2	+2

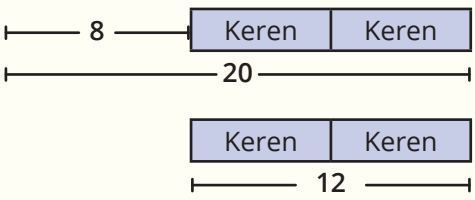
Strategy 3: **Draw a Diagram, and Reason Logically**

Scarlett and Keren shared **20** cards.

Scarlett got **8** cards more than Keren.



We can replace the bar representing Scarlett's cards, with (Keren + 8).



This is the same as noticing that, if we take **8** of Scarlett's cards away, Scarlett and Keren would each have the same number of cards.

Keren has **12 ÷ 2 = 6** cards.

Follow-Up: *Keren then swapped one card for two of Scarlett's cards. How many more cards does Scarlett have than Keren?* [6]

1B. What should be subtracted, so that the answer is the child's age?

Strategy 1: **Build a Table, and Find a Pattern**

Suppose the child is **7** years old. We can play out what happens during the trick.

Clown says	Child thinks
Think of your age.	7
Double it.	14
Add **20**.	34
Halve your answer.	17
Subtract ...	

If the final answer should be the child's age (**7**), the child should now subtract **17 – 7 = 10**.

Clown says	Child thinks
Think of your age.	7
Double it.	14
Add **20**.	34
Halve your answer.	17
Subtract **10**.	7

We can try the trick again for a child who is **8** years old.

Clown says	Child thinks	
Think of your age.	7	8
Double it.	14	16
Add **20**.	34	36
Halve your answer.	17	18
Subtract **10**.	7	8

Would it work for a **2** year old? What about a **100** year old person?

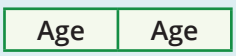
Clown says	Child thinks			
Think of your age.	7	8	2	100
Double it.	14	16	4	200
Add **20**.	34	36	24	220
Halve your answer.	17	18	12	110
Subtract **10**.	7	8	2	100

To end up with their age as the answer, the clown should ask the child to subtract **10**.

Strategy 2: **Draw a Diagram**

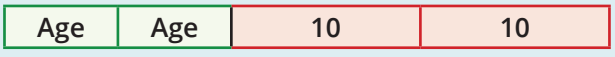
We can use a bar to represent the child's age.

The clown says: Double it.

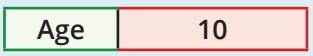


The clown says: Add **20**.

The clown says: Halve your answer.



After doing so, the result is Age + 10.



If the child then subtracts **10**, the result will be their age.

Follow-Up: *If the clown had said "Add 30" instead of "Add 20", what number would you need to subtract at the end?* [15]

1C. How many tiles will Joey need for the next grey border?

Strategy 1: **Draw a Diagram**

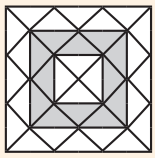
Joey begins by arranging four white tiles into a square.

Layers	1		
Tiles	4		

He surrounds the square with a border of **12** grey tiles.

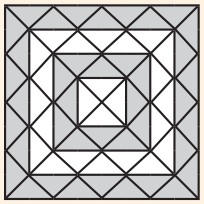
		+1	
Layers	1	2	
Tiles	4	12	
		+8	

He surrounds the grey tiles with a border of **20** white tiles.



		+1	+1	
Layers	1	2	3	
Tiles	4	12	20	
		+8	+8	

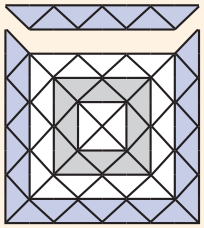
We can now work out the number of tiles in the next grey border.



		+1	+1	+1	
Layers	1	2	3	4	
Tiles	4	12	20		
		+8	+8	?	

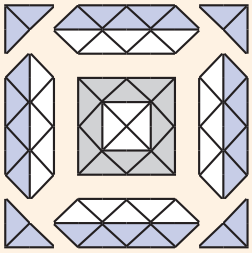
One way is to count the tiles along one side, and multiply it by **4**.

There are **4 × 7 = 28** tiles in the next grey border.



We can also see that the outside border has the same number of tiles as the one immediately within, plus **8** extra tiles in the corners.

There are **20 + 8 = 28** tiles in the next grey border.



Strategy 2: **Find a Pattern, and Draw a Diagram**

The first square has **4** tiles.

The second square contains **4 + 12 = 16** tiles.

The third square contains **16 + 20 = 36** tiles.

The number of tiles in each case is a square number.

Square	No. of Tiles
1st	$4 = 2^2$
2nd	$16 = 4^2$
3rd	$36 = 6^2$

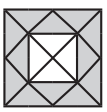
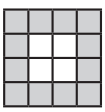
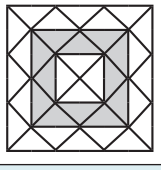
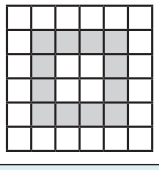
We could reasonably guess that there would be $8^2 = 64$ tiles in the **4th** square.

After recognising the square pattern, we might also notice that each triangle can be cut in half and re-assembled as a square.

We can use this idea to construct patterns from square tiles, that are the same size and have the same number of tiles as Joey is using.

After adding the next grey border, the **4th** square will contain $8^2 = 64$ tiles.

The next grey border uses **64 – 36 = 28** tiles.

Joey's tiles	Square tiles
	
	
	

Follow-Up: *What colour is the border that uses 44 tiles?* [Grey]

1D. How many small bottles do I have?

Strategy 1: **Build a Table, and Find a Pattern**

In the question, the capacity of a water bottle is stated in millilitres, but the total amount of water is given in litres.

To avoid confusion, we will convert the total amount of water into a quantity measured in millilitres.

In total, the bottles have a capacity of **2200** mL.

Litres	Millilitres
1	1000
2	2000
2.2	2200

Small bottles hold **150** mL.

If all **10** bottles were small, then the total capacity would be **10 × 150** mL = **1500** mL.

Small bottles	10				
Large bottles	0				
Total water (mL)	1500				

With **9** small and **1** large bottle, the total capacity would be **9 × 150 + 1 × 250 = 1600** mL.

With **8** small and **2** large bottles, the total capacity would be **8 × 150 + 2 × 250 = 1700** mL.

Exchanging a single **150** mL bottle for a **250** mL bottle increases the total capacity by **100** mL.

(Why is this?)

		−1	−1		
Small bottles	10	9	8		
Large bottles	0	1	2		
Total water (mL)	1500	1600	1700		
		+100	+100		

The total capacity of my bottles is **2200** mL.

This is **2200 – 1700 = 500** mL more than we have when there are **8** small bottles.

We need to exchange a further **500 ÷ 100 = 5** small bottles for large bottles.

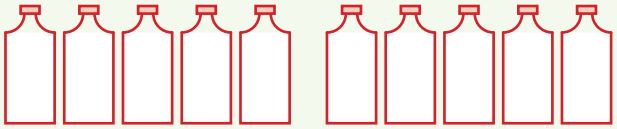
I have **8 – 5 = 3** small bottles in total.

		−1	−1		− 5 × 1	
Small bottles	10	9	8	...	3	
Large bottles	0	1	2	...	7	
Total water (mL)	1500	1600	1700	...	2200	
		+100	+100		+ 5 × 100	

Strategy 2: **Draw a Diagram**

Let's begin by assuming that all of the bottles have a capacity of **250** mL.

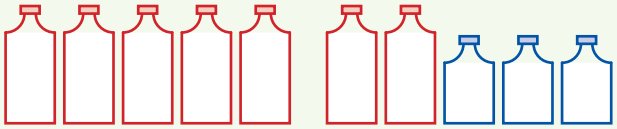
In total, they will hold **10 × 250 mL = 2500** mL.



Exchanging a **250** mL for a **150** mL bottle will reduce the total capacity by **250 – 150 = 100** mL.

We need to reduce the total capacity by **2500 – 2200 = 300** mL.

300 mL **= 3 × 100** mL, so we should exchange **3** bottles.



In total, I have **7** large bottles, and **3** small bottles.

Follow-Up: *If I had 20 bottles that together have a capacity of 3.1 litres, how many small bottles would there be?* [19]

1E. In how many different ways can Sara buy exactly 56 chocolate bars?

Strategy 1: **Build a Table**

Sara buys exactly **56** chocolate bars, in boxes of **6** or **10**.

Since **56 ÷ 10 = 5 remainder 6**, the largest possible number of boxes of **10** bars is **5**.

If Sara buys **5 × 10 = 50** bars, she will need **56 – 50 = 6** more bars to make up the total.

She can do this by buying one box of **6** bars.

No. of boxes of 10 bars	5					
No. of boxes of 6 bars	1					
Total bars	56					
No. of boxes in total	6					

If Sara buys **4 × 10 = 40** bars, she will need **56 – 40 = 16** more bars to make up the total.

If Sara buys **3 × 10 = 30** bars, she will need **56 – 30 = 26** more bars to make up the total.

Neither **16** nor **26** is divisible by **6**.

No. of boxes of 10 bars	5	4	3			
No. of boxes of 6 bars	1					
Total bars	56					
No. of boxes in total	6					

If Sara buys **2 × 10 = 20** bars, she will need **56 – 20 = 36** more bars to make up the total.

36 is divisible by **6**.

Sara can buy **36 ÷ 6 = 6** boxes of **6** bars.

No. of boxes of 10 bars	5	4	3	2	1	0
No. of boxes of 6 bars	1			6		
Total bars	56			56		
No. of boxes in total	6			8		

Continuing the table, we can see that there are no other ways to buy exactly **56** chocolate bars.

There are **2** different ways for Sara to buy exactly **56** chocolate bars.

Strategy 2: **Reason Arithmetically**

Chocolate bars come in boxes of **6** or **10**.

Multiples of **6** are:

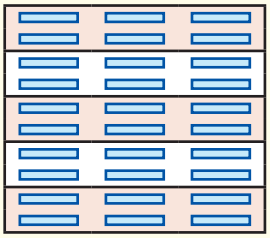
6, 12, 18, 24, 30, 36, 42, 48, 54, 60

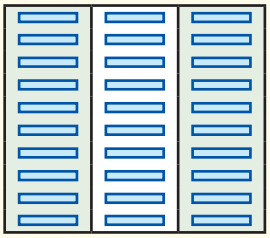
Multiples of **10** are:

10, 20, 30, 40, 50, 60, 70, 80, 90, 100

The lowest common multiple here is **30**.

Multiples of **30** are multiples of both **6** and **10**.



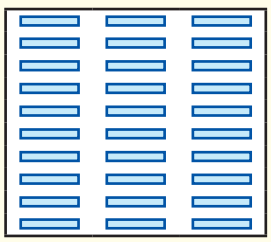


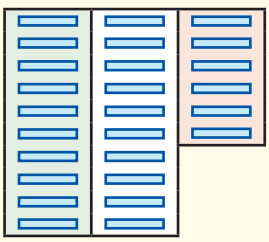
56 chocolate bars can be arranged as one group of **30**, with **56 – 30 = 26** chocolate bars remaining.

To buy exactly **26** chocolate bars, Sara would need to buy two boxes of **10**, and one box of **6**.

There are then **2** different ways that she could buy the other **30** bars.

In total, there are **2** ways that Sara could buy exactly **56** bars.





Follow-Up: *In how many different ways could Sara buy exactly 70 chocolate bars?* [3: 7 × 10, 4 × 10 + 5 × 6, 1 × 10 + 10 × 6]