2025 Maths Olympiads Division S Preparation Kit

Preparing for the APSMO Maths Olympiads

The purpose of this Preparation Kit is to provide students with an opportunity to familiarise themselves with the concepts, and terminology, that will subsequently be used in the four competition papers for 2025:

- Wednesday, 7 May 2025
- Wednesday, 11 June 2025
- Wednesday, 30 July 2025
- Wednesday, 10 September 2025

For each of the problems in this kit, a number of different solution methods are suggested, so that students can be exposed to multiple ways of approaching mathematical problems.

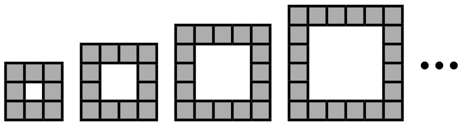
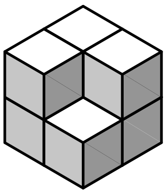
The first five questions are also each presented with a set of student work samples. The student samples demonstrate further solution methods, both correct and incorrect.

The kit additionally includes a reference sheet for relevant skills and terminology. This reference sheet can also be found in the Contest Preparation section of the Members' Portal.

Examples of how this kit may be used include:

- Reinforcing previously learned concepts and terminology
- Introducing new or different solution methods
- Providing diagrams that support a teacher's or student's explanations
- Offering problem-solving homework
- Supporting students' own study as a standalone resource

Further questions and solution methods can also be found in the APSMO resource books, available from www.apsmo.edu.au.

Questions	
A. Each figure in the sequence shown is entirely composed of 1 × 1 shaded squares.  If the pattern continues, how many 1 × 1 shaded squares will there be in the 20th figure?	*Write your answers in the boxes on the back.* ← *Keep your answers hidden by folding backwards on this line.*
B. Jason has eight small wooden cubes, each with an edge length of 1 cm. He arranges them to form a larger cube with an edge length of 2 cm. Then he removes one of the small cubes from a corner of the larger cube. What is the surface area, in square centimetres, of Jason's object? 	
C. Each person has some information about a secret number. • Amelia knows that the secret number is exactly divisible by 9. • Blake knows that the secret number is a 3-digit number with digits appearing in increasing order. • Chelsea knows the difference between the last and middle digits does not equal the difference between the middle and first digits. • Daniel knows that neither 1 nor 9 is used anywhere as a digit. What is the secret number?	
D. In the following cryptarithm, different letters represent different digits. What is the greatest possible value represented by *MEOW*? $\begin{array}{r} CAT \\ +\ CAT \\ \hline MEOW \end{array}$	
E. There are 25 prime numbers less than 100. Some of these are two-digit prime numbers. What fraction of these two-digit prime numbers contain the digit 9? Express the answer in its most simplified form.	

A.	*Fold here. Keep your answers hidden.*
B.	
C.	
D.	
E.	

Write your answers in the boxes on the back.

Keep your answers hidden by folding backwards on this line.

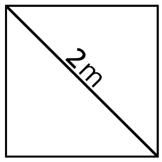
F. What is the probability that a randomly selected three-digit positive integer has no repeated digits?

Express your answer as a fraction in its lowest terms, in decimal form, and as a percentage.

G. Mrs Hicks marked out a square in the quadrangle.

The diagonals of this square are 2 metres in length.

What is the area of the square, in square metres?



H. 2^n means that 2 is multiplied by itself n times.

For example, 2^4 means $2 \times 2 \times 2 \times 2 = 16$.

The whole number $2^5 \times 3^4 \times 5^9$ has exactly K perfect square factors.

Find the whole number value of K.

I. Rory has four clear plastic pockets on his pencil case.

The pockets contain letter tiles that form "*RORY*".

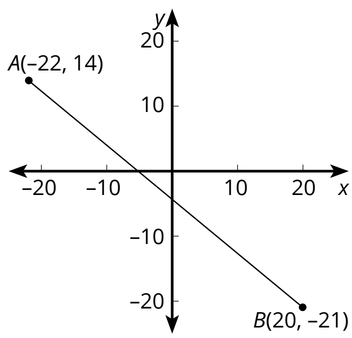
One day, Rory rearranged the tiles to form "*YRRO*".

How many different four-letter arrangements can be made using his tiles, including "*RORY*" and "*YRRO*"?

J. A lattice point is a point on a Cartesian plane where both the x and y co-ordinates are integers.

The interval $\overline{AB}$ joins the points $A(-22, 14)$ and $B(20, -21)$.

Including endpoints A and B, how many lattice points lie on $\overline{AB}$?

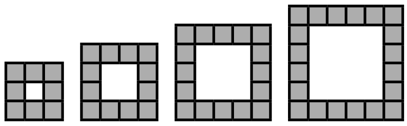


F.	*Fold here. Keep your answers hidden.*
G.	
H.	
I.	
J.	

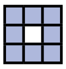
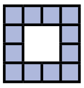
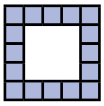
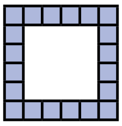
Example Solution A

Each figure in the sequence shown is entirely composed of 1 × 1 shaded squares.

If the pattern continues, how many 1 × 1 shaded squares will there be in the 20th figure?

 …

Strategy 1: Find a Pattern

Position in sequence (n)	1	2	3	4
Figure				
No. of shaded squares	8	12	16	20

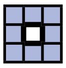
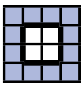
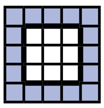
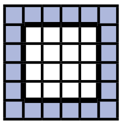
+4 +4 +4

As the length of the side of each figure increases by **1**, there is an increase of **4** shaded squares.

The number of shaded squares in the n^{th} figure is **(n + 1) × 4**.

Therefore, the number of shaded squares in the **20**th figure is **(20 + 1) × 4 = 84**.

Strategy 2: Consider the total number of squares, and the number of inner squares.

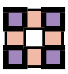
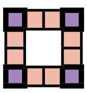
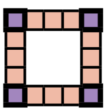
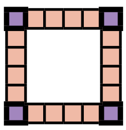
Position in sequence (n)	1	2	3	4
Figure				
Total Squares	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$	$6^2 = 36$
Inner Squares	$1^2 = 1$	$2^2 = 4$	$3^2 = 9$	$4^2 = 16$
No. of shaded squares	8	12	16	20

The number of shaded squares can be calculated by counting all of the squares, and then subtracting the number of inner squares.

The outer square has a side length that is **2** more than the position in the sequence.

The number of shaded squares in the **20**th figure is **$22^2 - 20^2 = 484 - 400 = 84$**.

Strategy 3: Consider the number of corner and non-corner shaded squares.

Position in sequence (n)	1	2	3	4
Figure				
Corner Squares	4	4	4	4
Non-Corner Squares	4 × 1 = 4	4 × 2 = 8	4 × 3 = 12	4 × 4 = 16
No. of shaded squares	8	12	16	20

There are **4** corner squares in each figure.

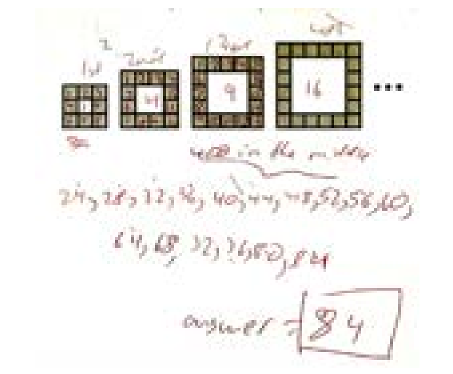
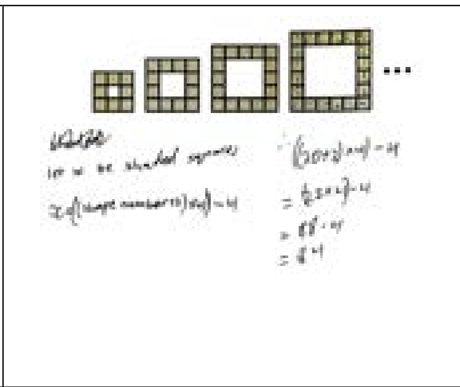
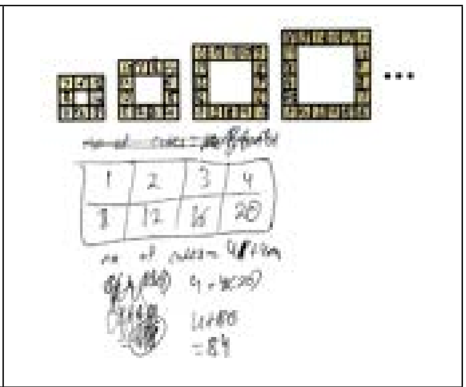
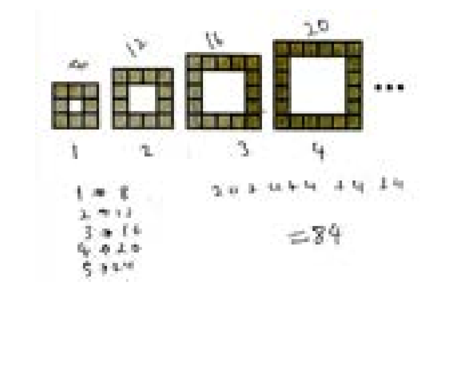
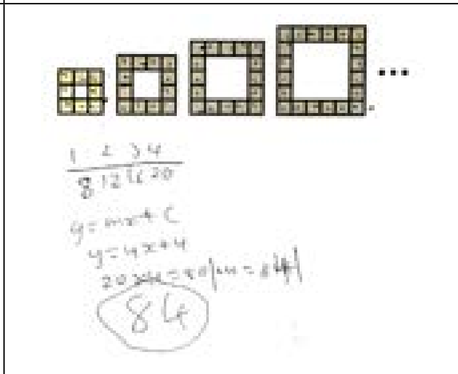
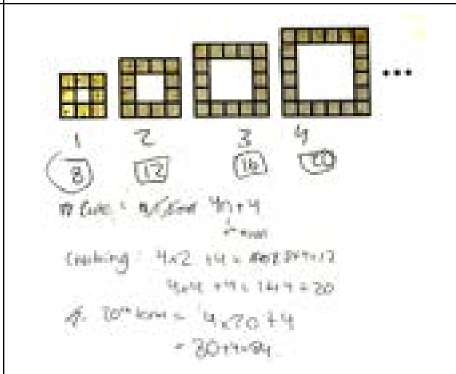
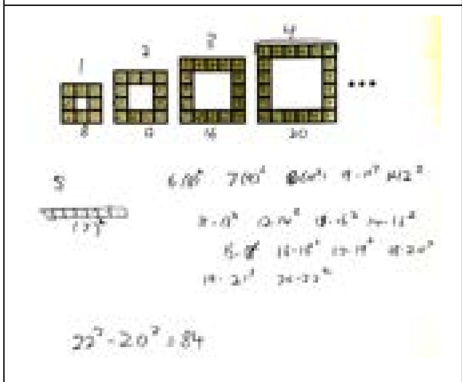
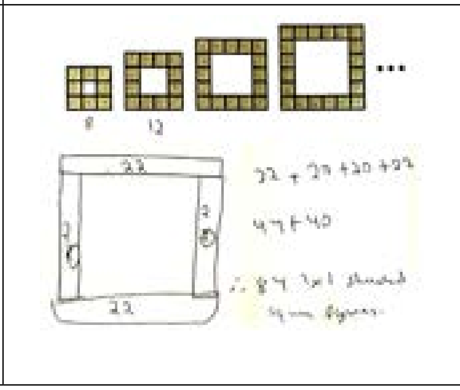
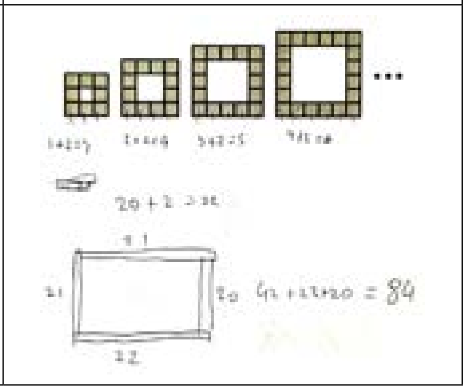
The number of shaded squares can then be found by adding the number of non-corner shaded squares in each figure, to the **4** corner squares.

The number of shaded squares in the **20**th figure is **4 + (4 × 20) = 84**.

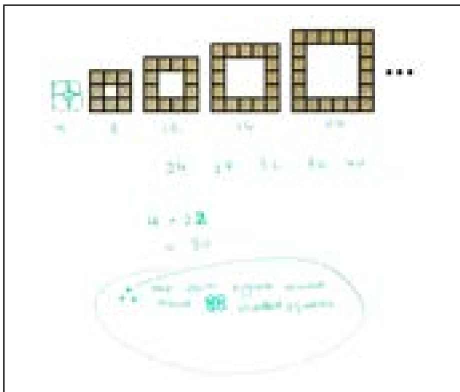
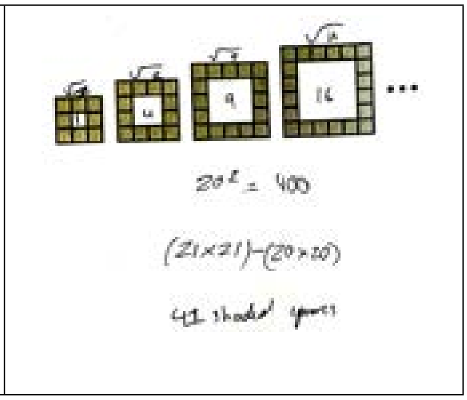
Answer: 84

Student Work Samples A

Students used various methods to solve this problem.

Students who did not get an answer of 84, often used methods that were valid and efficient - they just made mistakes when counting.

Answer: 84

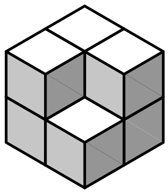
Example Solution B

Jason has eight small wooden cubes, each with an edge length of 1 cm.

He arranges them to form a larger cube with an edge length of 2 cm.

Then he removes one of the small cubes from a corner of the larger cube.

What is the surface area, in square centimetres, of the remaining object (including its bottom surface)?

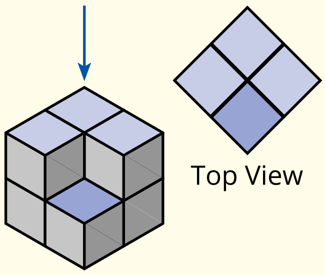


Strategy: Divide a Complex Shape

Let's consider how the object looks from each direction.

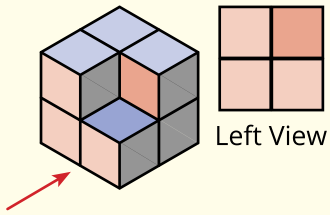
Looking straight down at the top of the cube, we would see three squares on the top level and one square that is a bit further away.

That's 4 square centimetres of surface area.



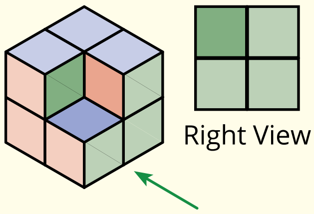
Looking at the cube from the left side, we would see three squares on the near face and one square that is a bit further away.

Again, that's 4 square centimetres of surface area.



Looking at the cube from the right side, we would see three squares on the near face and one square that is a bit further away.

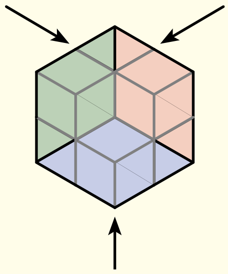
Once again, that's 4 square centimetres of surface area.



A cube has six faces.

We know that the 3 faces that we cannot see were not affected by the removal of the 1 cm^3 cube.

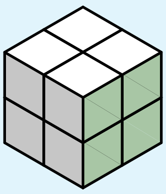
So those faces will each have 4 square centimetres of surface area.



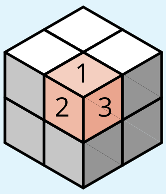
Therefore the total surface area is **6 × 4 = 24** square centimetres.

Strategy: Divide a Complex Shape (Alternate Method)

Jason's original object was made up of eight cubes.

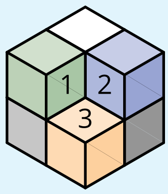


Each of the eight cubes contributes three faces to the surface area of the object.



When Jason removed one cube, he:

- removed the three faces that had been contributed by that cube, and
- also exposed three faces that had previously been hidden by that cube.



The 8 cm^3 cube had six 4 cm^2 faces, for a total surface area of 6 × 4 = 24 cm^2.

Alternatively, with 8 cubes in the original object, each contributing three 1 cm^2 faces to the surface area, the original object had a total surface area of 8 × 3 cm^2 = 24 cm^2.

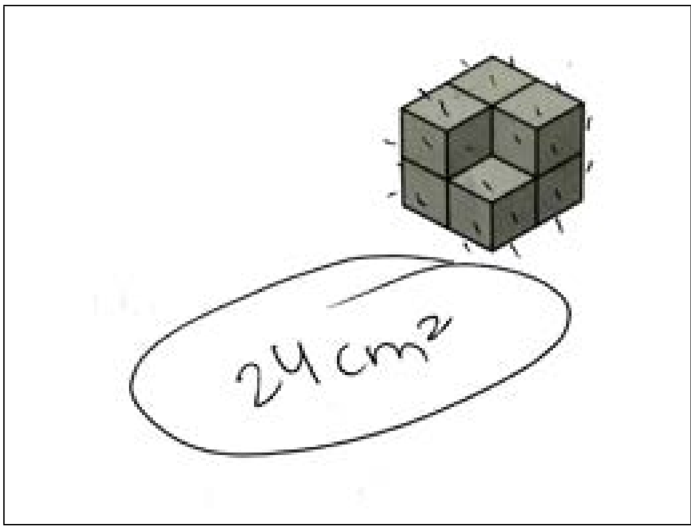
After removing one of the 1 cm^3 cubes in the object, the total surface area remains the same.

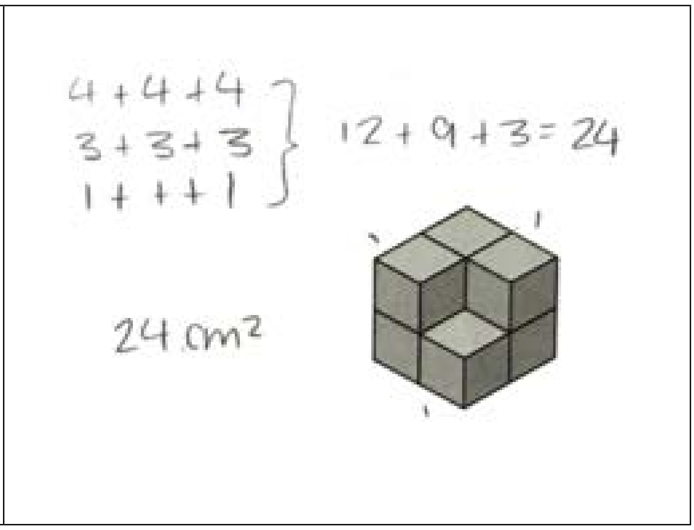
Therefore the exposed surface area of the object is **24** cm^2.

Answer: 24 cm^2

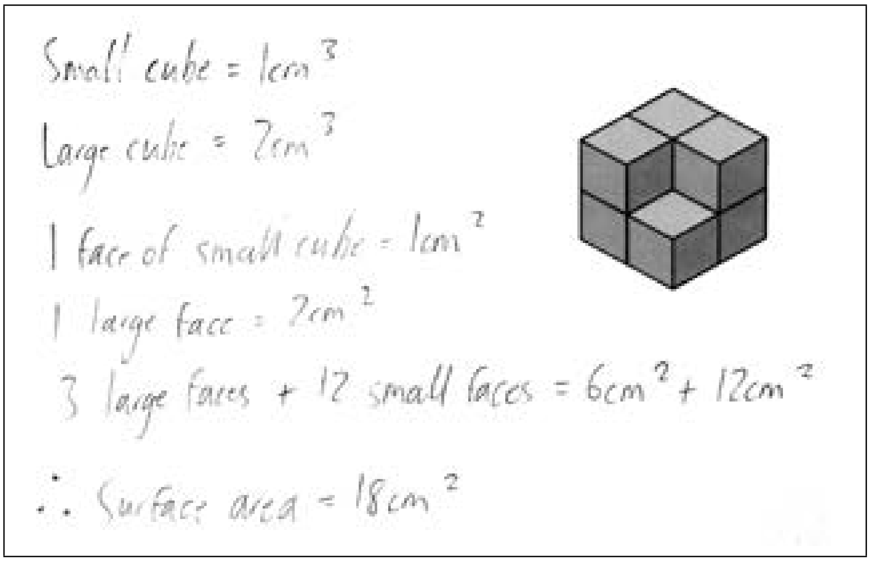
Student Work Samples B

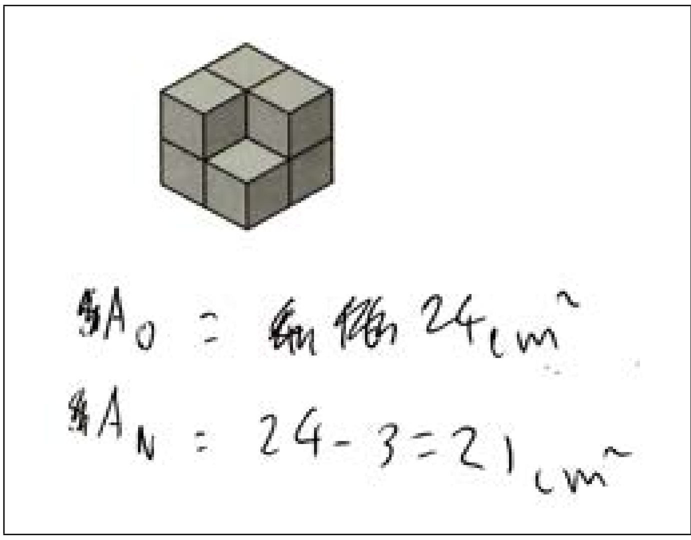
Students used various methods when attempting to solve this problem.

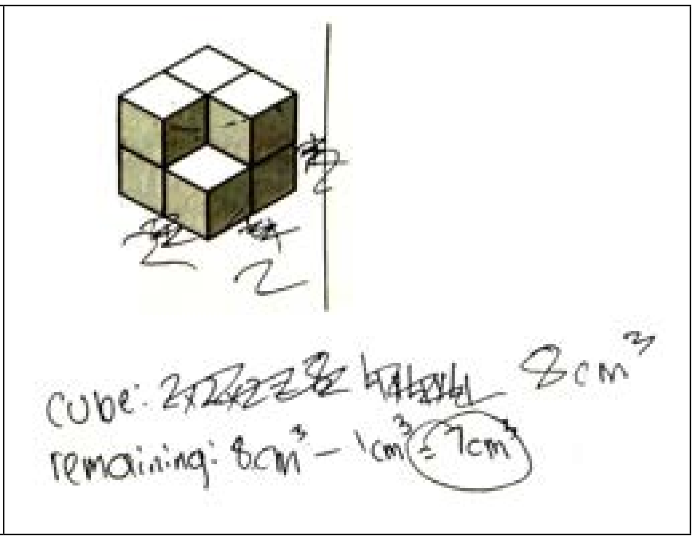




Can you suggest the reasoning that was used by the students who made a mistake?







Answer: 24 cm^2

Example Solution C

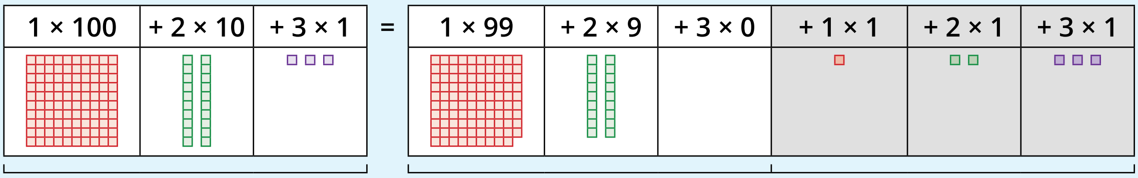
- Amelia knows that the secret number is exactly divisible by 9.
- Blake knows that the secret number is a 3-digit number with digits appearing in increasing order.
- Chelsea knows the difference between the last and middle digits does not equal the difference between the middle and first digits.
- Daniel knows that neither 1 nor 9 is used anywhere as a digit.

What is the secret number?

Strategy: Apply the process of elimination.

Amelia knows that the secret number is exactly divisible by **9**.

To work out if the number **123** is divisible by **9**, we can consider the value of each of the digits.



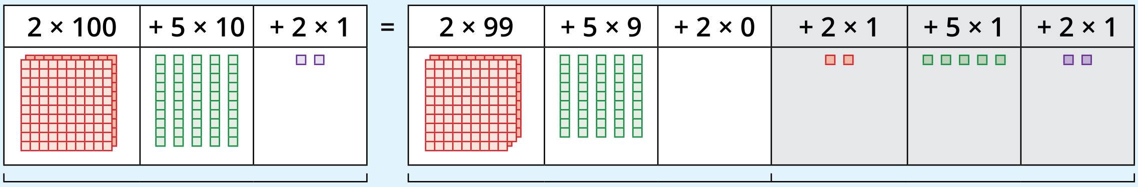
123 is equal to **(1 × 100) + (2 × 10) + (3 × 1)**.

Since **99**, **9**, and **0** are all divisible by **9**, we know that **(1 × 99) + (2 × 9) + (3 × 0)** is divisible by **9**.

Therefore, the number would be divisible by 9 if **(1 × 1) + (2 × 1) + (3 × 1)** were divisible by **9**.

We can use this method to determine if any number is divisible by **9**.

For example, for **252**:



252 is equal to **(2 × 100) + (5 × 10) + (2 × 1)**.

(2 × 99) + (5 × 9) + (2 × 0) is divisible by **9**.

Since **2 + 5 + 2** is divisible by **9**, **252** is divisible by **9**.

If a number's digit sum is divisible by **9**, then that number is also divisible by **9**.

Amelia's information means that the sum of the digits must be **9** or **18**.

Blake's information limits the number to between **123** and **789** inclusive.

Daniel's information further restricts the number to between **234** and **678** inclusive.

We can now list combinations where the digit sum is either **9** or **18**, neither **1** nor **9** is used anywhere as a digit, and the tens digit is greater than the hundreds digit.

H	T	O
2	3	4
2	4	3
2	5	2
~~2~~	~~6~~	~~1~~
2	7	0
2	8	8

H	T	O
3	4	2
~~3~~	~~5~~	~~1~~
3	6	0
3	7	8
3	8	7

H	T	O
~~4~~	~~5~~	~~9~~
4	6	8
4	7	7
4	8	6

H	T	O
5	6	7
5	7	6
5	8	5

H	T	O
6	7	5
~~6~~	~~8~~	~~4~~

After removing numbers where the ones digit is not greater than the tens digit, we are left with **234**, **378**, **468** or **567**.

Finally, Chelsea's information eliminates all but **378**.

The secret number is **378**.

H	T	O
2	3	4
~~2~~	~~4~~	~~3~~
~~2~~	~~5~~	~~2~~
~~2~~	~~6~~	~~1~~
~~2~~	~~7~~	~~0~~
~~2~~	~~8~~	~~8~~

H	T	O
~~3~~	~~4~~	~~2~~
~~3~~	~~5~~	~~1~~
~~3~~	~~6~~	~~0~~
3	7	8
~~3~~	~~8~~	~~7~~

H	T	O
~~4~~	~~5~~	~~9~~
4	6	8
~~4~~	~~7~~	~~7~~
~~4~~	~~8~~	~~6~~

H	T	O
5	6	7
~~5~~	~~7~~	~~6~~
~~5~~	~~8~~	~~5~~

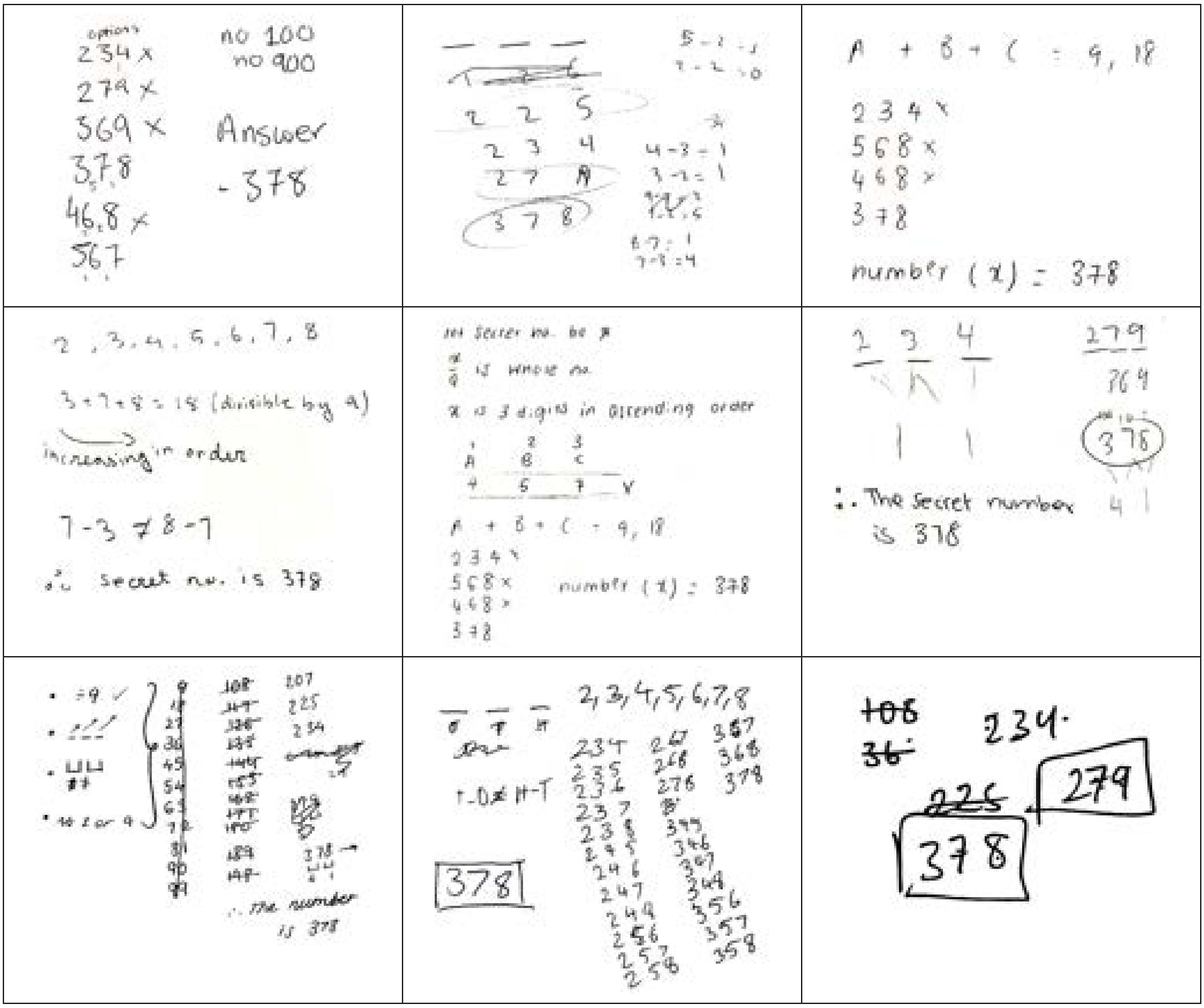
H	T	O
~~6~~	~~7~~	~~5~~
~~6~~	~~8~~	~~4~~

Answer: 378

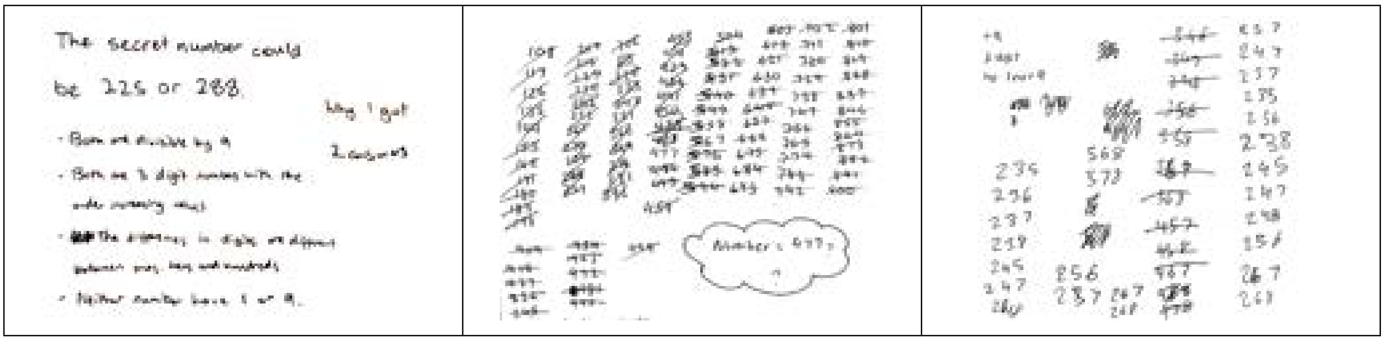
Student Work Samples C

The following students solved the problem correctly.

Can you identify the piece of information that each student used first?



These students did not have 378 as their answer. Identify the error in each case.



Answer: 378

MATHS OLYMPIAD

Example Solution D

A cryptarithm is a mathematical puzzle where digits in a calculation have been replaced by letters.

Different letters represent different digits.

What is the greatest possible value represented by *MEOW*?

```
    C A T
  + C A T
  -------
  M E O W
```

Strategy: Eliminate All But One Possibility

To find the greatest possible value for ***MEOW***, we might begin by considering the greatest possible value for ***M***.

Suppose ***M*** represented 9.
Then, the greatest possible value for ***C*** would be 8.

```
    8 A T
  + 8 A T
  -------
  9 E O W
```

Clearly, having ***C*** = 8 makes it impossible for ***M*** to be 9.

The greatest possible value for ***M*** is 1.

```
    8 A T
  + 8 A T
  -------
  1 7 O W
```

Using the preceding argument, we can see that the greatest possible value for ***MEOW*** requires us to maximise the value for ***C***.

Let's set ***C*** to 9.

Since 9 is taken, the greatest possible value for ***E*** would then be 8.

```
    9 A T
  + 9 A T
  -------
  1 8 O W
```

Since 9 and 8 are taken, the greatest possible value for ***O*** would be 7.

```
    9 A T
  + 9 A T
  -------
  1 8 7 W
```

It's not possible for ***A*** + ***A*** = 7, but with trading from the ones place, we can have ***A*** + ***A*** + 1 = 7.

```
      1
    9 3 T
  + 9 3 T
  -------
  1 8 7 W
```

9, 8, and 7 are now taken.

The greatest possible value for ***W*** is 6.

```
      1
    9 3 T
  + 9 3 T
  -------
  1 8 7 6
```

If ***W*** is 6, then with the trading requirement we will have ***T*** + ***T*** = 16.

That cannot be right, because ***E*** = 8. So ***T*** cannot be 8.

```
      1
    9 3 (8)
  + 9 3 (8)
  ---------
  1 [8] 7 6
```

The next greatest possible value for ***W*** is 5. If so, we will have ***T*** + ***T*** = 15.

This is not possible, since ***T*** cannot be 7.5.

```
      1
    9 3 T
  + 9 3 T
  -------
  1 8 7 5
```

The following values are likewise not possible:

- ***T*** + ***T*** = 14. ***T*** cannot be 7.
- ***T*** + ***T*** = 13. ***T*** cannot be 6.5.

```
      1
    9 3 T
  + 9 3 T
  -------
  1 8 7 W
```

The next greatest possible value for ***W*** is 2.

If so, we will have ***T*** + ***T*** = 12, and so ***T*** = 6.

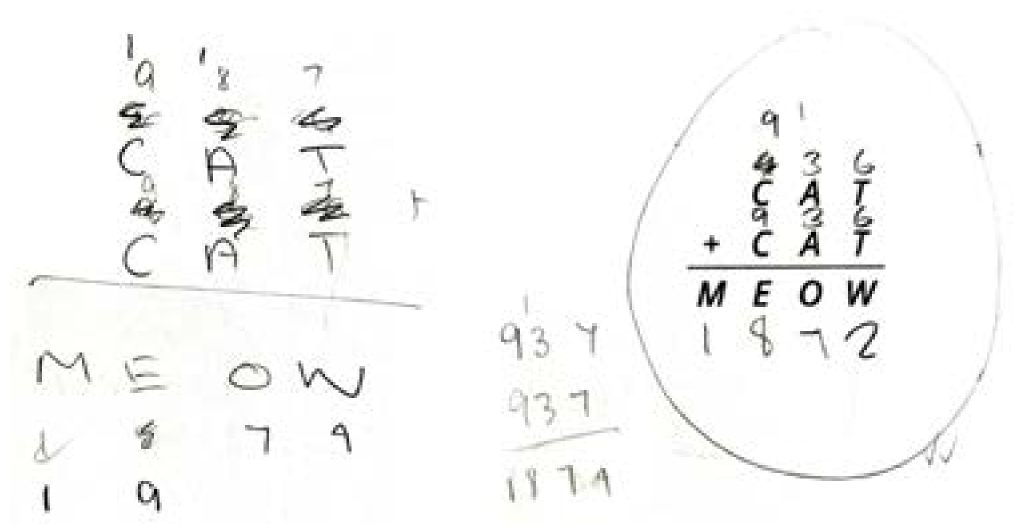
```
      1
    9 3 6
  + 9 3 6
  -------
  1 8 7 2
```

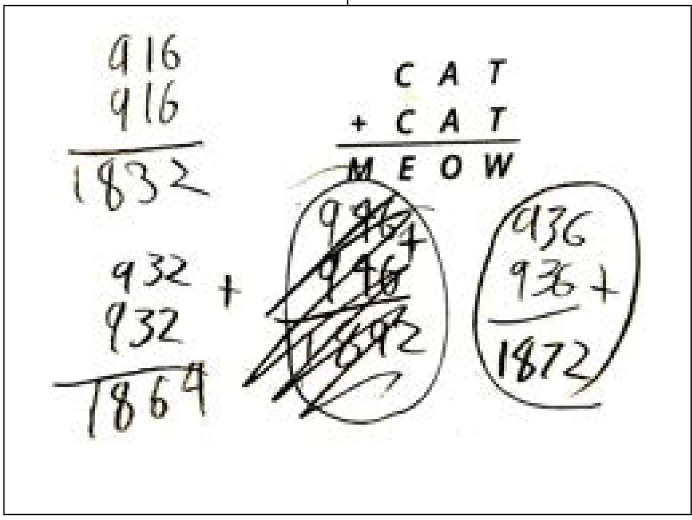
The greatest possible value for ***MEOW*** is **1872**.

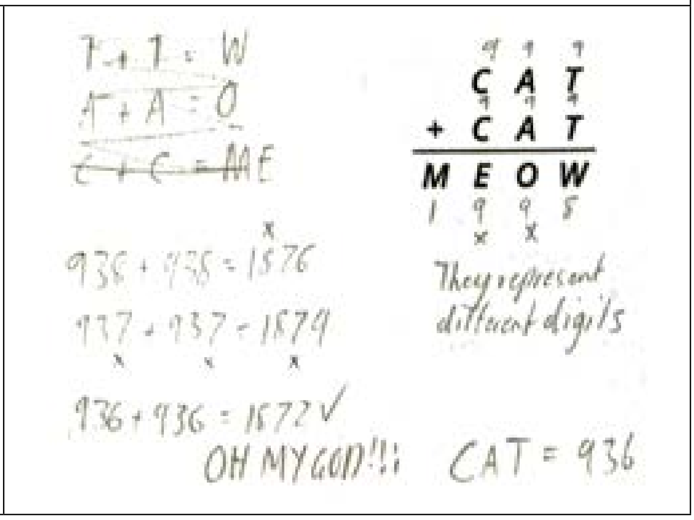
Answer: 1872

Student Work Samples D

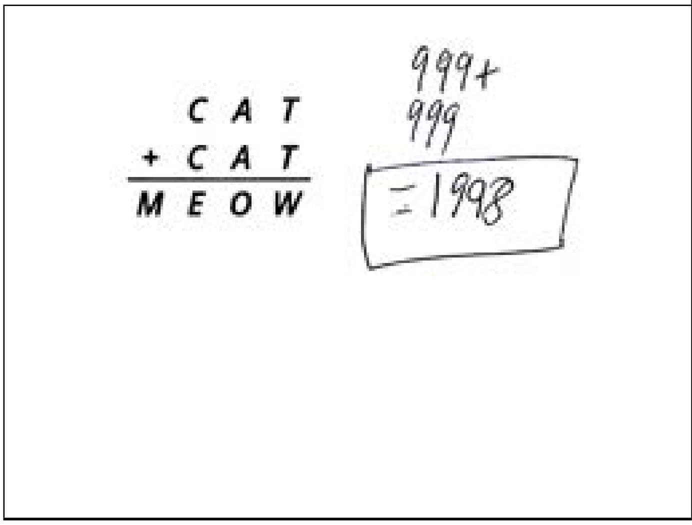
Many students quickly realised that a Guess, Check and Refine approach was suitable for solving this problem.

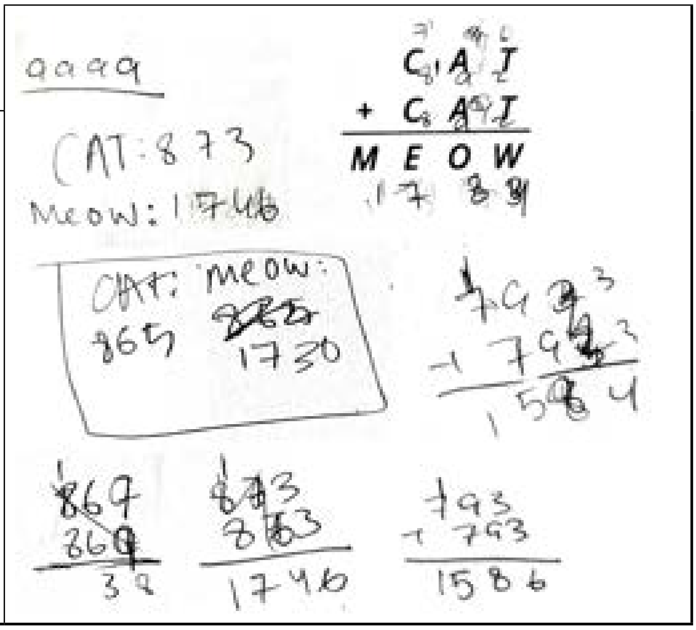






The following work samples resulted in incorrect answers. Describe the error in each case.





Answer: 1872

Example Solution E

There are 25 prime numbers less than 100.

Some of these are two-digit prime numbers.

What fraction of two-digit prime numbers contain the digit 9?

Strategy: Make an Organised List

We can begin by considering two-digit numbers that contain the digit **9**.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Eliminate multiples of 2 and 3.

1	**2**	**3**	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	25	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	35	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	49	~~50~~
~~51~~	~~52~~	53	~~54~~	55	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	65	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	77	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	85	~~86~~	~~87~~	~~88~~	89	~~90~~
91	~~92~~	~~93~~	~~94~~	95	~~96~~	97	~~98~~	~~99~~	~~100~~

Eliminate multiples of 5 and 7.

1	**2**	**3**	~~4~~	**5**	~~6~~	**7**	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

All composite numbers under 10^2 must have at least one factor that is less than **10**.

By considering every prime number less than **10**, and eliminating all of their multiples, we can be sure that we have eliminated all of the two-digit composite numbers.

All of the two-digit numbers that remain on the grid must be prime.

There are 21 two-digit prime numbers.

There are 6 two-digit primes that contain the digit **9**.

1	**2**	**3**	~~4~~	**5**	~~6~~	**7**	~~8~~	~~9~~	~~10~~
11	~~12~~	**13**	~~14~~	~~15~~	~~16~~	**17**	~~18~~	**19**	~~20~~
~~21~~	~~22~~	**23**	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	**29**	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	**37**	~~38~~	~~39~~	~~40~~
41	~~42~~	**43**	~~44~~	~~45~~	~~46~~	**47**	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	**53**	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	**59**	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	**67**	~~68~~	~~69~~	~~70~~
71	~~72~~	**73**	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	**79**	~~80~~
~~81~~	~~82~~	**83**	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	**89**	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	**97**	~~98~~	~~99~~	~~100~~

The fraction of two-digit prime numbers that contain the digit **9** is $\frac{6}{21} = \frac{2}{7}$.

Answer: $\frac{2}{7}$

Student Work Samples E

Students provided a range of different answers for this problem.

For students who made a mistake, identify the errors in their answers.

Answer: $\frac{2}{7}$

Example Solution F

What is the probability that a randomly selected three-digit positive integer has no repeated digits?

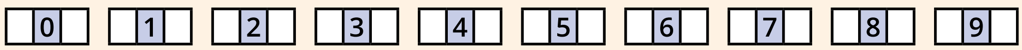
Express your answer both as a fraction in its lowest terms, in decimal form, and as a percentage.

Strategy: Count in an Organised Way

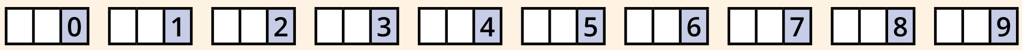
We begin by finding the number of 3-digit positive integers.

With 9 possible values in the hundreds place,

10 possible values in the tens place,



and 10 possible values in the ones place,

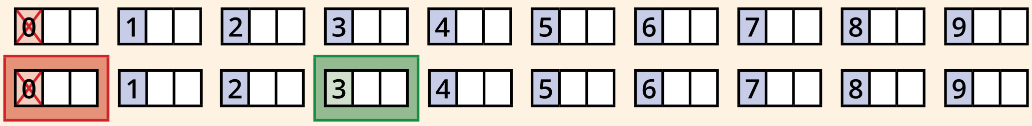


there are $9 \times 10 \times 10 = 900$ different 3-digit positive integers.

To find how many of these 3-digit positive integers have no repeated digits:

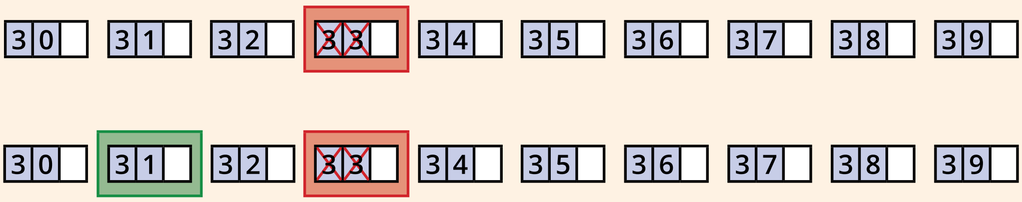
There are 9 possible values in the 100s place.

Let's suppose that the hundreds digit is 3.

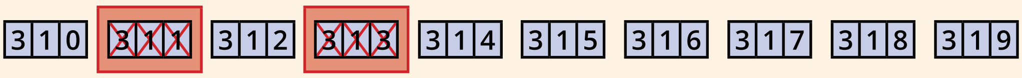


If we want no repeated digits, there are now just 9 possible values in the tens place.

Let's suppose that the tens digit is 1.



If we want no repeated digits, there are now just 8 possible values in the ones place.



There are $9 \times 9 \times 8 = 648$ 3-digit positive integers that have no repeated digits.

The probability that a randomly selected 3-digit positive integer has no repeated digits, is $\frac{9 \times 9 \times 8}{9 \times 10 \times 10} = \frac{648}{900}$.

Fraction in Lowest Terms

$$\frac{9 \times 9 \times 8}{9 \times 10 \times 10} = \frac{9}{9} \times \frac{9 \times 2 \times 2 \times 2}{5 \times 2 \times 5 \times 2}$$
$$= 1 \times \frac{9 \times 2}{5 \times 5} \times \frac{2 \times 2}{2 \times 2}$$
$$= 1 \times \frac{18}{25} \times 1$$
$$= \frac{18}{25}$$

Decimal and Percentage

$$\frac{9 \times 9 \times 8}{9 \times 10 \times 10} = \frac{9}{9} \times \frac{72}{100}$$
$$= 1 \times 72\%$$
$$= 1 \times 0.72$$
$$= 0.72$$

As a fraction, the probability is $\frac{18}{25}$.

As a decimal, the probability is **0.72**.

As a percentage, the probability is **72%**.

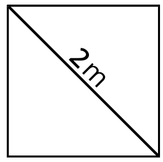
Answer: $\frac{18}{25}$; 0.72, 72%

Example Solution G

Mrs Hicks marked out a square in the quadrangle.

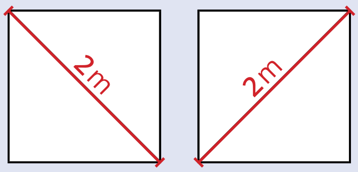
The diagonals of this square are 2 metres in length.

What is the area of the square, in square metres?

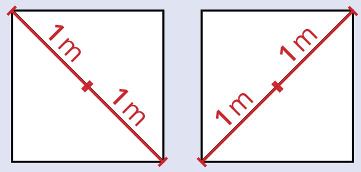


Strategy: Divide The Shape

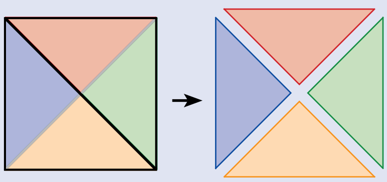
The length of each diagonal, from one vertex to the opposite vertex, is 2 metres.



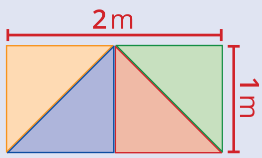
So the length of each line from the vertex to the centre of the square is 2 ÷ 2 = 1 metre.



We can begin by dividing the square into four triangles, along the two diagonals.



These four triangles can be reassembled into a rectangle that is 2 metres long and 1 metre high.



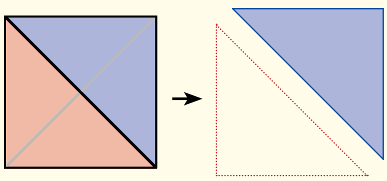
The area of the resulting rectangle is 2 × 1 = 2 square metres.

Since we have used all of the pieces from the original square, this rectangle must have the same area.

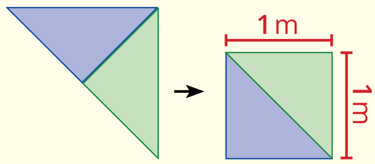
Therefore the area of the original square is **2** square metres.

Strategy: Divide The Shape (Alternate Approach)

Suppose we divide the square in half, along a diagonal.



This half can be divided into two triangles, and then rearranged to form a square.



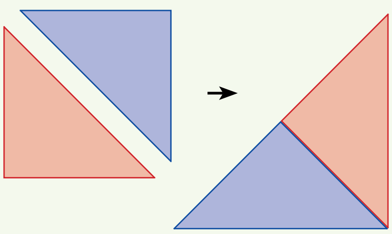
The area of the resulting square is 1 × 1 = 1 square metre.

This square is half of the area of the original square.

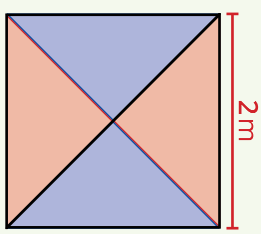
Therefore the area of the original square is 2 × 1 = **2** square metres.

Strategy: Divide The Shape (Another Approach)

If we divide the square in half along a diagonal, and rearrange it to form a triangle,



doubling that triangle forms a square with sides of length 2 metres.



The area of the resulting square is 2 × 2 = 4 square metres.

This square is double the area of the original square.

Therefore the area of the original square is 4 ÷ 2 = **2** square metres.

Answer: 2 (m^2)

Example Solution H

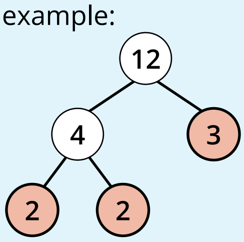
2^n means that 2 is multiplied by itself n times. For example, 2^4 means $2 \times 2 \times 2 \times 2 = 16$.

The whole number $2^5 \times 3^4 \times 5^9$ has exactly K perfect square factors. Find the whole number value of K.

Strategy: Consider the characteristics of a perfect square.

Every whole number can be expressed in terms of prime factors.

For example:

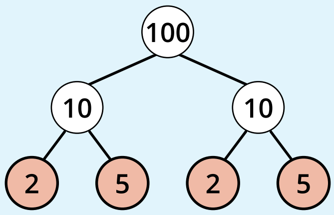


$12 = 2 \times 2 \times 3 = 2^2 \times 3^1$

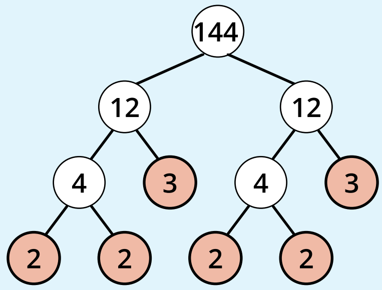
A perfect square results from multiplying a number (the square root) by itself.

The square root must be able to be expressed in terms of prime factors.

Therefore, a number is a perfect square if each of its prime factors occurs an even number of times.



$100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$



$144 = 2^4 \times 3^2$

Since $2^5 \times 3^4 \times 5^9 = \underbrace{2 \times 2 \times 2 \times 2 \times 2}_{\text{5 times}} \times \underbrace{3 \times 3 \times 3 \times 3}_{\text{4 times}} \times \underbrace{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}_{\text{9 times}}$,

a perfect square factor of $2^5 \times 3^4 \times 5^9$ must have:
- 2 exactly 0, 2, or 4 times;
- 3 exactly 0, 2, or 4 times; and
- 5 exactly 0, 2, 4, 6, or 8 times.

We can begin by listing perfect square factors with one prime factor value:

- $2^0, 2^2, 2^4$ (1, 4, 16);
- $3^0, 3^2, 3^4$ (1, 9, 81);
- $5^0, 5^2, 5^4, 5^6$, and 5^8 (1, 25, 625, 15625, 390625).

Note that $2^0 = 3^0 = 5^0 = 1$.

So far, there are 9 perfect square factors.

There are 20 with two prime factor values.

	2^2	2^4
3^2	$2^2 \times 3^2$	$2^4 \times 3^2$
3^4	$2^2 \times 3^4$	$2^4 \times 3^4$

	2^2	2^4
5^2	$2^2 \times 5^2$	$2^4 \times 5^2$
5^4	$2^2 \times 5^4$	$2^4 \times 5^4$
5^6	$2^2 \times 5^6$	$2^4 \times 5^6$
5^8	$2^2 \times 5^8$	$2^4 \times 5^8$

	3^2	3^4
5^2	$3^2 \times 5^2$	$3^4 \times 5^2$
5^4	$3^2 \times 5^4$	$3^4 \times 5^4$
5^6	$3^2 \times 5^6$	$3^4 \times 5^6$
5^8	$3^2 \times 5^8$	$3^4 \times 5^8$

There are 16 with three prime factor values.

	$2^2 \times 3^2$	$2^2 \times 3^4$	$2^4 \times 3^2$	$2^4 \times 3^4$
5^2	$2^2 \times 3^2 \times 5^2$	$2^2 \times 3^4 \times 5^2$	$2^4 \times 3^2 \times 5^2$	$2^4 \times 3^4 \times 5^2$
5^4	$2^2 \times 3^2 \times 5^4$	$2^2 \times 3^4 \times 5^4$	$2^4 \times 3^2 \times 5^4$	$2^4 \times 3^4 \times 5^4$
5^6	$2^2 \times 3^2 \times 5^6$	$2^2 \times 3^4 \times 5^6$	$2^4 \times 3^2 \times 5^6$	$2^4 \times 3^4 \times 5^6$
5^8	$2^2 \times 3^2 \times 5^8$	$2^2 \times 3^4 \times 5^8$	$2^4 \times 3^2 \times 5^8$	$2^4 \times 3^4 \times 5^8$

In total, there are $9 + 20 + 16 = 45$ perfect square factors.

Alternatively, we can consider all combinations with three prime factor values, including 2^0, 3^0, and 5^0.

	$2^0 \times 3^0$	$2^0 \times 3^2$	$2^0 \times 3^4$	$2^2 \times 3^0$	$2^2 \times 3^2$	$2^2 \times 3^4$	$2^4 \times 3^0$	$2^4 \times 3^2$	$2^4 \times 3^4$
5^0	$2^0 \times 3^0 \times 5^0$	$2^0 \times 3^2 \times 5^0$	$2^0 \times 3^4 \times 5^0$	$2^2 \times 3^0 \times 5^0$	$2^2 \times 3^2 \times 5^0$	$2^2 \times 3^4 \times 5^0$	$2^4 \times 3^2 \times 5^0$	$2^4 \times 3^2 \times 5^0$	$2^4 \times 3^4 \times 5^0$
5^2	$2^0 \times 3^0 \times 5^2$	$2^0 \times 3^2 \times 5^2$	$2^0 \times 3^4 \times 5^2$	$2^2 \times 3^0 \times 5^2$	$2^2 \times 3^2 \times 5^2$	$2^2 \times 3^4 \times 5^2$	$2^4 \times 3^2 \times 5^2$	$2^4 \times 3^2 \times 5^2$	$2^4 \times 3^4 \times 5^2$
5^4	$2^0 \times 3^0 \times 5^4$	$2^0 \times 3^2 \times 5^4$	$2^0 \times 3^4 \times 5^4$	$2^2 \times 3^0 \times 5^4$	$2^2 \times 3^2 \times 5^4$	$2^2 \times 3^4 \times 5^4$	$2^4 \times 3^2 \times 5^4$	$2^4 \times 3^2 \times 5^4$	$2^4 \times 3^4 \times 5^4$
5^6	$2^0 \times 3^0 \times 5^6$	$2^0 \times 3^2 \times 5^6$	$2^0 \times 3^4 \times 5^6$	$2^2 \times 3^0 \times 5^6$	$2^2 \times 3^2 \times 5^6$	$2^2 \times 3^4 \times 5^6$	$2^4 \times 3^2 \times 5^6$	$2^4 \times 3^2 \times 5^6$	$2^4 \times 3^4 \times 5^6$
5^8	$2^0 \times 3^0 \times 5^8$	$2^0 \times 3^2 \times 5^8$	$2^0 \times 3^4 \times 5^8$	$2^2 \times 3^0 \times 5^8$	$2^2 \times 3^2 \times 5^8$	$2^2 \times 3^4 \times 5^8$	$2^4 \times 3^2 \times 5^8$	$2^4 \times 3^2 \times 5^8$	$2^4 \times 3^4 \times 5^8$

In total, there are $3 \times 3 \times 5 = 45$ perfect square factors.

Answer: 45

Example Solution I

Rory has four clear plastic pockets on his pencil case. The pockets contain letter tiles that form "*RORY*".

One day, Rory rearranged the tiles to form "*YRRO*".

How many different four-letter arrangements can be made using his tiles, including "*RORY*" and "*YRRO*"?

Strategy: Make an Organised List

We can list the options alphabetically.

Beginning with arrangements starting with *O*:

- We have *R*, *R*, *Y* left.
- As there is only one *Y*, we just need to think about the different places the *Y* can be.

O	R	R	Y
O	R	Y	R
O	Y	R	R

Next, starting with *R*:

- We have *O*, *R*, *Y* left.
- What options are possible if the second letter is *O*?

R	O	R	Y
R	O	Y	R

- What options are possible if the second letter is the other *R*?

R	R	O	Y
R	R	Y	O

- What options are possible if the second letter is *Y*?

R	Y	O	R
R	Y	R	O

Finally, starting with *Y*:

- We have *O*, *R*, *R* left.
- Since the only one that is different is *O*, we just need to think about the different places the *O* can be.

Y	O	R	R
Y	R	O	R
Y	R	R	O

Since we counted them in an organised way, we can be sure that we have found all of the possible arrangements.

Rory can use his tiles to make **12** different four-letter arrangements.

Strategy: Count in an Organised Way

We can begin by pretending that all of the letter tiles are different. So, instead of ***RORY***, we will imagine that the letter tiles have the letters "***RОry***".

There are four clear plastic pockets on Rory's pencil case.

Since we have 4 different tiles, there would be 4 choices for the first pocket.

1st pocket could be:

	R			
or	O			
or	r			
or	Y			

Suppose we placed the ***r*** in the first pocket.

Then there would be 3 remaining options for the second pocket.

Whichever letter we chose for the first pocket, there would be 3 options remaining for the second pocket.

2nd pocket could be:

	r	R		
or	r	O		
or	r	Y		

There would then be 2 different options remaining for the 3rd pocket,

3rd pocket could be:

	r	Y	R	
or	r	Y	O	

and then just 1 option left for the last pocket.

r	Y	R	O

With 4 ways to choose the first tile, 3 ways for the second, 2 ways for the third and 1 way for the fourth, there are $4 \times 3 \times 2 \times 1 = 24$ ways to arrange these tiles.

However, the **24** combinations would include both ***RОry*** and ***rОRY***, and ***YRrO*** and ***YrRO***, and so on. With two different Rs, we have ended up double-counting each arrangement.

There are $24 \div 2 = 12$ different four-letter arrangements for Rory's tiles.

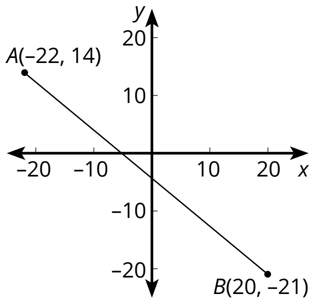
Answer: 12

Example Solution J

A lattice point is a point on a Cartesian plane where both the *x* and *y* co-ordinates are integers.

The interval $\overline{AB}$ joins the points *A*(–22, 14) and *B*(20, –21).

Including endpoints *A* and *B*, how many lattice points lie on $\overline{AB}$?

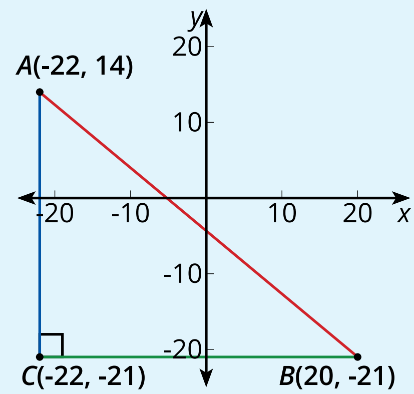


Strategy 1: Consider the Gradient of AB

We begin by constructing the point ***C*** that has the same *x*-value as ***A***, and the same *y*-value as ***B***.

This results in a right-angled triangle ***ABC***.

We can use this triangle to determine the gradient of ***AB***.

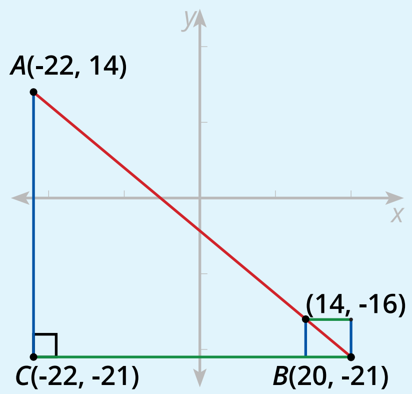


The rise *CA* is 14 – (–21) = 35.

The run *BC* is –22 – 20 = –42.

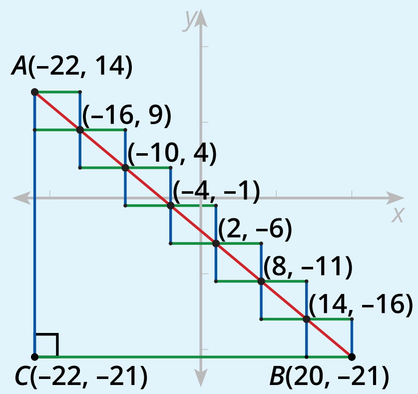
Therefore the gradient of *AB* is

$\frac{CA}{BC} = \frac{35}{-42} = -\frac{5}{6}$.



Since the gradient of *AB* is $-\frac{5}{6}$, then there will be a lattice point 6 units to the left and 5 units up from *B*(20, –21).

This means that there is a lattice point at (14, –16).



We can now continue to locate lattice points that are 6 units to the left and 5 units up from the previous lattice point.

We can see that, including ***A*** and ***B***, there are **8** lattice points on $\overline{AB}$.

Strategy 2: Build a Table, and Find a Pattern

The gradient of the line through *A*(-22, 14) and *B*(20, -21) is

$$\frac{(-21) - 14}{20 - (-22)} = -\frac{5}{6}.$$

Since this line passes through *A*(-22, 14), the equation of the line would be:

$$y - 14 = -\frac{5}{6}(x - (-22))$$

$$y = -\frac{5}{6}x - \frac{13}{3}$$

To find lattice points for $-22 \le x \le 20$ on the line $y = -\frac{5}{6}x - \frac{13}{3}$, consider the *y*-value for every integer value of *x*.

x	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9
y	14	$13\frac{1}{6}$	$12\frac{1}{3}$	$11\frac{1}{2}$	$10\frac{2}{3}$	$9\frac{5}{6}$	9	$8\frac{1}{6}$	$7\frac{1}{3}$	$6\frac{1}{2}$	$5\frac{2}{3}$	$4\frac{5}{6}$	4	$3\frac{1}{6}$

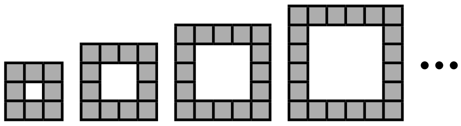
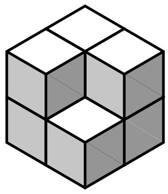
We can see that there is a pattern. A lattice point is occurring for every 6th integral *x*-value.

x	-22	...	-16	...	-10	...	-4	...	2	...	8	...	14	...	20
y	14	...	9	...	4	...	-1	...	-6	...	-11	...	-16	...	-21

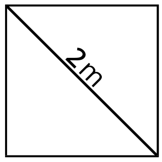
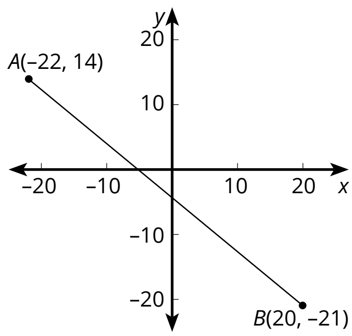
Therefore there are **8** lattice points on $\overline{AB}$.

Answer: 8

Answers

A. **Answer: 84**	Each figure in the sequence shown is entirely composed of 1 × 1 shaded squares.  If the pattern continues, how many 1 × 1 shaded squares will there be in the 20th figure?
B. **Answer: 24** cm^2	Jason has eight small wooden cubes, each with an edge length of 1 cm. He arranges them to form a larger cube with an edge length of 2 cm. Then he removes one of the small cubes from a corner of the larger cube.  What is the surface area, in square centimetres, of Jason's object?
C. **Answer: 378**	Each person has some information about a secret number. • Amelia knows that the secret number is exactly divisible by 9. • Blake knows that the secret number is a 3-digit number with digits appearing in increasing order. • Chelsea knows the difference between the last and middle digits does not equal the difference between the middle and first digits. • Daniel knows that neither 1 nor 9 is used anywhere as a digit. What is the secret number?
D. **Answer: 1872**	In the following cryptarithm, different letters represent different digits. $\begin{array}{r} C\;A\;T \\ +\;C\;A\;T \\ \hline M\;E\;O\;W \end{array}$ What is the greatest possible value represented by *MEOW*?
E. **Answer:** $\frac{2}{7}$	There are 25 prime numbers less than 100. Some of these are two-digit prime numbers. What fraction of these two-digit prime numbers contain the digit 9? Express the answer in its most simplified form.

Answers

F. **Answer:** $\frac{18}{25}$**; 0.72, 72%**	What is the probability that a randomly selected three-digit positive integer has no repeated digits? Express your answer as a fraction in its lowest terms, in decimal form, and as a percentage.
G. **Answer: 2** (m^2)	Mrs Hicks marked out a square in the quadrangle. The diagonals of this square are 2 metres in length. What is the area of the square, in square metres? 
H. **Answer: 45**	2^n means that 2 is multiplied by itself n times. For example, 2^4 means $2 \times 2 \times 2 \times 2 = 16$. The whole number $2^5 \times 3^4 \times 5^9$ has exactly K perfect square factors. Find the whole number value of K.
I. **Answer: 12**	Rory has four clear plastic pockets on his pencil case. The pockets contain letter tiles that form "*RORY*". One day, Rory rearranged the tiles so that they formed "*YRRO*". How many different four-letter arrangements can be made using his tiles, including "*RORY*" and "*YRRO*"?
J. **Answer: 8**	A lattice point is a point on a Cartesian plane where both the x and y co-ordinates are integers. The interval $\overline{AB}$ joins the points $A(-22, 14)$ and $B(20, -21)$. Including endpoints A and B, how many lattice points lie on $\overline{AB}$? 

MATHS OLYMPIAD

Basic Terms

• Sum	• Difference	• Product	• Quotient
• Value	• Multiple	• Factor	• Remainder
• Fraction	• Decimal Fraction	• Percentage	• Ratio
• Square / Perfect Square	• Square Root	• Cube / Perfect Cube	• Cube Root

- "Or" is inclusive: "***a*** or ***b***" means "***a*** or ***b*** or both".
- An ellipsis (...) indicates that some information has been omitted intentionally.
  Read "**1 + 2 + 3 + ...**" as "one plus two plus three and so on, without end."
  Read "**1 + 2 + 3 + ... + 10**" as "one plus two plus three and so on up to ten."

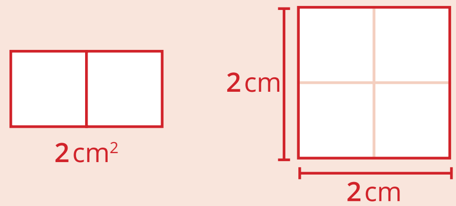
Units of Measurement

Familiarity with units of measurement is assumed, including conversions from one unit to another:

- **Time**: seconds ↔ minutes ↔ hours ↔ days
- **Length**: millimetres ↔ centimetres ↔ metres ↔ kilometres
- **Area**: mm^2 ↔ cm^2 ↔ m^2 ↔ km^2
- **Volume/Capacity**: mm^3 ↔ cm^3 ↔ m^3; millilitres ↔ litres
- **Mass**: grams ↔ kilograms
- **Angles**: degrees (°)

Units of measurement must be correct if given in an answer.

To avoid confusion, read cm^2 as "square centimetres", not "centimetres squared".



2 square centimetres

Possible misinterpretation

Presenting Answers

Unless otherwise specified in a problem, **equivalent numbers or expressions are acceptable**.

- For example, $3\frac{1}{2}$, $\frac{7}{2}$, and 3.5 are equivalent. $3\frac{2}{4}$ and $\frac{70}{20}$ are not in lowest terms and will not be accepted.

After reading a problem, it is useful to indicate the nature of the answer, before commencing the solution strategy. For example:

- "*A* = ____ , *B* = ____ ."
- **"The largest number is ____ ."**
- **"The [sum | difference | product | quotient] is ____ ."**
- **"The probability, as a [fraction | decimal | percentage], is ____ ."**
- **"The perimeter is ____ centimetres."**
- **"The area is ____ square units."**
- **"The average speed is ____ kilometres per hour."**

Number

Digits and Integers

A **digit** is any one of the ten numerals **0, 1, 2, 3, 4, 5, 6, 7, 8, 9.**

- **358** is a three-digit number.

The **lead digit** (leftmost digit) of a number is not counted as a digit if it is **0**.

- **0358** is a three-digit number.

Terminal zeroes of a number are the zeroes to the right of the last nonzero digit.

- **30 500** has two terminal zeroes.

Whole numbers: **{ 0, 1, 2, 3, …. }.**

Counting numbers, or **Positive Integers**: **{ 1, 2, 3, … }.**

Integers: **{ …, –2, –1, 0, 1, 2, 3, … }.**

- Positive numbers, negative numbers, non-negative numbers, and non-positive numbers are terms that may appear in Division S problems.

Consecutive Numbers are counting numbers that differ by **1**.

- **83, 84, 85, 86, 87.**

Consecutive Even Numbers are multiples of **2** that differ by **2.**

- **36, 38, 40, 42.**

Consecutive Odd Numbers are non-multiples of **2** that differ by **2.**

- **57, 59, 61, 63.**

Factors and Divisibility

Suppose $A = B \times C$, and A, B, and C are all **counting numbers** **(1, 2, 3, …).**

- **6** = **2** × **3**.

Then, A is divisible by B, and A is a multiple of B.

- **6** is divisible by **2.**
- **6** is a multiple of **2**

Likewise, A is divisible by C, and A is a multiple of C.

- **6** is divisible by **3.**
- **6** is a multiple of **3**

Both B and C are factors of A.

- **2** and **3** are factors of **6.**

A **prime number** is a counting number with exactly two factors, **1** and itself.

- **2, 3, 5, 7, 11, 13, …**

A **composite number** is a counting number which has at least three different factors.

- **4, 6, 8, 9, 10, 12, …**

The number **1** is neither prime nor composite since it has exactly one factor.

A number is **factored completely** when it is expressed as a product of only prime numbers.

- $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
  $= 2^4 \times 3^2$.

The **Highest Common Factor (HCF)** of two counting numbers is the largest counting number that divides each of the two numbers, and the remainder is zero.

If the **HCF** of two numbers is **1**, then we say that the numbers are **relatively prime**.

- **HCF (12,18) = 6.**

The **Lowest Common Multiple (LCM)** of two counting numbers is the smallest number that each of the given numbers divides, and the remainder is zero.

- **LCM (12,18) = 36.**

Fractions

For common or simple fractions $\frac{a}{b}$,

- a (the numerator) and b (the denominator) are both integers, and
- $b \neq 0$.

In a unit fraction, the numerator is 1.	In a proper fraction, $a < b$.	In an improper fraction, $a \geq b$.
• $\frac{1}{2}$ and $\frac{1}{100}$ are both unit fractions.	• $\frac{1}{2}$ and $\frac{5}{6}$ are both proper fractions.	• $\frac{3}{2}$ and $\frac{11}{8}$ are both improper fractions.

A complex fraction is a fraction whose numerator or denominator contains a fraction.

- $\frac{\frac{2}{3}}{5}$, $\frac{2}{\frac{3}{5}}$, $\frac{\frac{2}{3}}{\frac{5}{7}}$, $\frac{2+\frac{3}{5}}{5-\frac{1}{2}}$ are complex fractions.

The fraction $\frac{a}{b}$ is simplified (in lowest terms) if a and b have no common factor other than **1**, i.e. **HCF(*a*,*b*) = 1**.

- $\frac{1}{2}$ and $\frac{3}{2}$ are both expressed in lowest terms. $\frac{2}{4}$ and $\frac{30}{20}$ are not in lowest terms.
- Unless otherwise specified, fraction answers to Olympiad problems must be expressed in lowest terms.

A decimal or decimal fraction is a fraction whose denominator is a power of ten.

The decimal is written using decimal point notation.

- $0.07 = \frac{7}{100}$, $0.153 = \frac{153}{1000}$, $6.4 = 6\frac{4}{10}$ or $\frac{64}{10}$.

A recurring decimal, or repeating decimal, is a decimal fraction with a digit, or group of digits, that repeats forever.

- $\frac{1}{3} = 0.333... = 0.\dot{3} = 0.\overline{3}$
- $\frac{1}{6} = 0.1666... = 0.1\dot{6} = 0.1\overline{6}$
- $\frac{1}{7} = 0.142857142857... = 0.\dot{1}4285\dot{7} = 0.\overline{142857}$

A percentage is a fraction whose denominator is **100**. The percent sign represents the division by **100**.

- $9\% = \frac{9}{100}$, $125\% = \frac{125}{100}$, $0.3\% = \frac{0.3}{100}$ or $\frac{3}{1000}$.

Order of Operations

When an expression has more than one arithmetic symbol, certain operations occur before others.

There are a few ways to remember the order of operations, and mnemonics are often used (e.g. BIDMAS; PEMDAS).

However, it can also be useful to consider the intent when an arithmetic expression is constructed.

By convention, we observe the following priorities:

1. Perform operations in parentheses, braces, or brackets. The vinculum (line in a fraction) is also considered as a grouping symbol, similar to parentheses.
2. Evaluate exponents (indices).
3. Evaluate multiplication and division, from left to right.
4. Evaluate addition and subtraction, from left to right.

Example 1

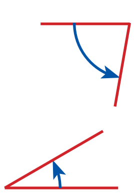
$30 + 6 \div 2 - 5 \times (9 - 7)$
$= 30 + 6 \div 2 - 5 \times 2$
$= 30 + 3 - 10$
$= 23$

Example 2

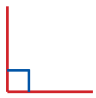
$20 - (8 + (1 + 2)^2)$
$= 20 - (8 + 3^2)$
$= 20 - (8 + 9)$
$= 20 - 17$
$= 3$

Two-Dimensional Figures

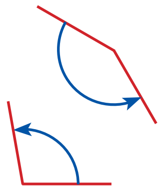
Acute angle
between 0° and 90°



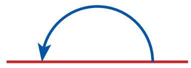
Right angle
90°



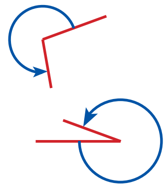
Obtuse angle
between 90° and 180°



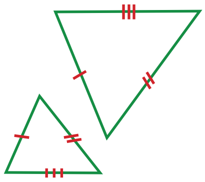
Straight angle
180°



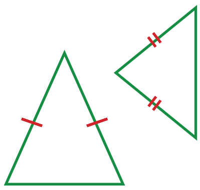
Reflex angle
between 180° and 360°



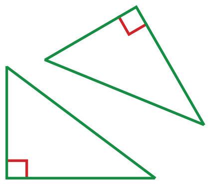
Scalene triangle



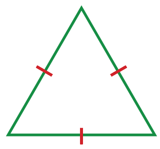
Isosceles triangle



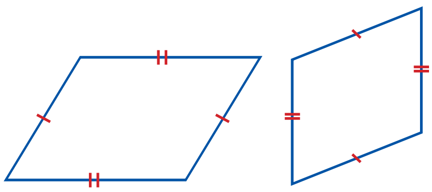
Right-angled triangle



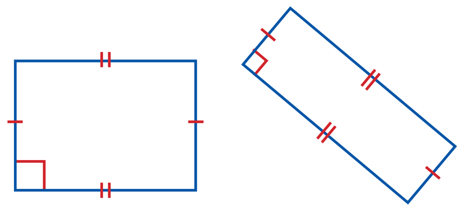
Equilateral triangle



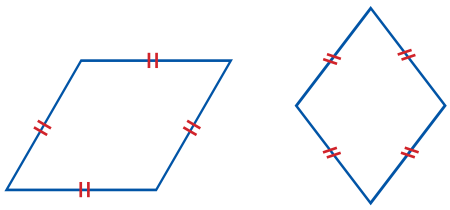
Parallelogram



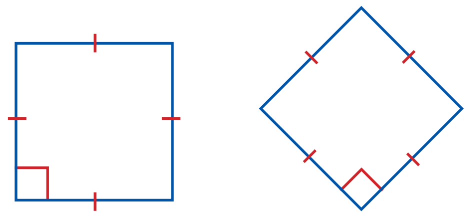
Rectangle



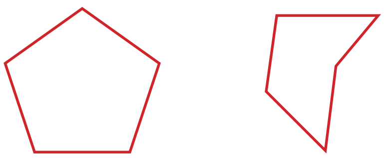
Rhombus



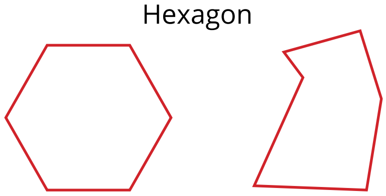
Square



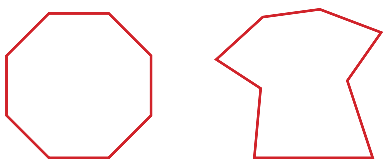
Pentagon



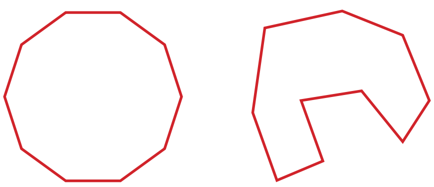
Hexagon



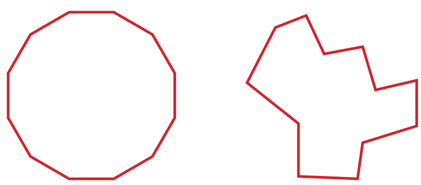
Octagon



Decagon

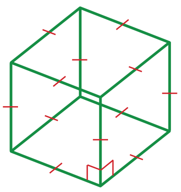


Dodecagon

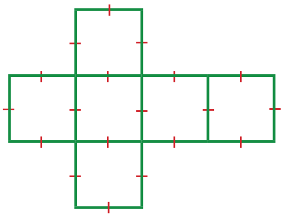


Three-Dimensional Objects

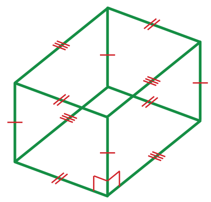
Cube



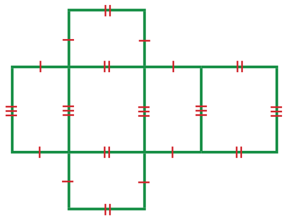
One possible net of a cube



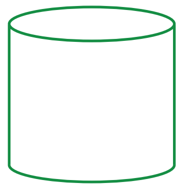
Rectangular Prism



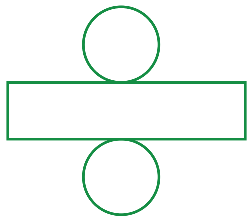
One possible net of a rectangular prism



Right Cylinder



One possible net of a right cylinder



Congruence and Similarity

Two geometric figures are **congruent** if they are identical.

- Congruent triangles coincide exactly when one is superimposed upon the other.
- Congruent plane figures have corresponding pairs of sides that are equal, and corresponding pairs of angles that are the same.

Two geometric figures are **similar** if their shape is the same, even though their size may be different.

- All squares are similar, and all circles are similar.

Classification of Geometric Figures

All equilateral triangles are isosceles, but only some isosceles triangles are equilateral.

A square is a rectangle with all sides congruent.

A square is also a rhombus with all angles congruent.

Within the USA/Canada, a trapezium is an irregular quadrilateral.
Outside the USA/Canada, a trapezium is a quadrilateral with at least one pair of parallel sides (known as a "trapezoid" within the USA/Canada).

Calendar Conventions

There was no year **0**. The first century spanned the years **1** to **100** inclusive.

- The **20th century** spanned the years **1901** to **2000** inclusive.
- The **21st century** spans the years **2001** to **2100** inclusive.

MATHS OLYMPIAD

Measures of Centre

The **mean**, **arithmetic mean**, or **average**, of a set of values is
- the sum of the values, divided by
- the number of values.

- For the set **{ 5, 5, 7, 11 }**, the mean is $\frac{5 + 5 + 7 + 11}{4}$ = **28 ÷ 4 = 7**.
- For the set **{ 7, 11, 23, 5, 5 }**, the mean is $\frac{7 + 11 + 23 + 5 + 5}{5}$ = **51 ÷ 5 = $10\frac{1}{5}$**.

The **median** is the value that is exactly in the middle of the set when it is ordered.
If there are an even number of values, then the median is the mean of the two middle values.

- For the set **{ 5, 5, 7, 11 }**, the median is **(5 + 7) ÷ 2 = 6.**
- For the set **{ 7, 11, 23, 5, 5 }**, we begin by ordering the set of values: **{ 5, 5, 7, 11, 23 }**.

  The median is the middle value, **7.**

The **mode** is the value that occurs the greatest number of times.

- For the set **{ 5, 5, 7, 11 }**, the mode is **5.**

A set with every value listed an equal number of times is said to have no mode.

- For the set **{ 5, 5, 7, 7, 8, 8 }**, there is no mode.

Probability

The probability of an event is a value that expresses how likely an event is to occur.

- If the event is impossible, then the probability is **0**.
- If the event is certain, then the probability is **1**.
- All probabilities are between **0** and **1** inclusive.

The probability is found by dividing the number of times an event does occur, by the total number of times the event can possibly occur.

- The probability of rolling an odd number on a die is $\frac{3}{6}$ or $\frac{1}{2}$.