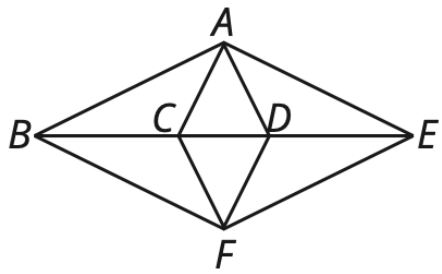
Triangleville has 6 intersections and 11 streets, as shown.

Without passing through the same intersection more than once, in how many ways can someone travel from *A* to *D*?



METHOD 2 ***Strategy:*** *Draw a tree diagram.*

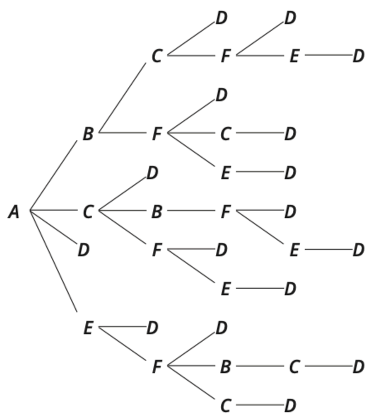
The tree diagram can help to count paths in an organised way.

Each vertex is linked to all of the neighbours that have not yet been visited.

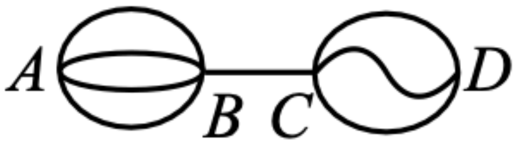
There are:

- **6** paths that start with ***AB***,
- **5** paths that start with ***AC***,
- Only **1** path for ***AD***, and
- **4** paths that start with ***AE***.

Thus, there are a total of **6 + 5 + 1 + 4 = 16** paths from ***A*** to ***D***.



Following only the paths shown, what is the number of different paths that go from *A* to *B* to *C* to *D* and touch each of those points exactly once?

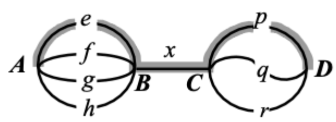


4B. <u>Method 1:</u> *<u>Strategy</u>: Count paths to each letter separately.*
For each of the 4 paths from *A* to *B*, there is 1 path from *B* to *C* and then 3 paths from *C* to *D*.

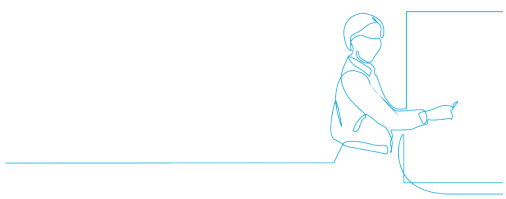
There are $4 \times 1 \times 3 =$ **12 different paths that go from *A* to *B* to *C* to *D*** and touch each point once.

<u>Method 2:</u> *<u>Strategy</u>: Make an organised list.*
Label the individual paths by naming the three segments travelled. One such path, shown by the thick lines is *<u>exp</u>*. Paths from *A* to *B* to C to D can be represented by a tree diagram or by the list at the right:

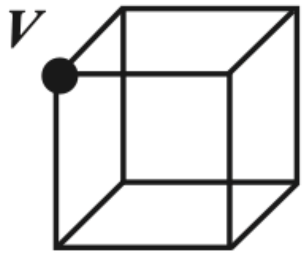
There are 12 different paths in all.



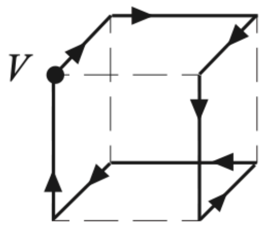
exp	*exq*	*exr*
fxp	*fxq*	*fxr*
gxp	*gxq*	*gxr*
hxp	*hxq*	*hxr*



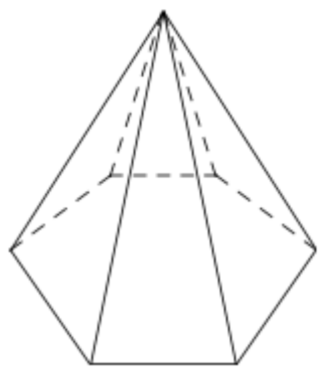
An ant sits at vertex V of a cube with edge of length 1 m. The ant moves along the edges of the cube and comes back to vertex V without visiting any other point twice. Find the number of metres in the length of the longest such path.



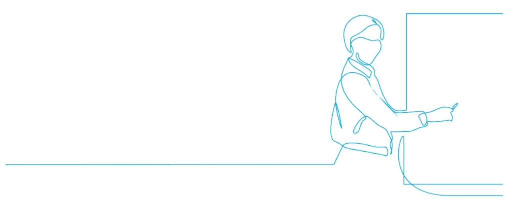
2E. *Strategy: Trace possible paths on the cube.*
Each possible path has a length of 4, 6, or 8 m. **The longest such path is 8 m.** One such path is shown in the diagram below.



The pyramid shown has 7 vertices, 12 edges, and 7 faces (one of which is a hexagon). At least one of the edges on each of the faces is to be colored red. What will be the least number of edges colored red?



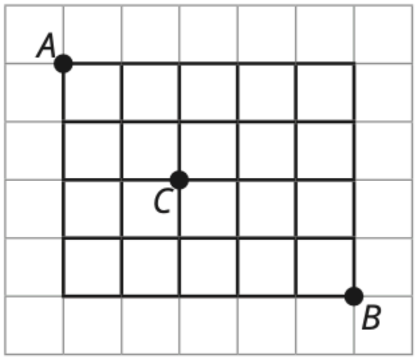
Method 2: *Strategy: Deform the pyramid and consider a "top" view.*
Looking down from the top you can see the hexagon and the six triangles. One possible coloring that would color at least one edge of each triangle and the hexagon can be seen in the diagram. Only **4** edges need to be colored red.



The diagram shows a map of the streets in a city where every block is an identical square.

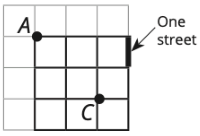
I need to travel along the streets from *A* to *B* via *C*, using the shortest possible distance.

In how many different ways can I make this trip?

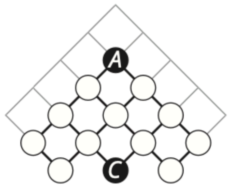


5D. ***Strategy:*** *Count in an organised way.*

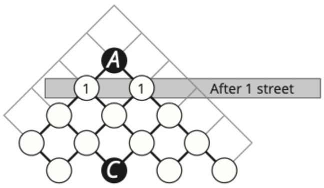
We shall define one street as being the path between two intersections,



and turn the map so that *A* is at the top.

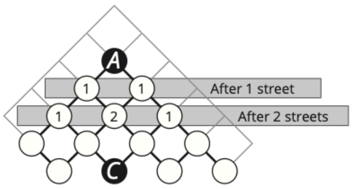


The first horizontal row would then include all of the intersections that occur after travelling along one street.

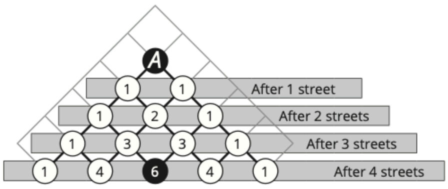


There is **1** way to reach each of those intersections.

The second horizontal row would include the intersections that occur after travelling along two streets.

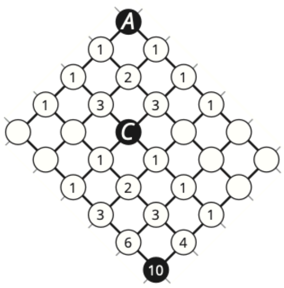


To count the paths to each intersection, we would add the number of paths that could be taken to reach each of the intersections that came immediately before.



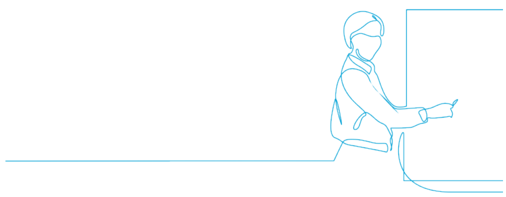
To travel from *A* to *C*, we can see that there are **6** paths that would lead to *C*.

To travel from *C* to *B*, we can see that there are **10** paths that would lead to *B*.

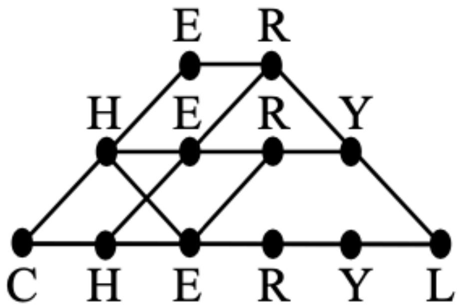


Therefore there are **6 × 10 = 60** paths from *A* to *B* via *C*.

[Note that this is Pascal's Triangle.]



Cheryl traces her name, CHERYL, by following the lines shown. She can change direction only at a letter. How many different paths can trace her name?



4E. Method 1: *Strategy: Show the number of ways to reach each point.*

To clarify the listing, use circled italicised letters for the points in the top path and bolded lower case letters for those in the middle path.

Figure 2 is the same as Figure 1. Figure 2 shows the number of paths from C to each point. From C there is only one way to reach points **H**, **h**, and (*e*). There are only two ways to reach points **E**, **R**, **Y**, and **e**. With three ways to reach point (*r*) and four ways to reach point **r**, there are seven ways to reach point y. Thus, **there are 9 paths that spell Cheryl.**

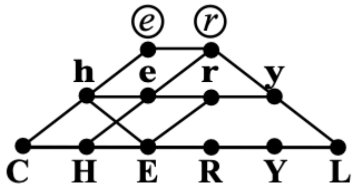


Figure 1

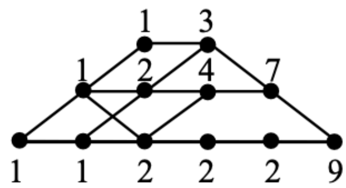
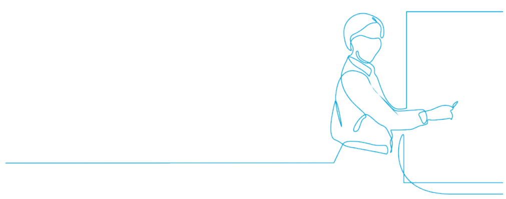
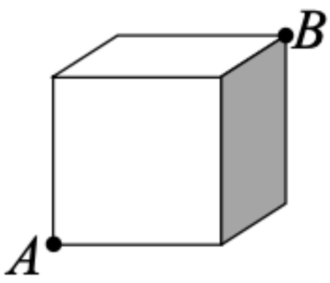


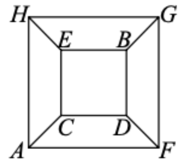
Figure 2



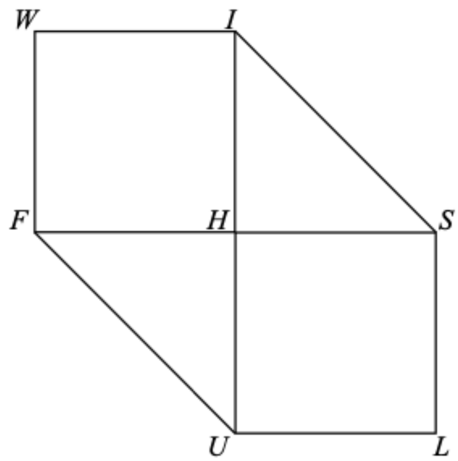
Given the cube shown at the right, an ant travels from vertex A to vertex B, always walking along an edge of the cube.
How many shortest paths are there from A to B?



1C. Method 1: *Strategy: Deform the cube and list all the possible paths.*
The shortest path for the ant to get from A to B consists of 3 edges. The list of all possible paths of length 3 is: *ACEB*, *ACDB*, *AFDB*, *AFGB*, *AHGB*, and *AHEB*. Thus there are exactly **6** shortest paths from A to B.



In the plane figure shown, you are only allowed to move along the lines (moving down, to the right, or diagonally downwards).
How many different possible paths can be taken to move from W to L?



Method 3: *Strategy: Find the possible paths to get to a point by adding the number of ways to get to the points that lead to it.*
There is one way to get to I from W and one way to get to F from W.
To get to H you need to go through I (one way) or F (one way) so $1 + 1 = 2$ ways from W to H.
To get to S you need to go through I (one way) or H (2 ways) so $1 + 2 = 3$ ways from W to S.
To get to U you need to go through F (one way from W) or H (2 ways from W) so $1 + 2 = 3$ ways to U from W.
To get to L you need to go through S (3 ways from W) or U (3 ways from W) so $3 + 3 =$ **6** ways from to W from L.