IMPORTANT

The information contained in this file is ONLY for the use of registered participants of the 2024 Australasian Problem Solving Mathematical Olympiads.

APSMO

2024 OLYMPIADS

MATHS OLYMPIAD

Organisation and Procedures

For full details, see the Members' Area

To ensure the integrity of the competition, the Olympiads must be administered under examination conditions.

DO	DO NOT
• Supervise students at all times • Seat students apart • Maintain silence • Provide blank working paper • Give time warnings when 3 minutes remain, and again when 1 minute remains • Collect, mark and retain the papers	• Print the Olympiad papers prior to the Olympiad Date • Read the questions aloud to the students • Interpret the questions for students • Permit any discussion or movement around the room • Permit the use of calculators or other electronic devices

- Olympiad papers are scored by the PICO using the *Solutions and Answers* sheet provided.
- Results should be submitted in the Members' Area within 7 days of the Olympiad.
- Original student answer sheets should be retained by the PICO until the end of the year.
- *Solutions and Answers sheets* are not to be handed out to students. They are a teaching resource for use in class ***after*** completion of the Olympiad paper.

Timing of the Olympiad

- The *Total Time Allowed* for the Olympiad is **30 minutes**.
- The time for each individual question is a guide for the students.

Absent Student Policy

A student who is legitimately absent on the Olympiad date, may sit the Olympiad under examination conditions on their first day back at school (if that date is within 2 weeks of the original Olympiad date). If these conditions cannot be met, the student must be marked as absent on the submitted results.

The Absent Student Policy is available in the **Contest Administration** section of the Members' Area.

APSMO

MATHS OLYMPIAD

2024 : DIVISION S

WEDNESDAY 12 JUNE 2024

OLYMPIAD 2

Total Time Allowed: ***30 Minutes***

Write your answers in the boxes on the back.

Keep your answers hidden by folding backwards on this line.

2A. Find the integer value of $\frac{N + 32}{N - 4}$ when $N = -5$.

2B. Solid rectangular prisms are formed by gluing together thirty-six identical cubes.

We will consider a prism that measures 1 × 3 × 12 to be the same as a prism that measures 3 × 1 × 12.

How many different rectangular prisms are possible?

2C. In the cryptarithm shown, different letters represent different digits, and no leading digit equals 0.

$$\begin{array}{r} A\ B\ C \\ +\ D\ E\ F \\ \hline 3\ 8\ 4 \end{array}$$

A, *B*, *C*, *D*, *E*, or *F* cannot be equal to 3, 8, or 4.

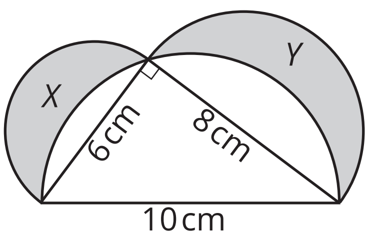
What other digit can *A*, *B*, *C*, *D*, *E*, or *F* not equal?

2D. When factored fully, $2024 = 2^3 \times p \times q$, where p and q are different prime numbers.

Find $p + q$.

2E. Semicircles with diameters 6cm, 8cm, and 10cm rest on the sides of a right-angled triangle as shown.

Find the number of square centimetres in the combined areas of regions *X* and *Y* (shaded grey).



* The area of a circle is equal to πr^2, where r is the radius of the circle.
You can use $\frac{22}{7}$ as an approximation for π.

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MATHS OLYMPIAD

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2024 : DIVISION S

WEDNESDAY 12 JUNE 2024

OLYMPIAD

2

2A.

Student Name:

2B.

Fold here. Keep your answers hidden.

2C.

2D.

2E.

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MATHS OLYMPIAD

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2024 : DIVISION S

WEDNESDAY 12 JUNE 2024

OLYMPIAD 2

Solutions and Answers

(Items in parentheses are not required)

For teacher use only. Not for Distribution.

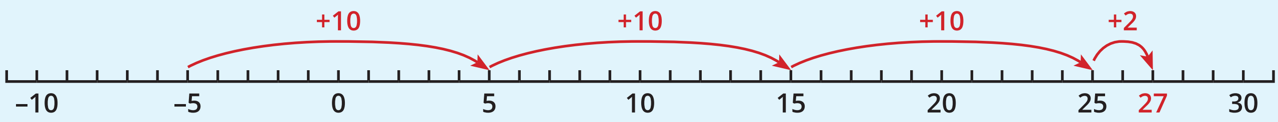
2A: –3	**2B:** 8	**2C:** 6	**2D:** 34	**2E:** 24 (cm²)

2A. The question is: Find the integer value of $\frac{N+32}{N-4}$ when $N = -5$.

METHOD: *Substitute the value of N into the expression.*

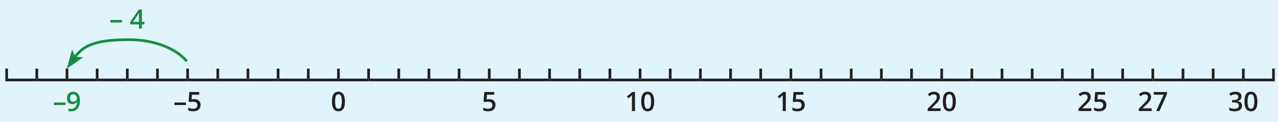
The value of the numerator is $N + 32$.

When $N = -5$, $\quad N + 32 = -5 + 32$
$\qquad\qquad\qquad\qquad = 27.$



The value of the denominator is $N - 4$.

When $N = -5$, $\quad N - 4 = -5 - 4$
$\qquad\qquad\qquad\qquad = -9.$



When $N = -5$, $\frac{N+32}{N-4} = \frac{27}{-9}$.

Strategy 1: Consider equivalent fractions.

Recognising that a value does not change if we multiply or divide it by **1**:

$$\frac{27}{-9} = \frac{3 \times 9}{-1 \times 9} = \frac{3}{-1} \times \frac{9}{9} = -3$$

Strategy 2: Reason algebraically.

Suppose that $\qquad x = \frac{27}{-9}$

Multiplying both sides by **–9**: $\qquad -9x = 27$

Since **–9** × **–3** = 27, $\qquad x = -3$

When $N = -5$, the integer value of $\frac{N+32}{N-4}$ is **–3**.

Follow-Up: *For how many values of N does* $\frac{N+32}{N-4}$ *have a positive integer value?* [9: 5, 6, 7, 8, 10, 13, 16, 22, 40]

2B. The question is: How many different rectangular prisms are possible?

METHOD 1: *Make an organised list.*

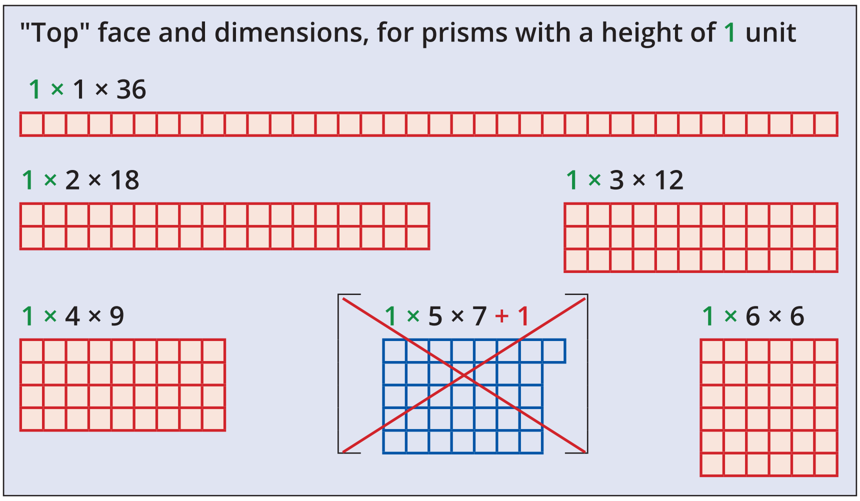
We shall define the side length of a cube as **1** unit, and begin by considering rectangular prisms that have a height of **1** unit.

For prisms with a height of **1** unit, the area of the "top" face would be **36** square units.

We can list these prisms in an organised way by considering every possible value for the width of the "top" face.

We can stop listing prisms when width exceeds the length, as those prisms have already been found.

Note that it is not possible to create a prism that is **5** units wide.

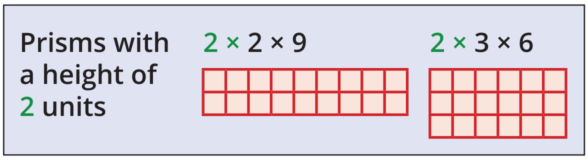


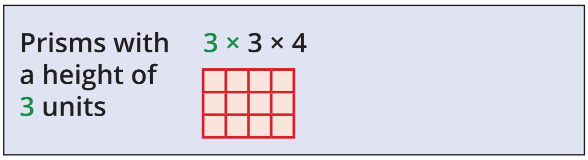
For prisms with a height of **2** units, the area of the "top" face would be **18** square units.

For prisms with a height of **3** units, the area of the "top" face would be **12** square units.

We do not need to consider widths that are less than the height, as those prisms have already been found.

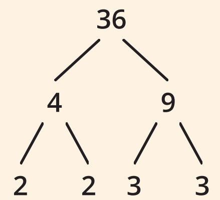
There are **8** possible rectangular prisms that can be constructed from **36** cubes.





METHOD 2: *Consider prime factors of 36.*

By constructing a factor tree, we can find the prime factors of **36**.



36 = 2 × 2 × 3 × 3.

Since **36** has just **2** distinct prime factors, we can use them in a table to list all of the factors of **36**.

×	1	2	2×2
1	1	2	4
3	3	6	12
3×3	9	18	36

With a complete set of factors, we can list the dimensions of each rectangular prism in an organised way.

One possible method might be to start with the longest side length in each case, and then only consider side lengths of decreasing size.

36 cubes can be used to construct **8** different rectangular prisms.

L	*W*	*H*
36	1	1
18	2	1
12	3	1
9	4	1
9	2	2
6	6	1
6	3	2
4	3	3

Follow-Up: *How many different rectangular prisms would be possible if there were 40 cubes?* [6]

2C. The question is: What other digit can *A*, *B*, *C*, *D*, *E*, or *F* not equal?

$$\begin{array}{r} A\;B\;C \\ +\;D\;E\;F \\ \hline 3\;8\;4 \end{array}$$

METHOD: *Eliminate all but one possibility.*

The digits represented by *A*, *B*, *C*, *D*, *E*, and *F* are all different.

None of the digits equals **3**, **8**, or **4**.

			X	X				X	
0	1	2	3	4	5	6	7	8	9

We also know that *A* and *D* do not equal **0**.

Since the sum has **3** in the hundreds place, it must be the case that *A* and *D* represent **1** and **2**, in either order.

Any other values for *A* and *D* would result in a sum that is greater than **300**.

$$\begin{array}{r} A\;B\;C \\ +\;D\;E\;F \\ \hline 3\;8\;4 \end{array} \rightarrow \begin{array}{r} 1\;B\;C \\ +\;2\;E\;F \\ \hline 3\;8\;4 \end{array}$$

	A	D	X	X				X	
0	1	2	3	4	5	6	7	8	9

Since the sum has **4** in the ones place, the sum of *C* and *F* must be either **4** or **14**.

Possible values for *C* and *F* are **0**, **5**, **6**, **7**, or **9**.

By considering all of the possible sums for two of these values, we see that the only result of **4** or **14** occurs when *C* and *F* represent **5** and **9**.

The sum of **5** and **9** is **14**, so there will be regrouping into the tens place.

$$\begin{array}{r} 1\;B\;C \\ +\;2\;E\;F \\ \hline 3\;8\;4 \end{array} \rightarrow \begin{array}{r} {}^{1}\;\; \\ 1\;B\;5 \\ +\;2\;E\;9 \\ \hline 3\;8\;4 \end{array}$$

	A	D	X	X	C			X	F
0	1	2	3	4	5	6	7	8	9

The sum has **8** in the tens place.

We have regrouping from adding *C* and *F* in the ones place, and we know that there is no regrouping into the hundreds place.

This means that the sum of *B* and *E* must be **7**.

With the only values remaining being **0**, **6**, and **7**, *B* and *E* must represent **0** and **7**.

$$\begin{array}{r} {}^{1}\;\; \\ 1\;B\;5 \\ +\;2\;E\;9 \\ \hline 3\;8\;4 \end{array} \rightarrow \begin{array}{r} {}^{1}\;\; \\ 1\;0\;5 \\ +\;2\;7\;9 \\ \hline 3\;8\;4 \end{array}$$

B	A	D	X	X	C		E	X	F
0	1	2	3	4	5	6	7	8	9

A and *D* must represent **1** and **2**, in either order.

B and *E* must represent **0** and **7**, in either order.

C and *F* must represent **5** and **9**, in either order.

The only digit that ***A***, ***B***, ***C***, ***D***, ***E***, and ***F*** cannot equal is **6**.

B	A	D	X	X	C		E	X	F
0	1	2	3	4	5	6	7	8	9

Follow-Up: *Given all the same restrictions as stated in the original question, including that the sum is 384, what is the greatest possible value for the product ABC × DEF ?* [36695]

2D. The question is: Find $p + q$, where p and q are both prime numbers, and $2024 = 2^3 \times p \times q$.

METHOD: *Make an organised list, and use number sense.*

We begin by finding the value of $p \times q$.

Strategy 1: Use a division algorithm.

Since

$2024 = 2^3 \times p \times q$,

we know that

$p \times q = 2024 \div 2^3$

$= 2024 \div 8$

$= 253$.

```
      2 5 3
8 ) 2 0 2 4
  - 1 6 0 0
      4 2 4
    - 4 0 0
        2 4
      - 2 4
          0
```

Strategy 2: Guess, Check and Refine.

Guess for $p \times q$	Guess × 8	Notes
200	1600	Too low
300	2400	Too high
250	2000	24 away
253	2024	Found it

Since $2024 = 253 \times 8$,

$p \times q = 253$.

Since the product of the two values is **253**, the square of one of the values must be less than **253**.

Listing prime numbers and their squares:

Prime Number	2	3	5	7	11	13	17
Square of Prime Number	4	9	25	49	121	169	289

Since $17 \times 17 > 253$, at least one of p or q must be less than **17**.

We shall say that $p < q$, and so we will begin by determining the value of p, which must be less than **17**.

$p \neq 2$, because **253** is not even.

$p \neq 5$, because **253** does not have **5** or **0** in the ones place.

Prime Number	~~2~~	3	~~5~~	7	11	13	~~17~~
Square of Prime Number	4	9	25	49	121	169	289

The possible values for p are 3, 7, 11, and 13.

To find if **253** is divisible by **3**, we can use the divisibility rule for **3**.

$2 + 5 + 3 = 10$, which is not divisible by **3**, and so **253** is not divisible by **3**.

We may also notice that **253** is a multiple of **11**, since

$253 = 10 \times 23 + 1 \times 23$

$= 230 + 23$.

Alternatively, using a division algorithm, we find that:

- **253** is not divisible by **3**, **7**, or **13**, as there are remainders.
- $253 = 11 \times 23$.

```
      8 4          3 6           2 3             1 9
3 ) 2 5 3    7 ) 2 5 3    11 ) 2 5 3    13 ) 2 5 3
  - 2 4 0      - 2 1 0       - 2 2 0       - 1 3 0
      1 3          4 3           3 3         1 2 3
    - 1 2        - 4 2         - 3 3       - 1 1 7
        1            1             0             6
```

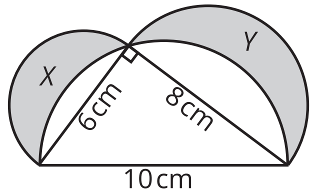
The value of $p + q$ is $11 + 23 = 34$.

Follow-Up: *What is the least integer greater than 2024 that can be factored as $2^3 \times p \times q$, where p and q are both prime?* [2072]

APSMO
2024 : DIVISION S
WEDNESDAY 12 JUNE 2024

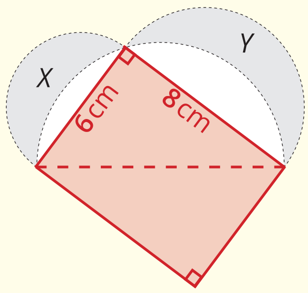
OLYMPIAD 2

2E. The question is: Find the number of square centimetres in the combined areas of regions *X* and *Y*.



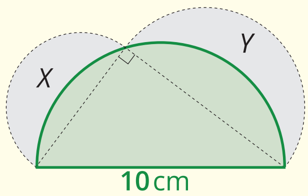
METHOD 1: *Determine the areas of the different regions.*

The area of the triangle is half of the area of a rectangle with length **8** cm and width **6** cm.



$A_{Triangle} = \frac{1}{2} \times 8\text{ cm} \times 6\text{ cm}$
$= 24\text{ cm}^2$.

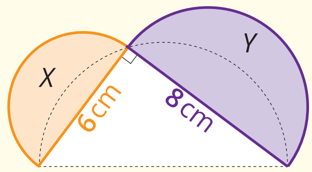
The semicircle with diameter **10** cm is half of the area of a circle with diameter **10** cm.



The radius of the semicircle is $\frac{1}{2} \times 10\text{ cm} = 5\text{ cm}$.

$A_{Semi10} = \frac{1}{2} \times \pi r^2$
$= \frac{1}{2} \times \pi \times 5^2$
$= \frac{25\pi}{2}\text{ cm}^2$.

The same method is used to calculate the areas of semicircles ***X*** and ***Y***.



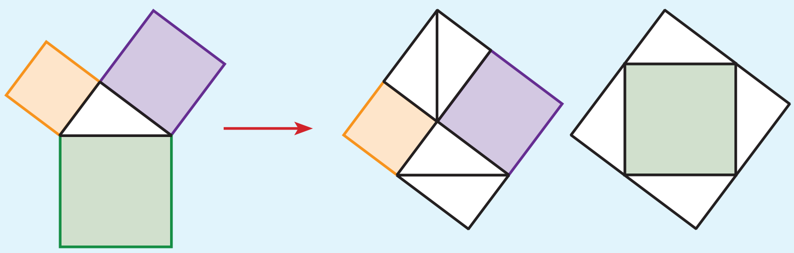
$A_{Semi6} = \frac{1}{2} \times \pi \times 3^2$
$= \frac{9\pi}{2}\text{ cm}^2$.

$A_{Semi8} = \frac{1}{2} \times \pi \times 4^2$
$= 8\pi\text{ cm}^2$.

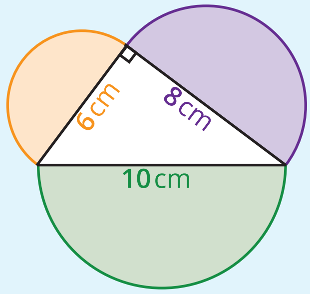
The combined areas ***X*** and ***Y*** is equal to $A_{Semi6} + A_{Semi8} + A_{Triangle} - A_{Semi10} = \frac{9\pi}{2} + 8\pi + 24 - \frac{25\pi}{2} = \mathbf{24}\text{ cm}^2$.

METHOD 2: *Recognise the application of Pythagoras' Theorem.*

Pythagoras' Theorem states that, for a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of squares of the lengths of other two sides.



Since the area of a semicircle is proportional to the area of the bounding square, then, by using Pythagoras' Theorem, we can determine that the area of the semicircle on the hypotenuse is equal to the sum of the semicircles on the other two sides.



As noted in the reasoning for Method 1, the combined areas *X* and *Y* is equal to $A_{Semi6} + A_{Semi8} + A_{Triangle} - A_{Semi10}$.

Therefore, the combined areas ***X*** and ***Y*** is equal to $A_{Triangle} = \mathbf{24}\text{ cm}^2$.

Follow-Up: *Suppose the diameters were 8 cm, 15 cm, and 17 cm. Find the number of square centimetres in the combined areas of regions X and Y.* [60]