APSMO

2024 OLYMPIADS

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APSMO

2024 OLYMPIADS

MATHS OLYMPIAD

Organisation and Procedures

For full details, see the Members' Area

To ensure the integrity of the competition, the Olympiads must be administered under examination conditions.

DO

- Supervise students at all times
- Seat students apart
- Maintain silence
- Provide blank working paper
- Give time warnings when 3 minutes remain, and again when 1 minute remains
- Collect, mark and retain the papers

DO NOT

- Print the Olympiad papers prior to the Olympiad Date
- Read the questions aloud to the students
- Interpret the questions for students
- Permit any discussion or movement around the room
- Permit the use of calculators or other electronic devices

- Olympiad papers are scored by the PICO using the *Solutions and Answers* sheet provided.
- Results should be submitted in the Members' Area within 7 days of the Olympiad.
- Original student answer sheets should be retained by the PICO until the end of the year.
- *Solutions and Answers sheets* are not to be handed out to students. They are a teaching resource for use in class ***after*** completion of the Olympiad paper.

Timing of the Olympiad

- The *Total Time Allowed* for the Olympiad is **30 minutes**.
- The time for each individual question is a guide for the students.

Absent Student Policy

A student who is legitimately absent on the Olympiad date, may sit the Olympiad under examination conditions on their first day back at school (if that date is within 2 weeks of the original Olympiad date). If these conditions cannot be met, the student must be marked as absent on the submitted results.

The Absent Student Policy is available in the **Contest Administration** section of the Members' Area.

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2024 : DIVISION S

WEDNESDAY 8 MAY 2024

OLYMPIAD 1

Total Time Allowed: ***30 Minutes***

Write your answers in the boxes on the back.

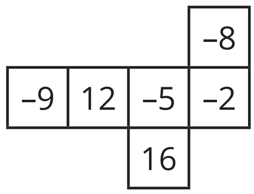
Keep your answers hidden by folding backwards on this line.

1A. How many integers are both greater than $(-3)^3$ and less than $(2)^4$?

1B. The given net is made up of six numbered squares.

The net is folded to form a cube.

What is the greatest sum of numbers on three faces which meet at a common vertex?



1C. In a cryptarithm, different letters represent different digits, and no leading digit equals 0.

Find all possible values of the whole number *COW*.

$$\begin{array}{r} M\ O\ O \\ +\ M\ O\ O \\ \hline C\ O\ W \end{array}$$

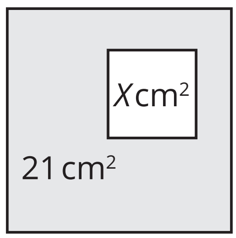
1D. Two nested squares, with integer side lengths, are arranged as shown.

The area of the shaded region is $21\,\text{cm}^2$.

The area of the small square is $X\text{cm}^2$.

Find the sum of all of the possible values for X.

Diagram not drawn to scale.



1E. A bag contains six coins.

Each coin has a whole number on the front, and a different whole number on the back, as shown in the given chart.

Front	2	3	5	7	10	12
Back	8	9	11	13	16	18

The bag is shaken and all six coins are spilled onto a table.

How many different sums are possible for the six numbers that are facing upwards?

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MATHS OLYMPIAD

OLYMPIAD

1

1A.

Student Name:

1B.

Fold here. Keep your answers hidden.

1C.

1D.

1E.

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MATHS OLYMPIAD

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2024 : DIVISION S

WEDNESDAY 8 MAY 2024

OLYMPIAD 1

Solutions and Answers

(Items in parentheses are not required)

For teacher use only. Not for Distribution.

1A: 42	**1B:** 23	**1C:** 398, 598, 798	**1D:** 104	**1E:** 7

1A. The question is: How many integers are both greater than $(-3)^3$ and less than $(2)^4$?

When multiplying signed numbers, it may help to think of subtraction as "taking away".

By reasoning logically, we can deduce that:

- $(+a) \times (+b) = +ab$
- $(+a) \times (-b) = -ab$
- $(-a) \times (+b) = -ab$
- $(-a) \times (-b) = +ab$.

We begin by finding the result of adding some positive and negative numbers.	–1 + +1 + –1 + +1	= 0
Adding **+1** increases the total by **1**. Therefore, **+(+1)** is the same as **+1**.	–1 + +1 + –1 + +1 + +1	= 1
Adding **–1** decreases the total by **1**. Therefore, **+(–1)** is the same as **–1**.	–1 + +1 + –1 + +1 + –1	= –1
Taking away **+1** decreases the total by **1**. Therefore, **–(+1)** is the same as **–1**.	–1 + +1 + –1	= –1
Taking away **–1** increases the total by **1**. Therefore, **–(–1)** is the same as **+1**.	–1 + +1 + +1	= 1

From the above reasoning, we can determine that: $(-3)^3 = -3 \times -3 \times -3 = +9 \times -3 = -27$. $(2)^4 = 2 \times 2 \times 2 \times 2 = 4 \times 4 = 16$.

Strategy 1: *Draw a diagram.*

We can use a number line to find integers that are both greater than **–27** and less than **16**.

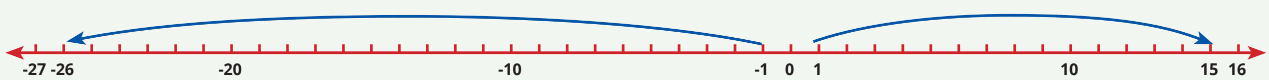
The smallest (most negative) number that satisfies the requirement is **–26**.

- There are **26** integers from **–1** to **–26** inclusive.

The greatest number that satisfies the requirement is **+15**.

- There are **15** integers from **1** to **15** inclusive.

Including the integer **0**, there are **26 + 15 + 1 = 42** integers that are greater than **–27** and less than **16**.



Strategy 2: *Find a pattern, and perform the arithmetic operation.*

The difference between **3** and **8** is **8 – 3 = 5**.

There are **5 – 1 = 4** integers that are greater than **3** and less than **8**:

3, 4, 5, 6, 7, 8.

The difference between **–2** and **1** is **1 – (–2) = 3**.

There are **3 – 1 = 2** integers that are greater than **–2** and less than **1**:

–2, –1, 0, 1.

The number of integers that occur between two integer values is the difference between those values, minus **1**.

The difference between **–27** and **16** is **16 – (–27) = 16 + 27 = 43**.

There are **43 – 1 = 42** integers that are greater than **–27** and less than **16**.

Follow-Up: *How many integers lie between $(-3)^5$ and $(-5)^3$?* [117]

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OLYMPIAD

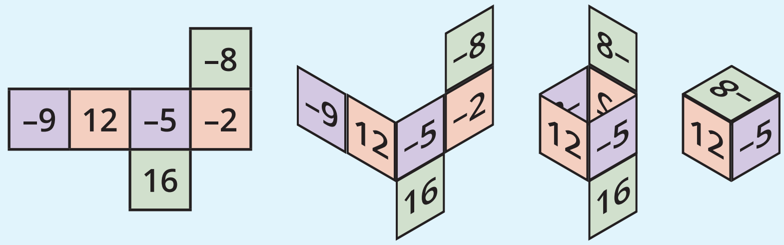
1

1B. The question is: What is the greatest sum of numbers on three faces which meet at a common vertex?

Strategy 1: *Fold the cube.*

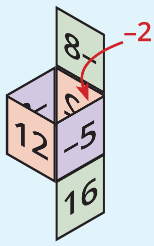
When we fold the net to form the cube, we can see that the values –9, 12, –5 and –2 will be arranged in a "band" around the cube.

The values –8 and 16 are on opposite faces on either side of this band.



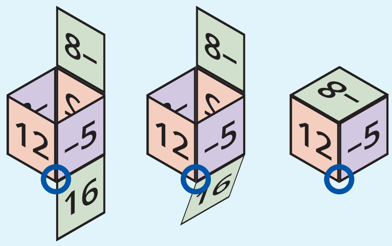
The three greatest values on the faces of the cube are 16, 12, and –2.

We can see that 12 and –2 are on opposite faces and so do not meet at a vertex.



After –2, the greatest value on the faces of the cube is –5.

We can see that 16, 12, and –5 do meet at a common vertex.

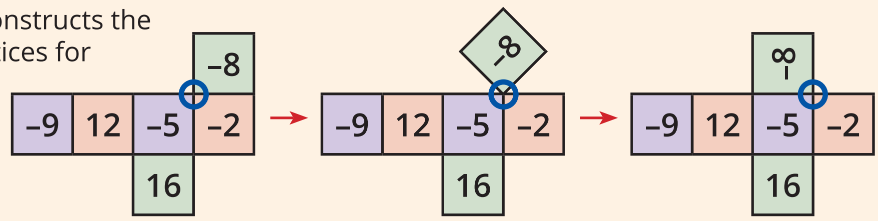


The greatest sum on three faces which meet at a common vertex is 16 + 12 + (–5) = 23.

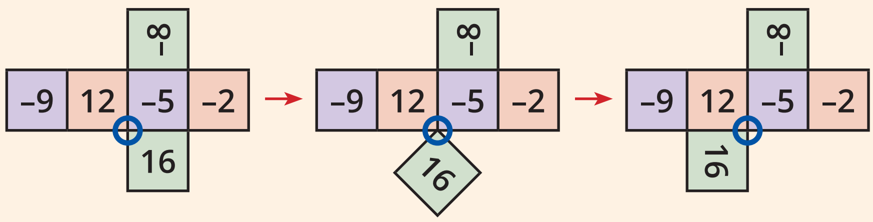
Strategy 2: *Convert to a more convenient form.*

To create a different net that constructs the same cube, we can identify vertices for which three faces are known.

In a valid net for this cube, any two of those faces can be joined.

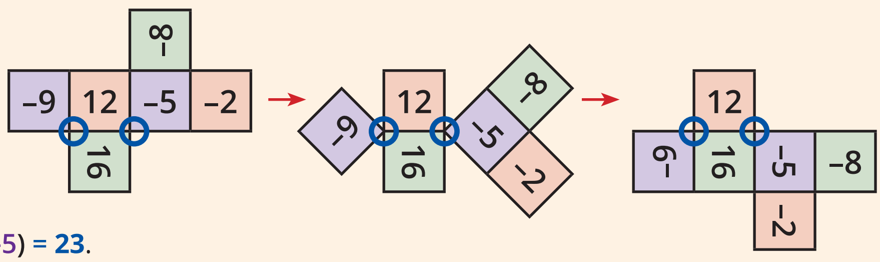


Since we want the greatest sum, we can begin by creating a common edge between the two greatest values, 16 and 12.



There are two vertices that are common to both the 16 face and the 12 face.

One is shared by the faces (16, 12, –9) and the other is shared by (16, 12, –5).



The greatest sum is 16 + 12 + (–5) = 23.

Follow-Up: *What is the least sum of three numbers that meet at a common vertex?* [–19]

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OLYMPIAD 1

1C. The question is: Find all possible values of the whole number *COW*.

Strategy: *Reason Logically, and Make an Organised List.*

We know that:

- Different letters represent different digits.
- No leading digit equals zero.

	0	1	2	3	4	5	6	7	8	9
M	X									
O										
C	X									
W										

$$\begin{array}{r} M\ O\ O \\ +\ M\ O\ O \\ \hline C\ O\ W \end{array}$$

Since we have ***O*** **+** ***O*** **=** ***W*** in the ones column, and ***O*** **+** ***O*** **=** ***O*** in the tens column, it must be the case that ***O*** **+** ***O*** requires regrouping and application of place value.

Therefore, ***O*** cannot be **0**, **1**, **2**, **3**, or **4**.

	0	1	2	3	4	5	6	7	8	9
M	X									
O	X	X	X	X	X					
C	X									
W										

$$\begin{array}{r} M\ O\ O \\ +\ M\ O\ O \\ \hline C\ O\ W \end{array}$$

Suppose ***O*** = 5. Then, ***O*** + ***O*** = 10, so:

- ***W*** = 0, and
- from the addition in the tens place, ***O*** = 1.

This is impossible, so ***O*** ≠ **5**.

	0	1	2	3	4	5	6	7	8	9
M	X									
O	X	X	X	X	X	X				
C	X									
W										

$$\begin{array}{r} {\scriptstyle 1\ \ 1}\ \ \\ M\ 5\ 5 \\ +\ M\ 5\ 5 \\ \hline C\ 1\ 0 \end{array}$$

Similar reasoning can be used to eliminate 6, 7, and 8 as possible values for ***O***.

	0	1	2	3	4	5	6	7	8	9
O	X	X	X	X	X	X	X	X	X	

$$\begin{array}{r} {\scriptstyle 1\ \ 1}\ \ \\ M\ 6\ 6 \\ +\ M\ 6\ 6 \\ \hline C\ 3\ 2 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\ \ 1}\ \ \\ M\ 7\ 7 \\ +\ M\ 7\ 7 \\ \hline C\ 5\ 4 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\ \ 1}\ \ \\ M\ 8\ 8 \\ +\ M\ 8\ 8 \\ \hline C\ 7\ 6 \end{array}$$

The one remaining possible value for ***O*** = 9:

- ***W*** = 8, and
- from the addition in the tens place, ***O*** = 9.

Since we know that ***W*** = 8 and ***O*** = 9, we also know that neither ***M*** nor ***C*** can be **8** or **9**.

	0	1	2	3	4	5	6	7	8	9
M	X								X	X
O	X	X	X	X	X	X	X	X	X	✓
C	X								X	X
W	X	X	X	X	X	X	X	X	✓	X

$$\begin{array}{r} {\scriptstyle 1\ \ 1}\ \ \\ M\ 9\ 9 \\ +\ M\ 9\ 9 \\ \hline C\ 9\ 8 \end{array}$$

We need to find all of the possible values for ***COW***.

M and ***C*** are now limited to values from **1** to **7** inclusive.

$$\begin{array}{r} {\scriptstyle 1\ \ 1}\ \ \\ 1\ 9\ 9 \\ +\ 1\ 9\ 9 \\ \hline 3\ 9\ 8 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\ \ 1}\ \ \\ 2\ 9\ 9 \\ +\ 2\ 9\ 9 \\ \hline 5\ 9\ 8 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\ \ 1}\ \ \\ 3\ 9\ 9 \\ +\ 3\ 9\ 9 \\ \hline 7\ 9\ 8 \end{array}$$

The possible values for ***COW*** are **398**, **598** and **798**.

Follow-Up: *Considering the cryptarithm at the right, what are the least and greatest possible values of the whole number FAST?* [1046, 1872]

$$\begin{array}{r} R\ U\ N \\ +\ R\ U\ N \\ \hline F\ A\ S\ T \end{array}$$

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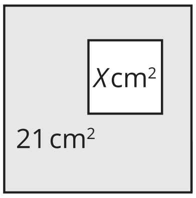
OLYMPIAD 1

1D. The question is: Find the sum of all of the possible values for X.



Strategy 1: *Build a table.*

The area of the small square is X cm².

Since the small square has integer side lengths, we know that X is a square number.

Possible area (X)	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256

The area of the large square is **(**X **+ 21)** cm².

We can see that there are two values for X **+ 21** that are also square numbers.

Possible area (X)	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256
X + 21	22	25	30	37	46	57	70	85	102	121	142	165	190	217	246	277

We also note that, for values of X greater than **100**, X **+ 21** is less than the next-largest square number.

This means that, for values of X greater than **100**, it is not possible for X **+ 21** to be a square number.

Possible area (X)	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256
X + 21	22	25	30	37	46	57	70	85	102	121	142	165	190	217	246	277

The sum of the possible values for X is **4 + 100 = 104**.

Strategy 2: *Construct a quadratic equation.*

Let the side length of the small square be n, and the side length of the large square be $n + k$.

The diagram indicates that $(n + k)^2 = n^2 + 21$:

$$n^2 + 2nk + k^2 = n^2 + 21$$

$$2nk + k^2 = 21$$

$$k^2 + 2nk - 21 = 0.$$

The factors of **21** are **1**, **3**, **7**, and **21**.

Since n represents a side length, it must be positive.

$$(k - 3)(k + 7) = k^2 + 4k - 21$$

$$2nk = 4k$$

$$n = 2$$

$(k + 3)(k - 7)$ gives the result $n = -2$, which is impossible.

$$(k + 21)(k - 1) = k^2 + 20k - 21$$

$$2nk = 20k$$

$$n = 10$$

$(k - 21)(k + 1)$ gives the impossible result $n = -10$.

The possible values for n are **2** and **10**.

X is the area of the small square with side length n, so the possible values for $X = n^2$ are **4** and **100.**

The sum of the possible values for X is **4 + 100 = 104**.

FOLLOW-UP: *In the original question, if the given area of the shaded region was 105, how many possible values would there be for X?* [4]

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MATHS OLYMPIAD

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2024 : DIVISION S

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OLYMPIAD 1

1E. The question is: How many different sums are possible for the six numbers that are facing upwards?

Front	2	3	5	7	10	12
Back	8	9	11	13	16	18

Strategy: *Find a pattern.*

We begin by supposing that all of the coins land with the front number showing.

One possible sum is **39**.

Front	2	3	5	7	10	12	$2 + 3 + 5 + 7 + 10 + 12 = 39$
Back	8	9	11	13	16	18	

Suppose only the first coin landed with the back number showing.

Another possible sum is **45**.

Front	2	3	5	7	10	12	$8 + 3 + 5 + 7 + 10 + 12 = 45$
Back	8	9	11	13	16	18	

We notice that the number on back of each coin is equal to the front number, plus **6**.

Regardless of how the coins land, the sum of all of the values must be equal to **39** plus a multiple of **6**.

Front	2	3	5	7	10	12
Back	8	9	11	13	16	18

Front	2	3	5	7	10	12	$2 + 3 + 5 + 7 + 10 + 12 + (2 \times 6) = 51$
Back	2+6	3+6	5+6	7+6	10+6	12+6	

Front	2	3	5	7	10	12	$2 + 3 + 5 + 7 + 10 + 12 + (3 \times 6) = 57$
Back	2+6	3+6	5+6	7+6	10+6	12+6	

⋮

The greatest possible sum is **$2 + 3 + 5 + 7 + 10 + 12 + (6 \times 6) = 75$.**

Front	2	3	5	7	10	12	$2 + 3 + 5 + 7 + 10 + 12 + (6 \times 6) = 75$
Back	2+6	3+6	5+6	7+6	10+6	12+6	

The possible sums are **39, 45, 51, 57, 63, 69, 75**.

The number of different possible sums is **7**.

Note that, by recognising that the two faces of each coin always differ by the same amount, we can determine the number of sums without calculating any of the sums.

Front	a	b	c	d	e	f	$a + b + c + d + e + f$ + (a multiple of k)
Back	$a+k$	$b+k$	$c+k$	$d+k$	$e+k$	$f+k$	

Any six coins with this property will result in **7** different possible sums: $a + b + c + d + e + f + nk$, where n is a value between **0** and **6** inclusive.

Follow-Up: *Using the original question, but replacing the coin that has 5 on the front and 11 on the back with a coin that has 4 on the front and 11 on the back, how many different sums are possible?* [12: 38, 44, 45, 50, 51, 56, 57, 62, 63, 68, 69, 75]