2024 Maths Olympiads Division J Preparation Kit

MATHS OLYMPIAD

Welcome to the APSMO Maths Olympiads for 2024

The format of the Maths Olympiads for 2024 will be:

Preparation Kit: Available from February, 2024

Competition 1: Wednesday May 8, 2024

Competition 2: Wednesday June 12, 2024

Competition 3: Wednesday July 31, 2024

Competition 4: Wednesday September 4, 2024

Preparing for the APSMO Maths Olympiads

The purpose of this Preparation Kit is to provide students with an opportunity to familiarise themselves with the concepts, and terminology, that will subsequently be used in the four competition papers for 2024.

The kit contains:

Pg. 2 - Preparation Olympiad Paper

Pg. 4 - Link to a Class Video - Strategies and Solutions for the Preparation Paper

Pg. 5 - Solutions to the Preparation Olympiad Paper

Pg. 11 - 10 Addition Preparation Questions

Pg. 13 - 10 Additional Preparation Questions (this time including hints)

Pg. 15 - Answers and solutions for the Additional Preparation Questions

Pg. 18 - Terminology

This kit may be used to:

- Reinforce previously learned concepts and terminology
- Introduce new or different solution methods
- Provide diagrams and animations that support teacher or student explanations
- Provide opportunity to collaboratively consider student work samples
- Support students' own study as a standalone resource

Further questions and solution methods can also be found in the APSMO resource books, available from www.apsmo.edu.au.

Suggested Time: ***30 Minutes***

A. Find the value of the following.

100 - 98 + 96 - 94 + 92 - 90 +... + 8 - 6 + 4 - 2

B. Amanda, Brittany, and Carly's ages are prime numbers.

They are all younger than 25.

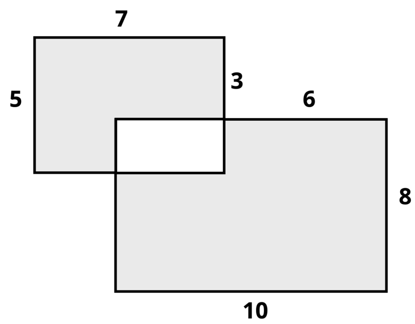
Amanda is the youngest.

The sum of the ages of Amanda and Brittany is equal to Carly's age.

How old is Amanda?

C. The length of each segment in the overlapping rectangles is given, in centimetres.

Find the sum of the areas of the shaded regions, in square centimetres.



D. The average mass of a group of children is 50 kilograms.

Todd, who has a mass of 62 kilograms, then joins the group.

This raises the average mass of the group to 52 kilograms.

How many children were in the original group?

E. Each letter in the following cryptarithm represents a different digit from the others.

What is the greatest value that WHAT can have?

		B	A	T
		F	A	T
+		C	A	T
	W	H	A	T

Write your answers in the boxes on the back.

Keep your answers hidden by folding backwards on this line.

A.	*Fold Here. Keep your answers hidden.*
B.	
C.	
D.	
E.	

Please click on the image below to access video strategies and solutions for the 2024 Maths Olympiad Junior Preparation Paper.

Solutions and Answers

For teacher use only. Not for Distribution.

A: 50	**B:** 2	**C:** 99	**D:** 5	**E:** 2450

A. The question is:

Find the value of the following.

100 - 98 + 96 - 94 + 92 - 90 +... + 8 - 6 + 4 - 2

Strategy 1: *Look for a pattern.*

The numbers listed in the problem are all the even numbers from 100 to 2.

This means that the complete list has 50 numbers.

If we group the numbers into pairs, we see that that value of each pair in parentheses is 2:

(100 - 98) + (96 - 94) + (92 - 90) + ... + (4 - 2)

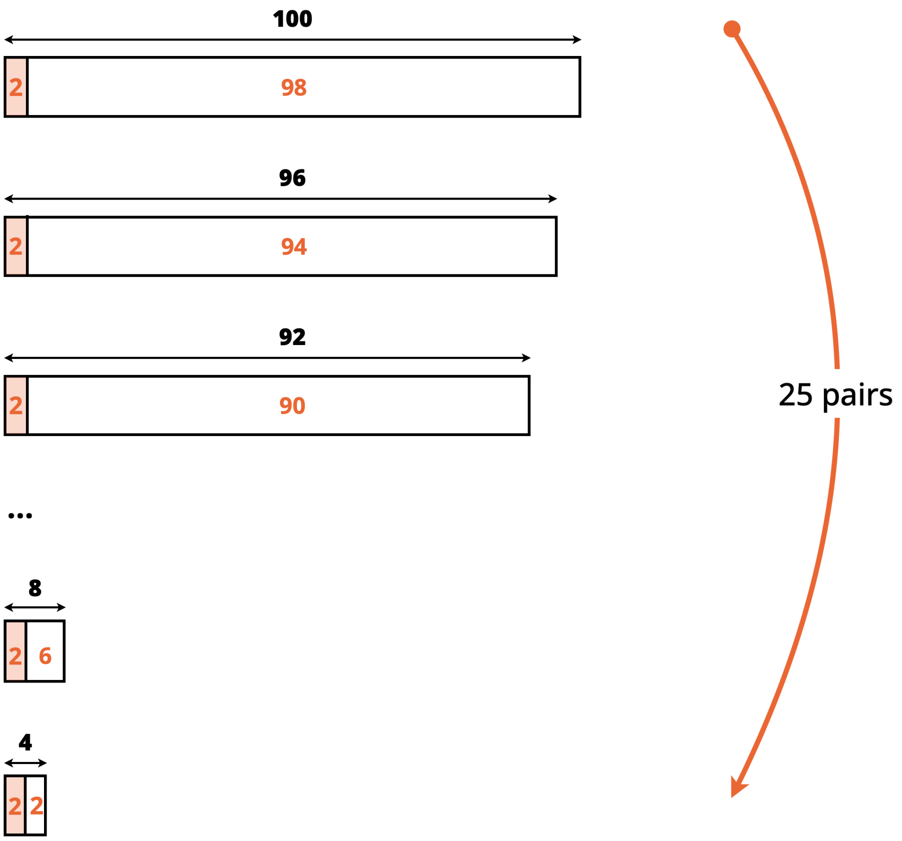
2 + 2 + 2 + ... 2

The 50 numbers will form 25 pairs.

Therefore, the value is **25 × 2 = 50.**

Strategy 2: *Draw a diagram.*

The following diagram demonstrates this pattern visually.



There are 25 pairs of numbers, each of which has a value of 2, that are to be added together.

Therefore, the value is **25 × 2 = 50.**

B. The question is:

Amanda, Brittany, and Carly's ages are prime numbers.

They are all younger than 25.

Amanda is the youngest.

The sum of the ages of Amanda and Brittany is equal to Carly's age.

How old is Amanda?

Strategy 1: *Use knowledge of prime numbers.*

The problem states that the sum of the ages of Amanda and Brittany is equal to Carly's age.

If Amanda's age and Brittany's age were both odd primes, Carly's age would be an even number, as the sum of two odd numbers is an even number.

This means that either Amanda's or Brittany's age must be an even number.

The only even prime number is 2, and this is the smallest prime number.

As the problem tells us that Amanda is the youngest, it must be Amanda's age.

Amanda is **2 years old**.

Strategy 2: *Guess, check and refine.*

We know that Amanda, Brittany, and Carly are all younger than 25 years.

We also know that their ages are all prime numbers.

List the prime numbers up to 25.

2	3	5	7	11	13	17	19	23

As 11 is in the middle of our list, let's start there.

Can we find another prime from the list to add to 11 so that the total is a prime number?

Let's start with prime numbers lower than 11.

7 + 11 = 18. 18 is *not* a prime number.

5 + 11 = 16. 16 is *not* a prime number.

3 + 11 = 14. 14 is *not* a prime number.

2 + 11 = 13. 13 **is** a prime number.

We can refine our method when we recognise that the total of 2 odd prime numbers will always be an even number.

2	3	5	7	11	~~13~~	~~17~~	~~19~~	~~23~~
2	3	5	7	11	~~13~~	~~17~~	~~19~~	~~23~~
2	3	5	7	11	~~13~~	~~17~~	~~19~~	~~23~~
2	3	5	7	11	13	~~17~~	~~19~~	~~23~~

We are told Carly's age, the eldest, is the sum of two prime numbers.

As Carly's age is a prime number, the age of one of the other girls must be an even number.

The only even prime is 2.

Knowing this, we can find the other potential ages of Brittany and Carly, but Amanda, who is the youngest, must be 2 years old for the problem to be solved.

Amanda is **2 years old.**

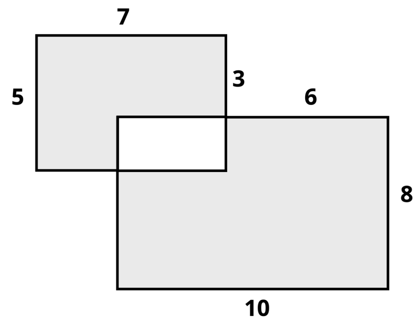
2	3	5	7	11	13	17	19	23
2	3	5	7	11	13	17	19	23
2	3	5	7	11	13	17	19	23
2	3	5	7	11	13	17	19	23

C. The question is:

The length of each segment in the overlapping rectangles is given, in centimetres.

Find the sum of the areas of the shaded regions, in square centimetres.

Strategy 1: *Calculate and subtract the area of the unshaded rectangle.*



To find the sum of the shaded areas, we need to calculate the area of the unshaded rectangle.

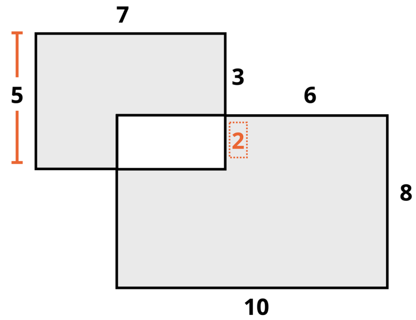
Step 1:

Using the lengths that are given, we can deduce the length and width of the unshaded rectangle.

The length of the upper rectangle is 5 centimetres.

This means that the length of the unshaded rectangle is

5 - 3 = 2 centimetres.



Step 2:

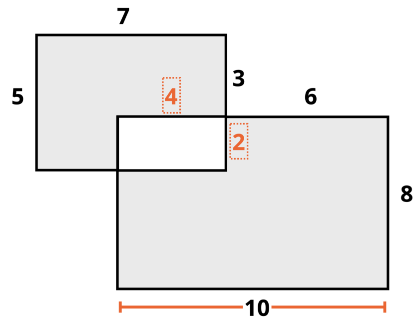
The width of the lower rectangle is 10 centimetres.

Therefore the width of the unshaded rectangle is

10 - 6 = 4 centimetres.

The area of the unshaded rectangle is;

2 × 4 = 8 square centimetres.

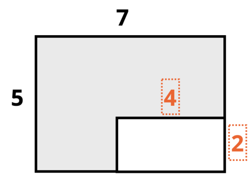


Step 3:

The area of the upper rectangle is 5 × 7 = 35 square centimetres.

Subtract the area of the unshaded rectangle (found in Step 2) to find the shaded area:

35 - 8 = 27 square centimetres.



Step 4:

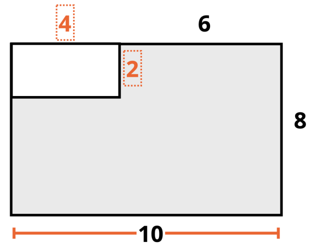
The area of the lower rectangle is 8 × 10 = 80 square centimetres.

Subtract the area of the unshaded rectangle (found in Step 2) to find the shaded area:

80 - 8 = 72 square centimetres.

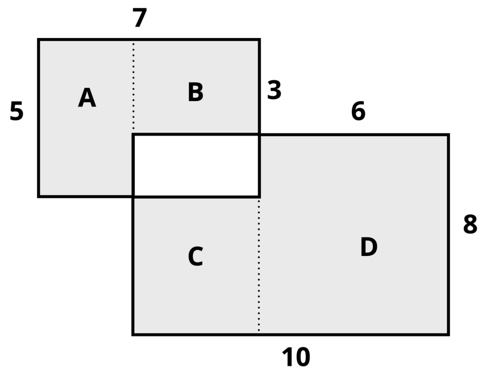
Add the two shaded areas to find the total area of the shaded regions:

27 + 72 = 99 square centimetres.

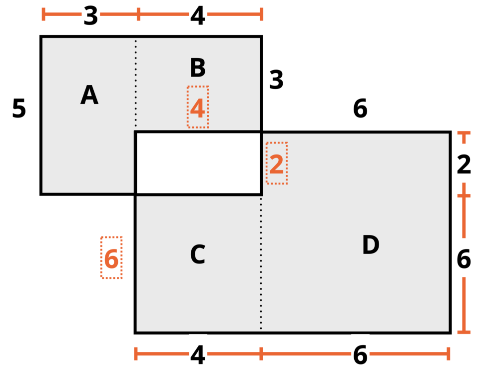


C. **Strategy 2:** *Divide a complex shape.*

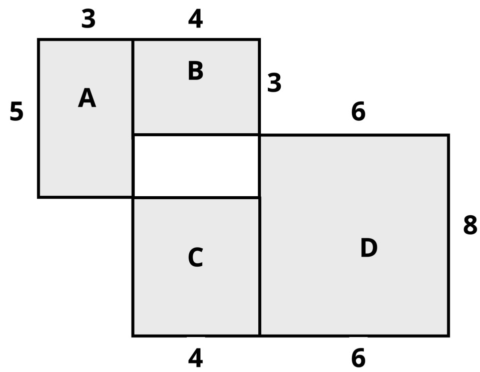
Use lines to divide the upper and lower rectangles and name each shaded region.



Using the lengths that are given, we can deduce the length and width of each of the the shaded regions.



We can now calculate the area of each shaded region:



A = 5 x 3 = 15 square centimetres

B = 3 x 4 = 12 square centimetres

C = 6 x 4 = 24 square centimetres

D = 8 x 6 = 48 square centimetres

The sum of the areas of the shaded regions is **99 square centimetres.**

D. The question is:

The average mass of a group of children is 50 kilograms.

Todd, who has a mass of 62 kilograms, then joins the group.

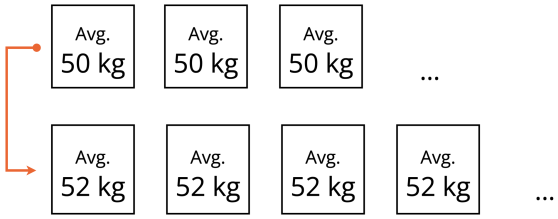
This raises the average mass of the group to 52 kilograms.

How many children were in the original group?

Strategy 1 : *Reason logically to find the number in the original group.*

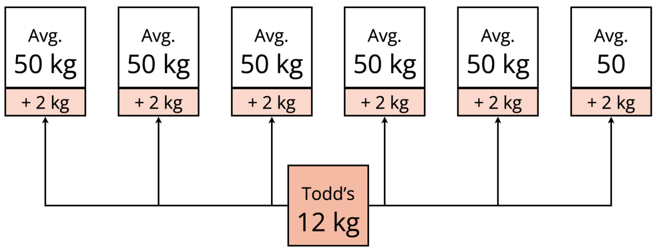
Initially, the average mass of the children is 50 kg.

The average increases to 52 kg when Todd's mass is added to the group.



We know that Todd's mass is 62 kilograms, 12 kilograms more than the initial average.

To increase the average mass of the children from 50 kilograms to 52 kilograms, Todd's extra 12 kilograms can be distributed equally among the children.



For the average to increase by 2 kilograms, Todd's 12 extra kilograms can be distributed to 6 children, including Todd.

Therefore, there were **5 children** in the group before Todd joined.

Strategy 2: *Compare Todd's mass to the new average mass.*

Todd's mass of 62 kilograms exceeds the new average mass of 52 kilograms by 10 kilograms.

This 10 kilograms increases the group's average from 50 kilograms to 52 kilograms.

Divide the 10 kilograms by the 2 kilogram increase to find the number of children in the original group:

10 ÷ 2 = 5.

There were **5 children in the original group.**

E. The question is:

Each letter in the following cryptarithm represents a different digit from the others.

What is the greatest value that WHAT can have?

$$\begin{array}{cccc} & B & A & T\\ & F & A & T\\ + & C & A & T\\ \hline W & H & A & T\end{array}$$

Strategy: *Use properties of whole numbers.*

The sum of T + T + T ends in T.

The only digits that have this property are 0 and 5.

If T = 5, then the sum of T + T + T = 15 and we would have to group the 1 in the tens column to get that 1 + A + A + A = A.

There is no digit that can make this possible.

Therefore T = 0.

$$\begin{array}{cccc} & & {\scriptstyle +1} & \\ & B & A & 5\\ & F & A & 5\\ + & C & A & 5\\ \hline W & H & \cancel{A} & 5\end{array} \qquad \begin{array}{cccc} & B & A & 0\\ & F & A & 0\\ + & C & A & 0\\ \hline W & H & A & 0\end{array}$$

Each letter must represent a different digit, and as T = 0, this means A = 5.

We group the tens and carry them to the hundreds column.

$$\begin{array}{cccc} & {\scriptstyle +1} & & \\ & B & 5 & 0\\ & F & 5 & 0\\ + & C & 5 & 0\\ \hline W & H & 5 & 0\end{array}$$

We are looking for the greatest value for WHAT so we should assign the largest digits to B, F and C.

However, if we assign 9, 8, and 7, the value of H will be a 5 (including the 1 in the hundreds column.)

$$\begin{array}{cccc} {\scriptstyle +2} & {\scriptstyle +1} & & \\ & 9 & 5 & 0\\ & 8 & 5 & 0\\ + & 7 & 5 & 0\\ \hline W & \cancel{5} & 5 & 0\end{array}$$

If we change 7 to the next greatest possible number, 6, we get the greatest possible sum.

Therefore the greatest value of WHAT is **2450**.

$$\begin{array}{cccc} {\scriptstyle +2} & {\scriptstyle +1} & & \\ & 9 & 5 & 0\\ & 8 & 5 & 0\\ + & 6 & 5 & 0\\ \hline 2 & 4 & 5 & 0\end{array}$$

Additional Preparation Questions 1 - 5

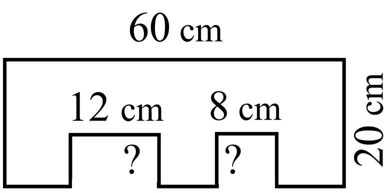
1 What is the value of the following?
55 – 11 + 44 – 22 + 33 – 33 + 22 – 44 + 11 – 55

2 Mrs. Saada is between 50 and 80 years old. If you divide her age by 9, the remainder is 1.
If you divide her age by 4, the remainder is 1. How old is Mrs. Saada?

3 A cricket chirps 6 times every 8 seconds.
At that rate, how many times does the cricket chirp in 2 minutes?

4 Amy, Brett, and Cate each secretly write down Z, U, or T.
What is the probability that Cate's letter is different from both Amy's letter and Brett's letter?

5 Two rectangles with equal heights are cut from a rectangular piece of paper as shown.
The area of the remaining piece of paper is 980 square centimetres.
What is the height of each cut, in square centimetres?



(*not drawn to scale*)

MATHS OLYMPIAD

Additional Preparation Questions 6 - 10

6 The average height of four adults is 180 centimetres.
Two of the adults are each 170 centimetres tall, and the third is 185 centimetres tall.
How tall, in centimetres, is the fourth adult?

7 The digits of a four-digit number are 1, 3, 6, and 9, but not necessarily in that order.
The thousands digit is prime.
The hundreds digit is 3 more than the tens digit.
What is the number?

8 Tracy's Trophies charges by the letter for engraving.
There is one fee for each vowel and a different fee for each consonant.
CAROL costs $31 to engrave.
GABRIEL costs $43 to engrave.
How many dollars does BRIDGET cost to engrave?

9 A 7×7 square is marked off into forty-nine 1×1 small squares.
Each of the small squares along the edges of the large square is painted blue.
How many small squares are painted blue?

10 Lin has 8 marbles. Each marble weighs either 20 grams or 40 grams or 50 grams. He has a different number of marbles (at least one) of each weight. What is the smallest possible total weight of Lin's marbles?

Additional Preparation Questions with Hints 1 - 5

1 What is the value of the following?
55 – 11 + 44 – 22 + 33 – 33 + 22 – 44 + 11 – 55

(Hint: Simplify the arithmetic by pairing like numbers.)

2 Mrs. Saada is between 50 and 80 years old.
If you divide her age by 9, the remainder is 1. If you divide her age by 4, the remainder is 1. How old is Mrs. Saada?

(Hint: Last year her age was a multiple of both 4 and 9.)

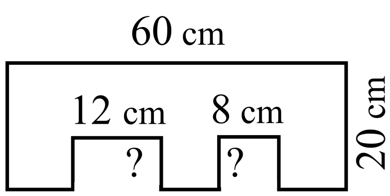
3 A cricket chirps 6 times every 8 seconds.
At that rate, how many times does the cricket chirp in 2 minutes?

(How many 8-second periods are there in 2 minutes?)

4 Amy, Brett, and Cate each secretly write down Z, U, or T.
What is the probability that Cate's letter is different from both Amy's letter and Brett's letter?

(List all possibilities. Circle those in which Cate's letter is different.)

5 Two rectangles with equal heights are cut from a rectangular piece of paper as shown.
The area of the remaining piece of paper is 980 square centimetres.
What is the height of each cut, in square centimetres?



(*not drawn to scale*)

(What was the area of the paper before the rectangles were cut out?)

MATHS OLYMPIAD

Additional Preparation Questions with Hints 6 - 10

6 The average height of four adults is 180 centimetres.
Two of the adults are each 170 centimetres tall, and the third is 185 centimetres tall.
How tall, in centimetres, is the fourth adult?

(What is the sum of the heights of the four adults?)

7 The digits of a four-digit number are 1, 3, 6, and 9, but not necessarily in that order.
The thousands digit is prime.
The hundreds digit is 3 more than the tens digit.
What is the number?

(Which of the choices can be in the thousands place?)

8 Tracy's Trophies charges by the letter for engraving.
There is one fee for each vowel and a different fee for each consonant.
CAROL costs $31 to engrave.
GABRIEL costs $43 to engrave.
How many dollars does BRIDGET cost to engrave?

(How many more consonants, vowels and dollars exist for GABRIEL than for CAROL?)

9 A 7×7 square is marked off into forty-nine 1×1 small squares.
Each of the small squares along the edges of the large square is painted blue.
How many small squares are painted blue?

(Draw the picture and count.)

10 Lin has 8 marbles.
Each marble weighs either 20 grams or 40 grams or 50 grams.
He has a different number of marbles (at least one) of each weight.
What is the smallest possible total weight of Lin's marbles?

(In how many ways can you find three different counting numbers whose sum is 8?)

MATHS OLYMPIAD

Additional Preparation Questions Solutions

1: 0	**2:** 73	**3:** 90	**4:** $\frac{4}{9}$	**5:** 11cm	**6:** 195cm	**7:** 3961	**8:** $45	**9:** 24	**10:** 230g

1 **Strategy:** *Rearrange the numbers to simplify the arithmetic.*

55 – 11 + 44 – 22 + 33 – 33 + 22 – 44 + 11 – 55 =

(55 – 55) + (44 – 44) + (33 – 33) + (22 – 22) + (11 – 11) = 0

The value is **0**.

2 **Strategy 1:** *Consider Mrs. Saada's age last year.*

Last year her age was a multiple of both 9 and 4 and thus a multiple of 36.

The only multiple of 36 between 50 and 80 is 72.

Then this year Mrs. Saada is **73** years old.

Strategy 2: *List the numbers that satisfy one of the conditions.*

It is faster to divide by 9 than by 4 because 9 produces fewer results.

The only numbers between 50 and 80 that are 1 more than a multiple of 9 are 55, 64, and 73.

Of these, only 73 is one more than a multiple of 4. Mrs. Saada is **73** years old.

3 **Strategy:** *Find the number of 8-second periods in the interval.*

Two minutes contain 120 seconds. Because 120 ÷ 8 = 15, a two-minute interval has 15 periods of 8 seconds each.

Because the cricket chirps 6 times in each period, the cricket chirps 15 × 6 = **90** times in two minutes.

4 **Strategy:** *Draw a table.*

Each has a choice of 3 letters. Amy and Brett could have the same letter.

The sample space has 3 × 3 × 3 = 27 possibilities.

Write them out as a chart as follows or as a tree diagram.

The instances when Cate's letter is different from both Amy's letter and from Brett's letter are boldfaced.

Cate chooses Z	ZZZ	ZZU	ZZT	ZUZ	**ZUU**	**ZUT**	ZTZ	**ZTU**	**ZTT**
Cate chooses U	**UZZ**	UZU	**UZT**	UUZ	UUU	UUT	**UTZ**	UTU	**UTT**
Cate chooses T	**TZZ**	**TZU**	TZT	**TUZ**	**TUU**	TUT	TTZ	TTU	TTT

The probability that Cate's letter is different from both Amy's and Brett's is $\frac{12}{27}$ or $\frac{4}{9}$.

Additional Preparation Questions Solutions

5 **Strategy:** *Find the total area of the missing pieces.*

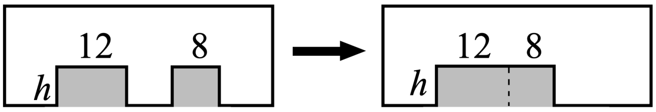
The original area of the piece of paper is 60 × 20 = 1200 square centimetres.

The total area of the regions cut out is then 1200 – 980 = 220 square centimetres.

Then:

Method 1: *Change the figure to create a simpler problem.*

“Slide” the two shaded cutout rectangles together, as shown.



None of the areas will change.

The total area of the cutouts is the same as the area of a single rectangle with base 12 + 8 = 20cm.

The area of this single rectangle is 220 so the height of each cut is 220 ÷ 20 = **11cm**.

Method 2: *Use algebraic reasoning.*

Let h = the height of each cut.

The areas of the cutout rectangles are 12h and 8h.

Add 12h and 8h:

Divide each side of the equation by 20:

The height of each cut is **11cm.**

$$12h + 8h = 220$$
$$20h = 220$$
$$h = 11$$

The height of each cut is 11 cm.

6 **Strategy:** *Find the total of the four heights.*

The four heights have an average of 180, so their total is 180 × 4 = 720cm.

The sum of the three known heights is 525cm, so **the fourth adult’s height is** 720 – 525 = **195cm**.

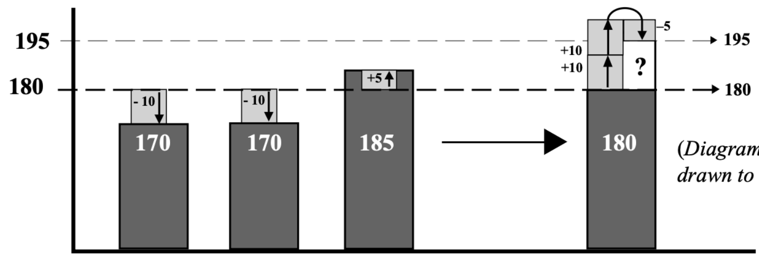
Strategy: Compare each height with the average.

Each of the heights of the first two adults are 10cm less than the given average.

The height of the third adult is 5cm *more* than the average.

The sum of these three heights is 15cm less than the average.

The fourth adult is **then 15cm more than the average, or 195cm tall,** as illustrated by the diagram.



(Diagram is not drawn to scale.)

7 **Strategy:** *Fulfil one requirement at a time.*

3 is the only prime number given .... 3 _ _ _

9 is 3 more than 6.......................... 3 9 6 _

1 is the only digit left...................... 3 9 6 1

The number is 3 961.

Additional Preparation Questions Solutions

8 **Strategy:** *Determine why GABRIEL costs more than CAROL.*

GABRIEL has 1 more consonant and 1 more vowel than CAROL.

Then 1 consonant and 1 vowel together cost 43 – 31 = $12.

2 consonants and 2 vowels together cost twice as much, $24.

But CAROL has 3 consonants and 2 vowels.

So the 1 extra consonant costs 31 - 24 = 7.

Then the 3 consonants in CAROL cost $21, and the 2 vowels cost 31 – 21 = $10.

1 vowel costs $5.

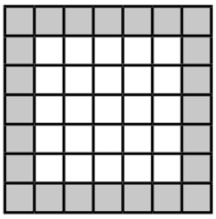
BRIDGET, with 5 consonants and 2 vowels, costs 5 x 7 + 2 x 5 = **$45** to engrave.

9 **Strategy 1:** *Draw a picture.*

Draw the 7 x 7 square.

Shade the small squares on the edges and count them.

24 of the small squares are painted blue.



Strategy 2: *Count the small squares along one edge.*

7 small squares along each of the 4 sides of the large square give a total of 28.

But each of the 4 corner squares has been counted twice.

Therefore, there are a total of 28 – 4 = **24** of the small squares are painted blue.

10 **Strategy:** *Find three different counting numbers with a sum of 8.*

The numbers of each type of marble can be 1, 3, and 4, or they can be 1, 2 and 5.

To get the least possible total weight, assign the greatest weight to the least number of marbles and the second greatest weight to the second least possible number of marbles.

Then the least total weight occurs when Lin has one 50 g marble, two 40 g marbles, and five 20 g marbles.

The smallest possible total weight of Lin's marbles is (1 × 50) + (2 × 40) + (5 × 20) = **230g**.

Basic Terms

• Sum	• Difference	• Product	• Quotient
• Value	• Multiple	• Factor	• Remainder
• Fraction	• Decimal Fraction	• Percentage	• Ratio
• Square / Perfect Square	• Square Root	• Cube / Perfect Cube	• Cube Root

- "Or" is inclusive: "*a* or *b*" means "*a* or *b* or both".
- An ellipsis (...) indicates that some information has been omitted intentionally.
  Read "**1 + 2 + 3 + ...**" as "one plus two plus three and so on, without end."
  Read "**1 + 2 + 3 + ... + 10**" as "one plus two plus three and so on up to ten."

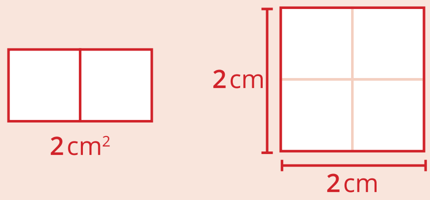
Units of Measurement

Familiarity with units of measurement is assumed, including conversions from one unit to another:

- **Time**: seconds ↔ minutes ↔ hours ↔ days
- **Length**: millimetres ↔ centimetres ↔ metres ↔ kilometres
- **Area**: mm^2 ↔ cm^2 ↔ m^2 ↔ km^2
- **Volume/Capacity**: mm^3 ↔ cm^3 ↔ m^3; millilitres ↔ litres
- **Mass**: grams ↔ kilograms
- **Angles**: degrees (°)

Units of measurement must be correct if given in an answer.

To avoid confusion, read cm^2 as "square centimetres", not "centimetres squared".



2 square centimetres | Possible misinterpretation

Presenting Answers

Unless otherwise specified in a problem, **equivalent numbers or expressions are acceptable**.

- For example, $3\frac{1}{2}$, $\frac{7}{2}$, and 3.5 are equivalent. $3\frac{2}{4}$ and $\frac{70}{20}$ are not in lowest terms and will not be accepted.

After reading a problem, it is useful to indicate the nature of the answer, before commencing the solution strategy. For example:

- **"*A* = ____ , *B* = ____ ."**
- **"The largest number is ____ ."**
- **"The [sum | difference | product | quotient] is ____ ."**
- **"The probability, as a [fraction | decimal | percentage], is ____ ."**
- **"The perimeter is ____ centimetres."**
- **"The area is ____ square units."**
- **"The average speed is ____ kilometres per hour."**

MATHS OLYMPIAD

Digits and Integers

A **digit** is any one of the ten numerals **0, 1, 2, 3, 4, 5, 6, 7, 8, 9**.

- **358** is a three-digit number.

The **lead digit** (leftmost digit) of a number is not counted as a digit if it is **0**.

- **0358** is a three-digit number.

Terminal zeroes of a number are the zeroes to the right of the last nonzero digit.

- **30 500** has two terminal zeroes.

Whole numbers: { 0, 1, 2, 3, ... }.

Counting numbers, or **Positive Integers**: { 1, 2, 3, ... }.

Integers: { ..., –2, –1, 0, 1, 2, 3, ... }.

- Positive numbers, negative numbers, non-negative numbers, and non-positive numbers are terms that may appear in Division S problems.

Consecutive Numbers are counting numbers that differ by **1**.

- **83, 84, 85, 86, 87.**

Consecutive Even Numbers are multiples of **2** that differ by **2**.

- **36, 38, 40, 42.**

Consecutive Odd Numbers are non-multiples of **2** that differ by **2**.

- **57, 59, 61, 63.**

Factors and Divisibility

Suppose $A = B \times C$, and A, B, and C are all **counting numbers** (1, 2, 3, ...).

- $6 = 2 \times 3$.

Then, A is divisible by B, and A is a multiple of B.

- **6** is divisible by **2**.
- **6** is a multiple of **2**

Likewise, A is divisible by C, and A is a multiple of C.

- **6** is divisible by **3**.
- **6** is a multiple of **3**

Both B and C are factors of A.

- **2** and **3** are factors of **6**.

A **prime number** is a counting number with exactly two factors, **1** and itself.

- **2, 3, 5, 7, 11, 13, ...**

A **composite number** is a counting number which has at least three different factors.

- **4, 6, 8, 9, 10, 12, ...**

The number **1** is neither prime nor composite since it has exactly one factor.

A number is **factored completely** when it is expressed as a product of only prime numbers.

- $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
  $= 2^4 \times 3^2$.

The **Highest Common Factor (HCF)** of two counting numbers is the largest counting number that divides each of the two numbers, and the remainder is zero.

If the **HCF** of two numbers is **1**, then we say that the numbers are **relatively prime**.

- **HCF (12,18) = 6.**

The **Lowest Common Multiple (LCM)** of two counting numbers is the smallest number that each of the given numbers divides, and the remainder is zero.

- **LCM (12,18) = 36.**

Fractions

For **common** or **simple fractions** $\frac{a}{b}$,

- a (the **numerator**) and b (the **denominator**) are both integers, and
- $b \neq 0$.

In a **unit fraction**, the numerator is 1.	In a **proper fraction**, $a < b$.	In an **improper fraction**, $a \geq b$.
• $\frac{1}{2}$ and $\frac{1}{100}$ are both unit fractions.	• $\frac{1}{2}$ and $\frac{5}{6}$ are both proper fractions.	• $\frac{3}{2}$ and $\frac{11}{8}$ are both improper fractions.

A **complex fraction** is a fraction whose numerator or denominator contains a fraction.

- $\frac{\frac{2}{3}}{5}$, $\frac{2}{\frac{3}{5}}$, $\frac{\frac{2}{3}}{\frac{5}{7}}$, $\frac{2+\frac{3}{5}}{5-\frac{1}{2}}$ are complex fractions.

The fraction $\frac{a}{b}$ is **simplified** (in **lowest terms**) if a and b have no common factor other than 1 - i.e. HCF(a,b) = 1.

- $\frac{1}{2}$ and $\frac{3}{2}$ are both expressed in lowest terms. $\frac{2}{4}$ and $\frac{30}{20}$ are not in lowest terms.
- Unless otherwise specified, fraction answers to Olympiad problems must be expressed in lowest terms.

A **decimal** or **decimal fraction** is a fraction whose denominator is a power of ten.

The decimal is written using decimal point notation.

- $0.07 = \frac{7}{100}$, $0.153 = \frac{153}{1000}$, $6.4 = 6\frac{4}{10}$ or $\frac{64}{10}$.

A **recurring decimal**, or **repeating decimal**, is a decimal fraction with a digit, or group of digits, that repeats forever.

- $\frac{1}{3} = 0.333... = 0.\dot{3} = 0.\overline{3}$
- $\frac{1}{6} = 0.1666... = 0.1\dot{6} = 0.1\overline{6}$
- $\frac{1}{7} = 0.142857142857... = 0.\dot{1}4285\dot{7} = 0.\overline{142857}$

A **percentage** is a fraction whose denominator is 100. The **percent sign** represents the division by 100.

- $9\% = \frac{9}{100}$, $125\% = \frac{125}{100}$, $0.3\% = \frac{0.3}{100}$ or $\frac{3}{1000}$.

Order of Operations

When an expression has more than one arithmetic symbol, certain operations occur before others.

There are a few ways to remember the order of operations, and mnemonics are often used (e.g. **BIDMAS**; **PEMDAS**).

However, it can also be useful to consider the intent when an arithmetic expression is constructed.

By convention, we observe the following priorities:

1. Perform operations in **parentheses**, **braces**, or **brackets**. The **vinculum** (line in a fraction) is also considered as a grouping symbol, similar to parentheses.
2. Evaluate **exponents** (**indices**).
3. Evaluate **multiplication** and **division**, from left to right.
4. Evaluate **addition** and **subtraction**, from left to right.

Example 1

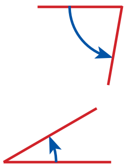
$30 + 6 \div 2 - 5 \times (9 - 7)$
$= 30 + 6 \div 2 - 5 \times 2$
$= 30 + 3 - 10$
$= 23$

Example 2

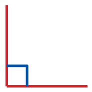
$20 - (8 + (1 + 2)^2)$
$= 20 - (8 + 3^2)$
$= 20 - (8 + 9)$
$= 20 - 17$
$= 3$

Two-Dimensional Figures

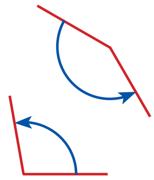
Acute angle
between 0° and 90°



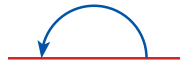
Right angle
90°



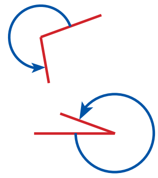
Obtuse angle
between 90° and 180°



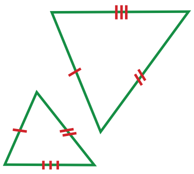
Straight angle
180°



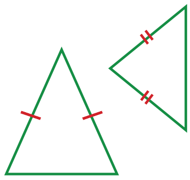
Reflex angle
between 180° and 360°



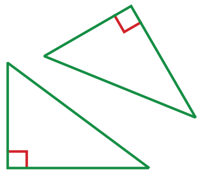
Scalene triangle



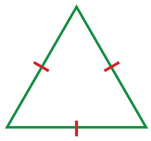
Isosceles triangle



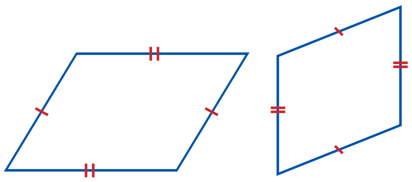
Right-angled triangle



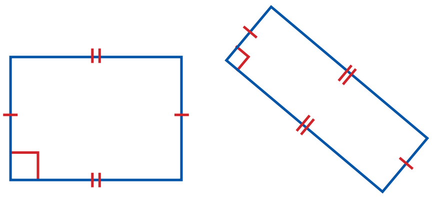
Equilateral triangle



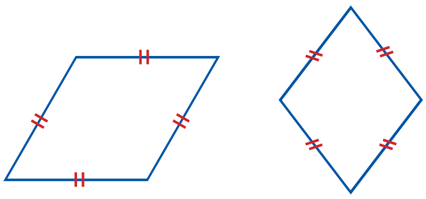
Parallelogram



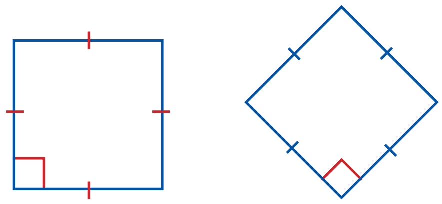
Rectangle



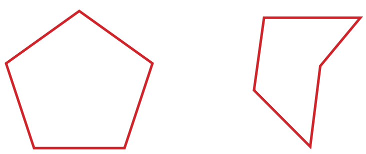
Rhombus



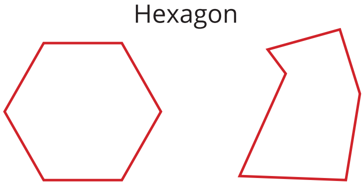
Square



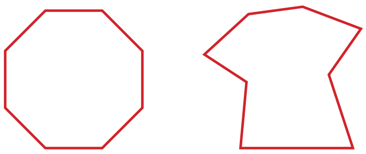
Pentagon



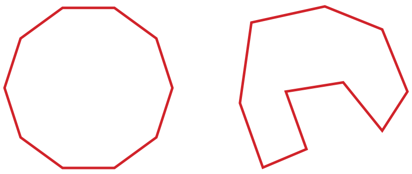
Hexagon



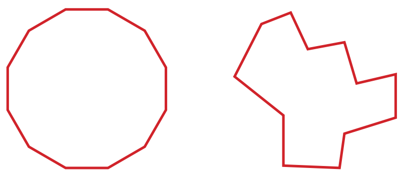
Octagon



Decagon

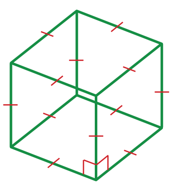


Dodecagon

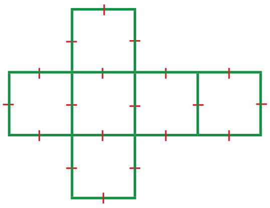


Three-Dimensional Objects

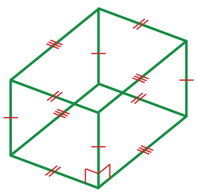
Cube



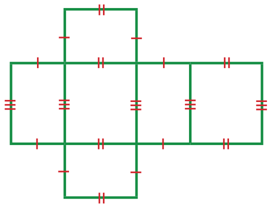
One possible net of a cube



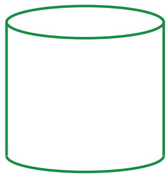
Rectangular Prism



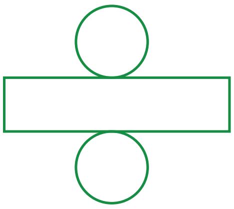
One possible net of a rectangular prism



Right Cylinder



One possible net of a right cylinder



Congruence and Similarity

Two geometric figures are **congruent** if they are identical.

- Congruent triangles coincide exactly when one is superimposed upon the other.
- Congruent plane figures have corresponding pairs of sides that are equal, and corresponding pairs of angles that are the same.

Two geometric figures are **similar** if their shape is the same, even though their size may be different.

- All squares are similar, and all circles are similar.

Classification of Geometric Figures

All equilateral triangles are isosceles, but only some isosceles triangles are equilateral.

A square is a rectangle with all sides congruent.

A square is also a rhombus with all angles congruent.

Within the USA/Canada, a trapezium is an irregular quadrilateral.
Outside the USA/Canada, a trapezium is a quadrilateral with at least one pair of parallel sides (known as a "trapezoid" within the USA/Canada).

Calendar Conventions

There was no year **0**. The first century spanned the years **1** to **100** inclusive.

- The **20th century** spanned the years **1901** to **2000** inclusive.
- The **21st century** spans the years **2001** to **2100** inclusive.

Measures of Centre

The mean, arithmetic mean, or average, of a set of values is

- the sum of the values, divided by
- the number of values.

- For the set **{ 5, 5, 7, 11 }**, the mean is $\frac{5+5+7+11}{4}$ = **28 ÷ 4 = 7**.
- For the set **{ 7, 11, 23, 5, 5 }**, the mean is $\frac{7+11+23+5+5}{5}$ = **51 ÷ 5 = $10\frac{1}{5}$**.

The median is the value that is exactly in the middle of the set when it is ordered.
If there are an even number of values, then the median is the mean of the two middle values.

- For the set **{ 5, 5, 7, 11 }**, the median is **(5 + 7) ÷ 2 = 6.**
- For the set **{ 7, 11, 23, 5, 5 }**, we begin by ordering the set of values: **{ 5, 5, 7, 11, 23 }**.

  The median is the middle value, **7.**

The mode is the value that occurs the greatest number of times.

- For the set **{ 5, 5, 7, 11 }**, the mode is **5.**

A set with every value listed an equal number of times is said to have no mode.

- For the set **{ 5, 5, 7, 7, 8, 8 }**, there is no mode.

Probability

The probability of an event is a value that expresses how likely an event is to occur.

- If the event is impossible, then the probability is **0**.
- If the event is certain, then the probability is **1**.
- All probabilities are between **0** and **1** inclusive.

The probability is found by dividing the number of times an event does occur, by the total number of times the event can possibly occur.

- The probability of rolling an odd number on a die is $\frac{3}{6}$ or $\frac{1}{2}$.