APSMO

2024 OLYMPIADS

IMPORTANT

The information contained in this file is ONLY for the use of registered participants of the 2024 Australasian Problem Solving Mathematical Olympiads.

APSMO
2024 OLYMPIADS

Organisation and Procedures

For full details, see the Members' Area

To ensure the integrity of the competition, the Olympiads must be administered under examination conditions.

DO	DO NOT
• Supervise students at all times • Seat students apart • Maintain silence • Provide blank working paper • Give time warnings when 3 minutes remain, and again when 1 minute remains • Collect, mark and retain the papers	• Print the Olympiad papers prior to the Olympiad Date • Read the questions aloud to the students • Interpret the questions for students • Permit any discussion or movement around the room • Permit the use of calculators or other electronic devices

- Olympiad papers are scored by the PICO using the *Solutions and Answers* sheet provided.
- Results should be submitted in the Members' Area within 7 days of the Olympiad.
- Original student answer sheets should be retained by the PICO until the end of the year.
- *Solutions and Answers sheets* are not to be handed out to students. They are a teaching resource for use in class ***after*** completion of the Olympiad paper.

Timing of the Olympiad

- The *Total Time Allowed* for the Olympiad is **30 minutes**.
- The time for each individual question is a guide for the students.

Absent Student Policy

A student who is legitimately absent on the Olympiad date, may sit the Olympiad under examination conditions on their first day back at school (if that date is within 2 weeks of the original Olympiad date). If these conditions cannot be met, the student must be marked as absent on the submitted results.

The Absent Student Policy is available in the **Contest Administration** section of the Members' Area.

APSMO

2024: DIVISION J
WEDNESDAY 4 SEPTEMBER 2024

MATHS OLYMPIAD

OLYMPIAD 4

Total Time Allowed: ***30 Minutes***
Calculators NOT Permitted

Write your answers in the boxes on the back.

Keep your answers hidden by folding backwards on this line.

4A. Let *AB* and *CD* represent two, 2-digit numbers with no digits in common.
What is the greatest possible product *AB* × *CD*?

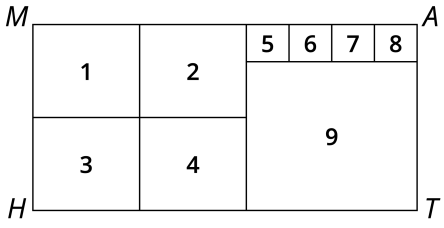
4B. The number 345 has 3 prime factors.
What is the sum of these three primes?

4C. In the kingdom of Mathsendorf, people use gems instead of coins.
There are 3 types of gems; diamonds, emeralds and rubies.
The conversion rates are:
5 diamonds = 6 emeralds and
2 emeralds = 15 rubies.

What is the least number of gems a person needs so that ⅓ of the total value is represented by diamonds, ⅓ of the total value is represented by emeralds, and ⅓ of the total value is represented by rubies?

4D. For the list of 5 integers 2, 0, 2, 4, *N*, the median is 1 less than the mean.
Find the value of *N*.

4E. Rectangle *MATH* is divided into nine squares as shown.
If the perimeter of rectangle *MATH* is divided by the perimeter of the square labelled number 5, what is the answer?



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OLYMPIAD

4

Student Name:

4A.

4B.

4C.

4D.

4E.

Fold Here. Keep your answers hidden.

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OLYMPIAD 4

Solutions and Answers

For teacher use only. Not for Distribution.

4A: 8352	**4B:** 31	**4C:** 56	**4D:** 7	**4E:** 7

4A. The question is:

Let *AB* and *CD* represent two, 2-digit numbers with no digits in common.
What is the greatest possible product *AB* × *CD*?

METHOD 1 *Strategy*: *Reason Logically.*

The 4 largest digits we can assign to *A, B, C* and *D* are 9, 8, 7 and 6.

Compare the products of 97 × 86 and 96 × 87 to find the greatest product.

The greatest product is 96 × 87 = **8352**

$$\begin{array}{rrrrr} & & 9^{5}_{4} & 7 \\ & \times & 8 & 6 \\ \hline & & 5^{1} & 8 & 2 \\ + & 7^{1} & 7 & 6 & 0 \\ \hline & 8 & 3 & 4 & 2 \end{array} \qquad \begin{array}{rrrrr} & & 9^{4}_{4} & 6 \\ & \times & 8 & 7 \\ \hline & & 6^{1} & 7 & 2 \\ + & 7^{1} & 6 & 8 & 0 \\ \hline & 8 & 3 & 5 & 2 \end{array}$$

METHOD 2 *Strategy*: *Use an Area Model Diagram.*

We can use area models to compare the products of 97 × 86 and 96 × 87 to find the greatest product.

The sum of the first area model is:

7200 + 630 + 480 + 42 = **8352**

The sum of the second area model is:

7200 + 540 + 560 + 42 = **8342**

The greatest product is shown by Area Model 1:

96 × 87 = **8352**

Area Model 1

×	90	6
80	7200	480
7	630	42

Area Model 2

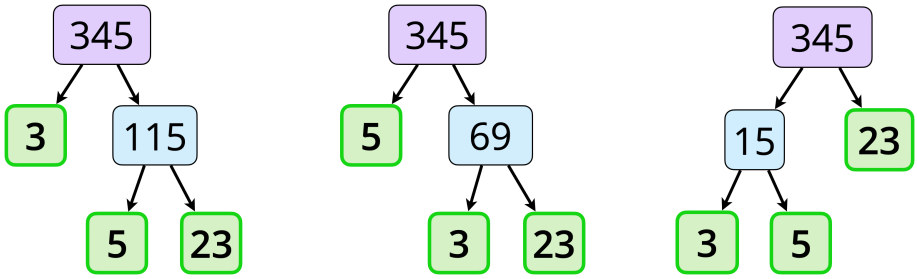
×	90	7
80	7200	560
6	540	42

4B. The question is:
The number 345 has 3 prime factors.
What is the sum of these three primes?

METHOD 1 *Strategy*: *Create Factor Trees.*

Find the factors of 345 using factor trees.

Identify the 3 prime numbers.



The sum of these 3 prime factors is 3 + 5 + 23 = **31**

METHOD 2 *Strategy*: *Prime Factorisation with the Division Method.*

To find the 3 different prime factors of 345, we can use the prime factorisation division method.

The first step is to divide 345 by the smallest prime number that divides into the number evenly.

345 is not divisible by 2. The smallest prime number that does divide evenly is 3.

Now we divide the quotient (115) by the smallest prime number by which it can be divided evenly.

115 is not divisible by 2 or 3.

The smallest prime number that does divide evenly is 5.

3	**345**
	115

115 ÷ 5 = 23

None of these prime numbers: 2, 3, 5, 7, 11, 13, 17 or 19 can divide into 23 evenly.

Therefore, the smallest prime number that can is 23.

3	**345**
5	115
	23
	1

Now we have 1 as the quotient.

We have completed the prime factorisation of 345.

The sum of these 3 prime factors is 3 + 5 + 23 = **31**

3	**345**
5	115
23	23
	1

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4C. The question is:

What is the least number of gems a person needs so that ⅓ of the total value is represented by diamonds, ⅓ of the total value is represented by emeralds, and ⅓ of the total value is represented by rubies?

METHOD 1 *Strategy*: *Draw a Table.*

The conversion rates given for the gems in the problem are 5 diamonds = 6 emeralds and 2 emeralds = 15 rubies.

There cannot be fewer than 15 rubies, or there will be a fractional number of emeralds.

We can draw a table to find that the least number of rubies needed for there to be a whole number of emeralds and diamonds is 45.

Add the number of rubies, emeralds and diamonds to find the total number of gems:

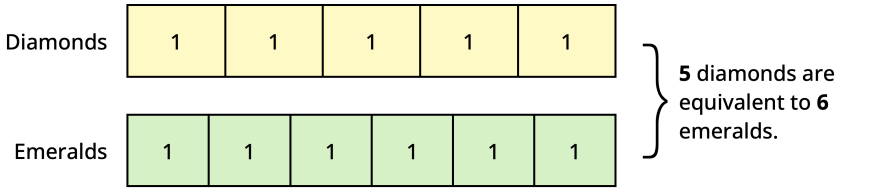
45 + 6 + 5 = **56**

Rubies	Emeralds	Diamonds
15	2	1 ⅔ ✗
22 ½ ✗	3	2 ½ ✗
30	4	3 ⅓ ✗
37 ½ ✗	5	4 ⅙ ✗
45	**6**	**5**

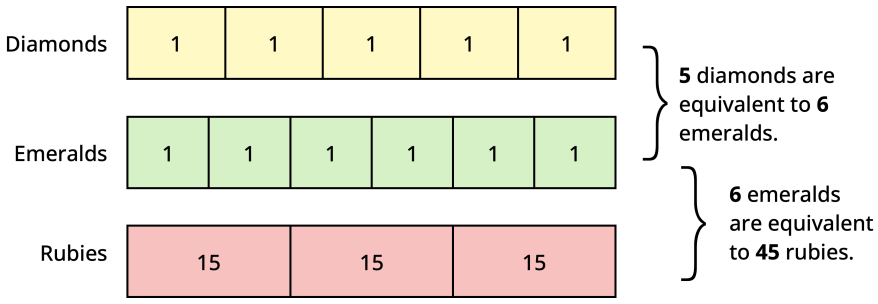
METHOD 2 *Strategy*: *Use Ratios to make Equivalent Values.*

There cannot be fewer than **6** emeralds, or there will be a fractional number of diamonds.

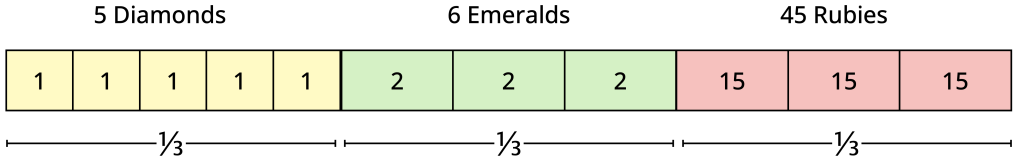
The least number of emeralds of equal value to **5** diamonds is **6** emeralds.



The least number of rubies of equal value to **6** emeralds is 45 rubies.



The least numbers of gems of equal value is 5 diamonds, 6 emeralds and 45 rubies is 5 + 6 + 45 = **56** gems.



4D. The question is:

For the list of 5 integers 2, 0, 2, 4, *N*, the median is 1 less than the mean.

Find the value of *N*.

METHOD 1 *Strategy*: *Guess, Check and Refine.*

The median of this list must be an integer, therefore the mean must also be an integer.

The mean is 0 + 2 + 2 + 4 + *N* divided by 5.

Therefore 8 + *N* must be divisible by 5.

Let's try some values of *N* to find an integer sum (that is divisible by 5) where the median is 1 less than the mean.

The median is 1 less than the mean when *N* is **7**.

The value of *N* is **7**.

N	Integer Sum	Mean	Median
2	8 + 2 = **10**	10 ÷ 5 = **2**	0, 2, 2, 2, 4
7	8 + 7 = **15**	15 ÷ 5 = **3**	0, 2, 2, 3, 4
12	8 + 12 = **20**	20 ÷ 5 = **4**	0, 2, 2, 4, 4

METHOD 2 *Strategy*: *Reason Logically.*

Reason logically to find the median:

If ***N*** is **0**, **1**, or **2**, the median is **2**:

0, 0, **2**, 2, 4

0, **1**, **2**, 2, 4

0, **2**, **2**, 2, 4

If ***N*** is **3** or **4**, the median is **2**:

0, 2, **2**, **3**, 4

0, 2, **2**, 4, **4**

If *N* is greater than **4**, the median is **2**:

0, 2, **2**, 4, ***N***

The median of the 5 integers must be **2**.

Therefore the mean of the 5 integers is **3**.

(0 + 2 + 2 + 4 + *N*) ÷ 5 = 3

(8 + *N*) ÷ 5 = 3

Rearrange the equation to find *N*.

(8 + *N*) = *3* × *5*

8 + *N* = *15*

N = *15* − *8*

N = **7**

The value of *N* is **7**.

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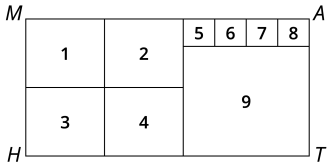
4E. The question is:

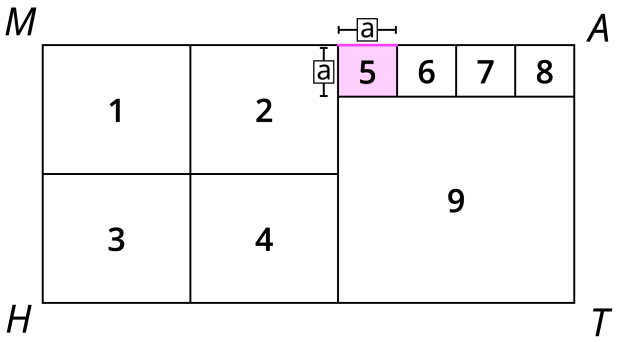
Rectangle *MATH* is divided into nine squares as shown.

If the perimeter of *MATH* is divided by the perimeter of the square with the number 5, what is the answer?



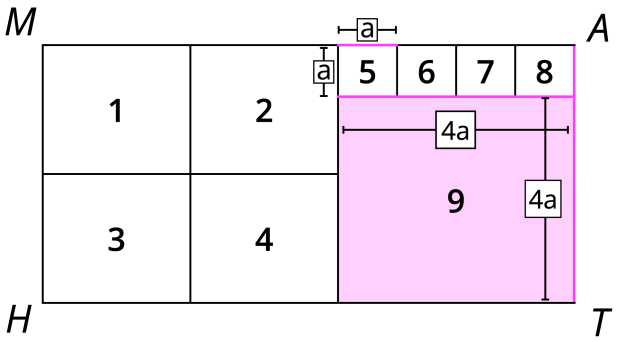
METHOD 1 *Strategy*: *Find the missing lengths and then calculate each perimeter.*

Start by identifying the square with the number 5 and marking its sides as *a*.

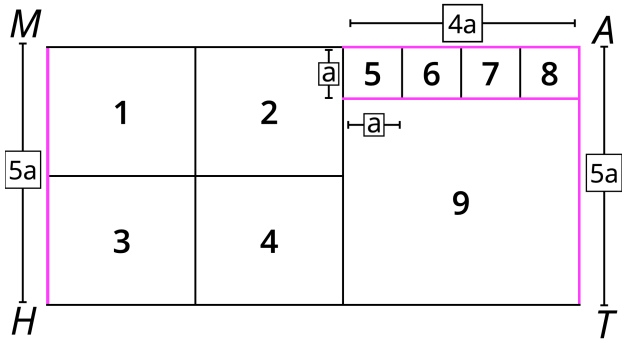


The **height** of the rectangle *MATH* is 5*a*. This includes the height of square 9 (4*a* units) and the height of square 8 (*a* units).

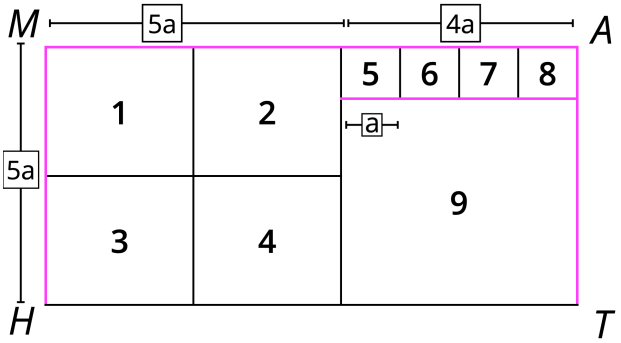
We can show that width of square 9 is 4*a*, as is its height.



The width of squares 1 and 2 is 5*a* as it equal to the height of squares 1 and 3.



The **width** of the rectangle *MATH* is 9*a*.



We now have the dimensions of rectangle *MATH* and can calculate its perimeter:

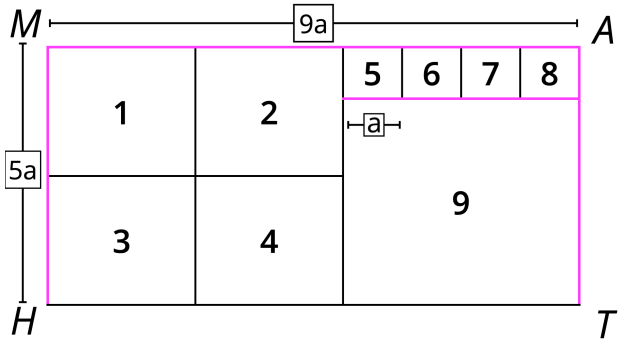
$5a + 9a + 5a + 9a = \mathbf{28a}$

We can also calculate the perimeter of square 5:

$a + a + a + a = \mathbf{4a}$

To divide the perimeter of the rectangle *MATH* by the perimeter of square 5 we calculate:

$28 \div 4 = \mathbf{7}$



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OLYMPIAD 4

Questions and Answers
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4A. **Follow Up 1:**

In the original problem, what is the least possible product *AB* × *CD*? **[230]**

Follow Up 2:

Let *ABC* and *DEF* represent two, 3-digit numbers with no digits in common.
What is the greatest possible product for *ABC* × *DEF*? **[843,500]**

4B. **Follow Up 1:**

Find the sum of all whole number factors of 345. **[576]**

Follow Up 2:

Consider the proper factors of 12: 1, 2, 3, 4, and 6. Their sum is 16, which is greater than 12 (the original number). Therefore, 12 is an "abundant" number and it is the least abundant number. What is the next greater abundant number? **[18].**

4C. **Follow Up 1:**

A customer handed the shopkeeper a 90-ruby banknote to pay for an item that cost 51 rubies. What is the least number of gems the customer could have received in change? **[5]**

Follow Up 2:

In GemWorld, *N* emeralds cost 1.2*N* rubies. If the market price of emeralds increases by 25%, then *N* rubies will cost *KN* clams. Find K. **[1.5]**

4D. **Follow Up:**

For the list of five integers 2, 0, 2, 3, *N*, the median is 1 more than the mean. Find *N*. **[-2]**

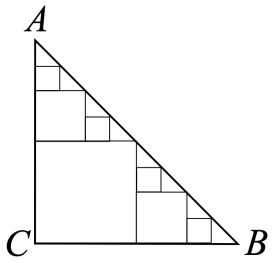
4E. **Follow Up 1:**

Three squares with dimensions 3 × 3, 4 × 4, and 5 × 5 are arranged, edge to edge, without overlap, to form a single irregular polygon. What is the smallest perimeter of this polygon? **[32]**

Follow Up 2:

The given figure consists of isosceles right triangles and squares.

What fraction of the total area of *ABC* is covered by squares? **[7/8]**



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What is the greatest possible product for *ABC* × *DEF*?

4B. Follow Up 1:

Find the sum of all whole number factors of 345.

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OLYMPIAD 4

4C. Original Question:

In the kingdom of Mathsendorf, people use gems instead of coins.
There are 3 types of gems; diamonds, emeralds and rubies.
The conversion rates are:
5 diamonds = 6 emeralds and
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Follow Up 1:

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For the list of five integers 2, 0, 2, 3, N, the median is 1 more than the mean. Find N.

4E. Follow Up 1:

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