APSMO

2024 OLYMPIADS

IMPORTANT

The information contained in this file is ONLY for the use of registered participants of the 2024 Australasian Problem Solving Mathematical Olympiads.

APSMO
2024 OLYMPIADS

Organisation and Procedures

For full details, see the Members' Area

To ensure the integrity of the competition, the Olympiads must be administered under examination conditions.

DO	DO NOT
• Supervise students at all times • Seat students apart • Maintain silence • Provide blank working paper • Give time warnings when 3 minutes remain, and again when 1 minute remains • Collect, mark and retain the papers	• Print the Olympiad papers prior to the Olympiad Date • Read the questions aloud to the students • Interpret the questions for students • Permit any discussion or movement around the room • Permit the use of calculators or other electronic devices

- Olympiad papers are scored by the PICO using the *Solutions and Answers* sheet provided.
- Results should be submitted in the Members' Area within 7 days of the Olympiad.
- Original student answer sheets should be retained by the PICO until the end of the year.
- *Solutions and Answers sheets* are not to be handed out to students. They are a teaching resource for use in class ***after*** completion of the Olympiad paper.

Timing of the Olympiad

- The *Total Time Allowed* for the Olympiad is **30 minutes**.
- The time for each individual question is a guide for the students.

Absent Student Policy

A student who is legitimately absent on the Olympiad date, may sit the Olympiad under examination conditions on their first day back at school (if that date is within 2 weeks of the original Olympiad date). If these conditions cannot be met, the student must be marked as absent on the submitted results.

The Absent Student Policy is available in the **Contest Administration** section of the Members' Area.

APSMO

2024: DIVISION J
WEDNESDAY 31 JULY 2024

OLYMPIAD 3

Total Time Allowed: ***30 Minutes***

Write your answers in the boxes on the back.

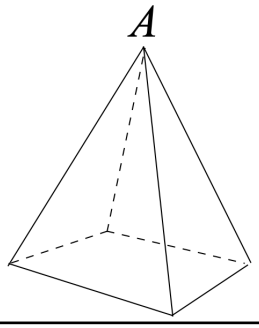
Keep your answers hidden by folding backwards on this line.

3A. On standard dice, the numbers 1 through 6 are used, one number on each face. The sums of opposite faces on the dice are the same.
What is the sum of all the numbers on two standard dice excluding the number that appears on the opposite face to 6?

3B. The only 2-digit perfect squares are 16, 25, 36, 49, 64, and 81.
The 4-digit passcode represented by *ABCD* has the following properties:
AB, *BC*, and *CD* are each perfect squares.
If ABCD is even what is the 4-digit passcode ABCD?

3C. Isabella wants to make a rectangular garden with an area of 108 square metres. She wants the length and width of the garden to be whole numbers when expressed in metres.
Let G be the largest possible perimeter of the garden and let L be the smallest possible perimeter of the garden.
Find the value of G – L in metres.

3D. A bug will walk along the edges of a pyramid, never retracing its steps.
It starts at vertex A, traces edges and visits every vertex exactly once until it returns to vertex A.
How many different pathways can the bug follow?



3E. In the cryptarithm shown, each letter represents a unique digit.
No "crypto-word" can begin with the digit '0'.
What is the largest possible sum?

$$\begin{array}{rcccc} & A & B & C & D \\ & B & C & D & A \\ & C & D & A & B \\ + & D & A & B & C \\ \hline \end{array}$$

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WEDNESDAY 31 JULY 2024

OLYMPIAD

3

Student Name:

3A.

3B.

3C.

3D.

3E.

Fold Here. Keep your answers hidden.

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2024: DIVISION J
WEDNESDAY 31 JULY 2024

OLYMPIAD 3

Solutions and Answers

For teacher use only. Not for Distribution.

3A: 40	**3B:** 8164	**3C:** 176	**3D:** 8	**3E:** 33,330

3A. The question is:

On standard dice, the numbers 1 through 6 are used, one number on each face.

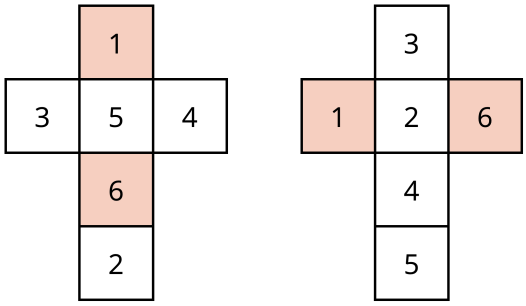
The sums of opposite faces on the dice are the same.

What is the sum of all the numbers on two standard dice excluding the number that appears on the opposite face to 6?

METHOD 1 *Strategy*: *Use Number Properties and Dice Nets.*

For the sums of the opposite faces on the dice to be the same, 3 is opposite 4 (opposite faces on a standard dice add to 7), 5 is opposite 2 and 1 is opposite 6.

We can sketch 2 dice nets where pairs of numbers adding to 7 are placed on opposite faces



The sum of these numbers excluding the number on the face opposite 6 is:

$6+5+4+3+2+6+5+4+3+2=$**40**

METHOD 2 *Strategy*: *Use Number Properties and Reason Logically.*

The only way for the sum of opposite faces on a standard dice to be the same is for the opposite faces to add to 7.

Therefore the number opposite any face with a 6 must be 1.

The sum of the numbers on a standard dice is $6+5+4+3+2+1=21$.

On two standard dice, the sum of all the numbers is 42.

To remove the numbers opposite any face with a 6, we calculate $42-(1+1)=$ **40**

3B. The question is:

The only 2-digit perfect squares are 16, 25, 36, 49, 64, and 81.

The 4-digit passcode represented by *ABCD* has the following properties: *AB*, *BC*, and *CD* are each perfect squares.

If *ABCD* is even what is the 4-digit passcode ABCD?

METHOD 1 *Strategy*: *Reason Logically.*

Consider *BC.*

BC must be a perfect square number.

B must also be the second digit in a perfect square number.

C must also be the lead digit in an even perfect square number.

16 is the only perfect square that can meet these conditions.

1 is the last digit of 81 and 6 is the first digit of 64, which is even.

BC can't be 64 because *CD* would need to be 49, and this would make the passcode odd.

The 4-digit passcode is **8164.**

A	*B*	*C*	*D*
	1	**6**	
8	**1**	**6**	4

A	*B*	*C*	*D*
	6	**4**	
1	**6**	**4**	~~9~~
3	**6**		

METHOD 2 *Strategy*: *Working Backwards using the Process of Elimination.*

ABCD is even.
This means ***CD*** could equal **16**, **36** or **64**.

If ***CD*** is **16**, ***BC*** must end with **1**. ***BC*** could be **81**.
If ***BC*** is **81**, ***AB*** must end with **8**.
This is not possible.

If ***CD*** is **36**, ***BC*** must end with **3**.
This is not possible.

If ***CD*** is **64**, ***BC*** must end with **6**. ***BC*** could be **36** or **16**.

If ***BC*** is **36**, ***AB*** must end with **3**.
This is not possible. A perfect square doesn't end in **3**.

If ***BC*** is **16**, ***AB*** must end with **1**.
AB must be **81**.

Therefore ***ABCD*** is **8164**.

3C. The question is:
Isabella wants to make a rectangular garden with an area of 108 square metres.
She wants the length and width of the garden to be whole numbers when expressed in metres.
Let G be the largest possible perimeter of the garden and let L be the smallest possible perimeter of the garden.
Find the value of G – L in metres.

METHOD 1 *Strategy*: *Identify Factors and Draw a Table.*

As the length and width of the garden are to be whole numbers of metres, and the area of the garden is 108 square metres, the length and the width must be factors of 108.

List the factors of 108:

1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54 and 108.

Draw a table, pairing factors that have a product of 108, and find the perimeter of each rectangle.

Length x	Width y	Perimeter $2x + 2y$
1	108	218
2	54	112
3	36	78
4	27	62
6	18	48
9	12	42

G is the largest possible perimeter of the garden. G = 218 metres.

L is the smallest possible perimeter of the garden. L = 42 metres.

The value of G − L is 218 − 42 = **176**

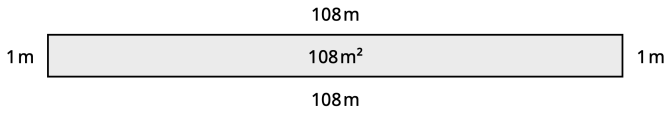
METHOD 2 *Strategy*: *Reason Logically.*

We know that the largest perimeter of a rectangle has the greatest difference in dimensions.

The factors 108 and 1 have the greatest difference.

The largest perimeter is $2\times(108+1) = 2\times109$

$= 218\,\text{m}$

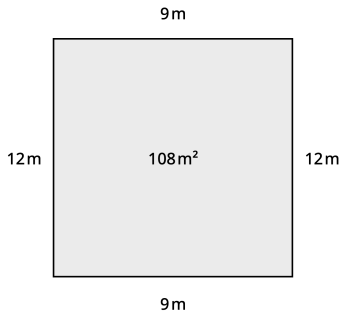


The smallest perimeter of a rectangle has the least difference in dimensions.

The factors 9 and 12 have the least difference.

The smallest perimeter is $2\times(9+12) = 2\times21$

$= 42\,\text{m}$



G is the largest possible perimeter of the garden. **G** = 218 metres.

L is the smallest possible perimeter of the garden. **L** = 42 metres.

G − L = **176** metres

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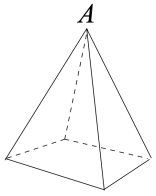
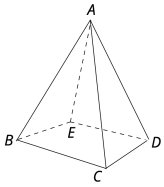
OLYMPIAD 3

3D. The question is:
A bug will walk along the edges of a pyramid, never retracing its steps. It starts at vertex A, traces edges and visits every vertex exactly once until it returns to vertex A.
How many different pathways can the bug follow?



METHOD 1 *Strategy*: *Create an Organised List.*

Start by naming each vertex.
From A, the bug can travel first to either B, C, D, or E before completing a full loop.
The bug cannot retrace its steps.



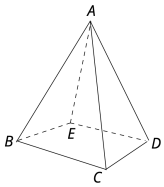
Create an organised list to find all possible pathways.

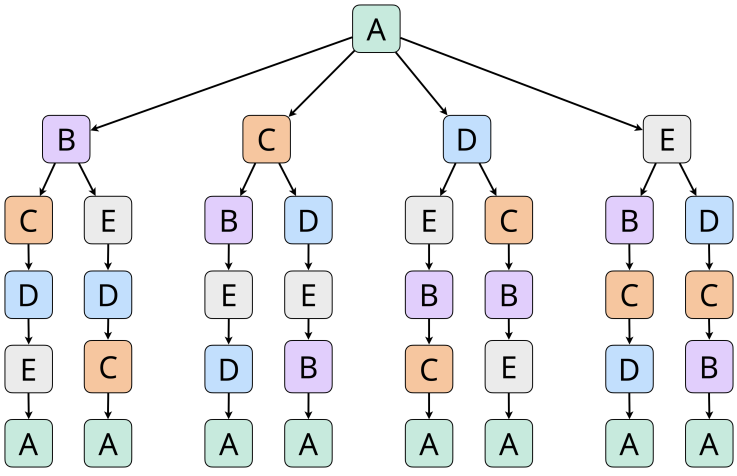
There are **8** pathways the bug can follow.

2 ways via **B**:	**A**	B	C	D	E	**A**
	A	B	E	D	C	**A**
2 ways via **C**:	**A**	C	B	E	D	**A**
	A	C	D	E	B	**A**
2 ways via **D**:	**A**	D	E	B	C	**A**
	A	D	C	B	E	**A**
2 ways via **E**:	**A**	E	B	C	D	**A**
	A	E	D	C	B	**A**

METHOD 2 *Strategy*: *Draw a Tree Diagram.*

Start by naming each vertex.
From A, the bug can travel first to either B, C, D, or E before completing a full loop.
The bug cannot retrace its steps.
Draw a tree diagram to find all possible pathways.





There are **8** pathways the bug can follow.

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OLYMPIAD 3

3E. The question is:

In the cryptarithm shown, each letter represents a unique digit.

No "crypto-word" can begin with the digit '0'.

What is the largest possible sum?

$$\begin{array}{cccc} A & B & C & D \\ B & C & D & A \\ C & D & A & B \\ + \; D & A & B & C \\ \hline \end{array}$$

METHOD 1 *Strategy*: *Use Number Properties and Reason Logically.*

A, *B*, *C* and *D* appear once in each column and once in each row of the cryptarithm.

Each time they appear in a different position.

The 4 largest single digits we can assign to *A*, *B*, *C* and *D* are 9, 8, 7 and 6.

Regardless of which letter they are assigned to, each digit will appear once in the thousands, once in the hundreds, once in the tens and once in the units.

Let's assign 9876 to *ABCD*.

The largest possible sum is **33,330.**

$$\begin{array}{cccc} {\scriptstyle 3} & {\scriptstyle 3} & {\scriptstyle 3} & \\ 9 & 8 & 7 & 6 \\ 8 & 7 & 6 & 9 \\ 7 & 6 & 9 & 8 \\ + \; 6 & 9 & 8 & 7 \\ \hline \mathbf{3} & \mathbf{3} & \mathbf{3} & \mathbf{3} \; \; \mathbf{0} \end{array}$$

METHOD 2 *Strategy*: *Reason Logically.*

The 4 largest single digits we can assign to *A*, *B*, *C* and *D* are 9, 8, 7 and 6.

The sum of these digits placed in the units column is $9+8+7+6=30$.

Their sum in the tens column is $90+80+70+60=300$.

Their sum in the hundreds column is $900+800+700+600=3000$.

Their sum in the thousands column is $9000+8000+7000+6000=30\,000$.

We can add these vertically

The largest possible sum is **33,330.**

$$\begin{array}{cccccc} & & & & 3 & 0 \\ & & & 3 & 0 & 0 \\ & & 3 & 0 & 0 & 0 \\ + & 3 & 0 & 0 & 0 & 0 \\ \hline & \mathbf{3} & \mathbf{3} & \mathbf{3} & \mathbf{3} & \mathbf{0} \end{array}$$

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OLYMPIAD 3

Questions and Answers
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3A. **Follow Up 1:**

Some dice have 8 triangular faces; they are called octahedral dice.
(An octahedron can be formed by gluing together the bases of two square pyramids.)
Each of the numbers 1 through 8, is used, one number per face, on an octahedral die.
The sum of opposite faces equals N. Find N. **[9]**

Follow Up 2:

What is the probability that N (from Follow Up 1) is the sum that appears on a single toss of 2 standard, octahedral dice? **[1/8]**

3B. **Follow Up 1:**

Suppose the 4-digit passcode is odd. What are the possible 4-digit passcodes? **[1649 and 3649]**

Follow Up 2:

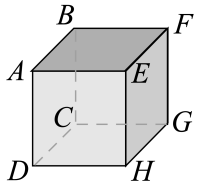
The 4-digit passcode represented by *ABCD* has the following properties: The first three digits are a 3-digit perfect square. The last three are also a 3-digit perfect square. The passcode is a palindrome. What is the passcode? **[1441]**

3C. **Follow Up 1:**

Using the given conditions in the question, how many gardens with whole number dimensions (in metres) are possible? Assume a garden with dimensions 9 x 12 metres is the same as a garden with dimensions 12 x 9 metres. **[6]**

3D. **Follow Up 1:**

A bug walks along the edges of a cube, never retracing its steps. It starts at vertex A and completes a full loop to visit every vertex exactly once until it returns to vertex A.
How many different pathways can the bug follow? **[12]**



3E. **Follow Up 1:** What is the smallest possible sum? **[11,110]**

Follow Up 2: How many different sums are possible? **[21]**

$$\begin{array}{r} A\,B\,C\,D \\ B\,C\,D\,A \\ C\,D\,A\,B \\ +\,D\,A\,B\,C \\ \hline \end{array}$$

3A. Follow Up 1:

Some dice have 8 triangular faces; they are called octahedral dice.

(An octahedron can be formed by gluing together the bases of two square pyramids.)

Each of the numbers 1 through 8, is used, one number per face, on an octahedral die.

The sum of opposite faces equals N.

Find N.

Follow Up 2:

What is the probability that N (from follow up 1) is the sum that appears on a single toss of 2 standard, octahedral dice?

3B. Follow Up 1:

The question was:

The only 2-digit perfect squares are 16, 25, 36, 49, 64, and 81.
The 4-digit passcode represented by *ABCD* has the following properties: *AB*, *BC*, and *CD* are each perfect squares.
If *ABCD* is even what is the 4-digit passcode *ABCD*?

Follow Up Question: What are the possible 4-digit **odd** passcodes?

Follow Up 2:

The 4-digit passcode represented by ABCD has the following properties:

The first three digits are a 3-digit perfect square number.

The last three digits are also a 3-digit perfect square number.

The passcode is a palindrome.

What is the passcode?

3C. Follow Up:

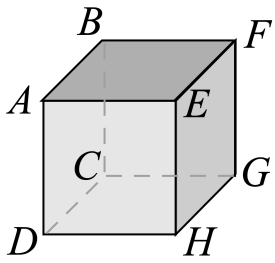
The question was:

Isabella wants to make a rectangular garden with an area of 108 square metres.
She wants the length and width of the garden to be whole numbers when expressed in metres.
Let G be the largest possible perimeter of the garden and let L be the smallest possible perimeter of the garden.
Find the value of G – L in metres.

Using the given conditions in the question, how many gardens with whole number dimensions in metres are possible?
Assume a garden with dimensions 9 x 12 metres is the same as a garden with dimensions 12 x 9 metres.

3D. Follow Up:

A bug walks along the edges of a cube, never retracing its steps.
It starts at vertex A and completes a full loop to visit every vertex exactly once until it returns to vertex A.
How many different pathways can the bug follow?



3E. Follow Up 1:

What is the smallest possible sum?

$$\begin{array}{r} A\,B\,C\,D \\ B\,C\,D\,A \\ C\,D\,A\,B \\ +\,D\,A\,B\,C \\ \hline \end{array}$$

Follow Up 2:

How many different sums are possible?

$$\begin{array}{r} A\,B\,C\,D \\ B\,C\,D\,A \\ C\,D\,A\,B \\ +\,D\,A\,B\,C \\ \hline \end{array}$$