





MPORTANT

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ORGANISATION AND **P**ROCEDURES

For full details, see the Members' Area

To ensure the integrity of the competition, the Olympiads must be administered under examination conditions.

DO

- Supervise students at all times
- Seat students apart
- Maintain silence
- Provide blank working paper
- Give time warnings when 3 minutes remain, and again when 1 minute remains
- Collect, mark and retain the papers

• Print the Olympiad papers prior to the Olympiad Date

DO NOT

- Read the questions aloud to the students
- Interpret the questions for students
- Permit any discussion or movement around the room
- Permit the use of calculators or other electronic devices
- Olympiad papers are scored by the PICO using the *Solutions and Answers* sheet provided.
- Results should be submitted in the Members' Area within 7 days of the Olympiad.
- Original student answer sheets should be retained by the PICO until the end of the year.
- *Solutions and Answers sheets* are not to be handed out to students. They are a teaching resource for use in class *after* completion of the Olympiad paper.

TIMING OF THE OLYMPIAD

- The *Total Time Allowed* for the Olympiad is **30 minutes**.
- The time for each individual question is a guide for the students.

ABSENT STUDENT POLICY

A student who is legitimately absent on the Olympiad date, may sit the Olympiad under examination conditions on their first day back at school (if that date is within 2 weeks of the original Olympiad date). If these conditions cannot be met, the student must be marked as absent on the submitted results.

The Absent Student Policy is available in the **Contest Administration** section of the Members' Area.





APSMO 2024: DIVISION J

WEDNESDAY 12 JUNE 2024

2

Total Time Allowed: 30 Minutes					
2A.	Aikan, Aiden, and Alden each have a different favourite single-digit number. The product of their numbers is 70. What is the sum of their numbers?	Write your answers in the boxes on the			
2B.	Mr Abacus has a maths club. 16 students come to the club. They are either 9 or 10 years old. There are as many 9 year olds as 10 year olds in the club. If the students add Mr Abacus' age to their combined ages, the sum is 199. How old is Mr Abacus?	Keep your answers hidden by folding backwards on			
2C.	Amelia and Cooper both drove 300 km from Sydney to Canberra. For the first 150 km, Amelia drove at an average speed of 60 km/hr. She drove the second 150 km at an average speed of 90 km/hr. Cooper drove the whole 300 km at an average speed of 75 km/hr. Cooper's trip took less time. How many minutes shorter was his trip than Amelia's?	this line.			
2D.	Figure 1 is a 5 unit x 8 unit rectangle made of cardboard. A 3 unit x 3 unit square from one of its corners was cut out and attached to the side of the card. Then, a 1 unit x 1 unit square was cut from another corner and attached to to side of the 3 x 3 square. What is the perimeter of the final shape seen in Figure 2?				
2E.	The positive whole numbers are written down in order. This forms a string of digits: 123456789101112131415				
	What is the 200th digit in that string?				

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2A.	Student Name:		
2B.	Fold H		
2C.	ere. Keep your answers		
2D.	hidden.		
2E.			



Solutions and Answers For teacher use only. Not for Distribution.							
	2A: 14	2B: 47	2C: 10	2D: 34	2E: 0		
2A.	The question is:						
	Aikan, Aiden, and	Alden each have a differei	nt favourite single-digit nu	mber. The product of the	ir numbers is 70.		
	What is the sum o	of their numbers?					
	METHOD 1 Strate	egy: Create a Factor Tree.					
	We know that the product of Aikan, Aiden and Alden's favourite numbers is 70.						
	We also know that their favourite numbers are single-digits.						
	We can create a factor tree of 70 to identify its single digit factors.						
	70 70 70 10 7 35 2 14 5 5 2 7 5 7 2						
	No matter what fa	actor tree we create, we fir	nd that the 3 single-digit n	umbers that multiply to gi	ive 70 are 2, 5, and 7.		
	The sum of Aikan	, Aiden and Alden's numbe	ers is:				
	2 + 5 + 7 = 14						

METHOD 2 Strategy: Use the Process of Elimination.

From the problem, we know that one single-digit and the product of the 2 other single-digits must multiply to 70.
We can use the process of elimination to identify the 3 single-digits, starting with the largest single digit number 9.
9 cannot be multiplied with another to reach 70.
8 is not a factor of 70.
7 is a factor of 70. It can be multiplied by 10.
10 has 2 factors that are single-digits, 5 and 2.
The sum of Aikan, Aiden and Alden's numbers is:
2 + 5 + 7 = 14



2B. The question is:

Mr Abacus has a maths club. 16 students come. They are either 9 or 10 years old. There are as many 9 year olds as 10 year olds in the club. If the students add Mr Abacus' age to their combined ages, the sum is 199. How old is Mr Abacus?

METHOD 1 Strategy: Use a Number Sentence and Reason Logically.

We are told that half of the 16 students are 9 years old, and the other half of the students are 10 years old.

Their ages added together:

8 × 9 + 8 × 10

= 72 + 80

= 152

When Mr Abacus' age is added to the total, the sum is 199.

We can subtract the combined students' age from the total sum to find Mr Abacus' age.

199 – 152 = **47**

Mr Abacus is **47 years old**.

METHOD 2 Strategy: Use a Bar Diagram.

We can use the information in the problem to create a bar diagram to determine the age of Mr Abacus.

We can show that the sum of the ages of the students and Mr Abacus is 199.



We can divide the 16 students into 2 equal groups.



The sum of the 9 year olds ages is 72 and the sum of the 10 year olds ages is 80.

The sum of all the students combined is 152.



Subtracting 152 from 199 will give Mr Abacus' age.







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2C. The question is:

Amelia and Cooper both drove 300 km from Sydney to Canberra. For the first 150 km, Amelia drove at an average speed of 60 km/hr. She drove the second 150 km at an average speed of 90 km/hr. Cooper drove the whole 300 km at an average speed of 75 km/h. Cooper's trip took less time. How many minutes shorter was his trip than Amelia's?

METHOD 1 Strategy: Use a Number Sentence and Reason Logically.

We can calculate the time taken by dividing the distance by the speed.

Amelia's trip:

First, Amelia drove at 60 km/h for 150 km.

We can calculate how far she drove each minute by dividing the speed by time:

 $60 \div 60 = 1$ km per minute.

150 km ÷ 1 = 150.

It will take Amelia 150 minutes for her to drive 150 km.

Next, Amelia drove at 90 km/h for 150 km.

We can calculate how far she drove each minute by dividing the speed by the time:

90 ÷ 60 = 1.5 km per minute.

150 km ÷ 1.5 = 100.

Therefore it will take Amelia **100 minutes** to drive the last 150 km.

Altogether, Amelia's trip took **150 + 100 = 250 minutes.**

METHOD 2 Strategy: Draw a Table.

We can draw a table to record Alice and Cooper's speed, distance travelled and time taken.

Cooper's trip:

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Cooper drove at 75 km/h for 300 km.

We can calculate how far he drove each minutes by dividing the speed by time:

75 ÷ 60 = 1.25 km per minute.

300 km ÷ 1.25 = 240.

Therefore Cooper's trip took 240 minutes.

Cooper's trip was **10 minutes shorter** than Amelia's trip.

Alice's Trip

Speed	60 km/h	60 km/h	60 km/h	90 km/h	90 km/h	90 km/h
Distance	60 km	120 km	150 km	195 km	285 km	300 km
Time	60 mins	120 mins	150 mins	180 mins	240 mins	250 min

Cooper's Trip

Speed	75 km/h	75 km/h	75 km/h	75 km/h
Distance	75 km	150 km	225 km	300 km
Time	60 mins	120 mins	180 mins	240 mins

Amelia's trip took 250 minutes. Cooper's trip took 240 minutes.

Cooper's trip was **10 minutes shorter** than Amelia's trip.



2D. The question is:

Figure 1 is a 5 unit x 8 unit rectangle made of cardboard. A 3 unit x 3 unit square from one of its corners was cut out from one of its corners and attached to the side of the card. Then, a 1 unit x 1 unit square was cut from another corner and attached to to side of the 3 x 3 square. What is the perimeter of the final shape seen in Figure 2?



METHOD 1 Strategy: Determine the lengths of the 16 sides of the region.

Using the measurements provided in the problem, we can use the diagram to label each side with its length. Then we can add the length of all 16 sides to find the total perimeter.



METHOD 2 *Strategy:* Form an outer rectangle with an equivalent perimeter.

Using the measurements provided in the problem, we can determine the length of the horizontal components of the perimeter. By moving all of the horizontal components of the perimeter out towards the edge of the original rectangle, we can calculate the horizontal length as **12 units**.





We can likewise determine the length of the vertical components of the perimeter.

By moving all of the vertical components of the perimeter out towards the edge of the original rectangle, we can calculate the vertical length as **5 units**.









OLYMPIAD

1E. The question is:

The positive whole numbers are written down in order.

This forms a string of digits:

123456789101112131415...

What is the 200th digit in that string?

METHOD 1 Strategy: Draw a table and count the number of digits in an organised way.

We can start by drawing a table to record the number of digits that single-digit counting numbers, 2 digit counting numbers, and 3 digit counting numbers require.

Counting numbers	1 → 9	10 → 99	100 → 999
Number of digits	9	180	900
Cumulative number of digits	9	189	2700
		20	1 D0th

The numbers 1 through to 9 require 9 digits.

The numbers 10 through to 99 require 180 digits.

Therefore the numbers 1 through to 99 require 189 digits.

We need another 11 digits to find the 200th digit in the string of digits.

We continue our string starting with 100, the first 3 digit counting number.

....10010110210....

The 200th digit is 0.





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Questions and Answers For teacher use only. Not for distribution.

2A. Follow Up 1:

Amelia and Blake each have a different favourite single-digit prime number. Chloe has a favourite 2-digit prime number. If the product of these 3 prime numbers is less than 100, what is their greatest possible sum? **[18]**

Follow Up 2:

How many different sets of 3 distinct, positive integers have a sum of 14? [10]

2B. Follow Up 1:

A school choir is made up of an equal number of 10, 11 and 12 year old students. Their director is 35 years old. The sum of the ages of the choir members and the director is 530. How many 10 year olds are in the choir? **[15]**

Follow Up 2:

In the Marble Club, the average number of marbles the members have is 100. When Person X joins the club with his 80 marbles, the average number of marbles per member becomes 96. How many people, including Person X, are in the Marble Club? **[5]**

2C. Follow Up 1:

Boris and Natasha start at the same place and at the same time on a 200 metre circular track and run in opposite directions. Boris runs at 5 metres per second and Natasha runs at 3 metres per second. In how many seconds will they first meet after starting? **[25]**

Follow Up 2:

Chloe regularly runs 200 metres at an average rate of 5 m/s. James wants to run 15% further than Chloe. If he runs his distance in the same amount of time Chloe runs hers, what must his average rate be? **[5.75 m/s]**

2D. Follow Up 1:

A square contains two squares, A and B, with areas of 16 square metres and 9 square metres, as shown. Find the number of metres in the perimeter of the shaded region. **[22 m]**



Find the area of the shaded region. [110 sq cm]





2E. The positive whole numbers are written down in order.

This forms a string of digits:

123456789101112131415...

Follow Up 1: What is the sum of the first 200 digits in the string?

Follow Up 2: What is the sum of the digits for all the whole numbers less than 100. [900]

[907]



2A. Follow Up 1:

Amelia and Blake each have a different favourite single-digit prime number. Chloe has a favourite 2-digit prime number.

If the product of these 3 prime numbers is less than 100, what is their greatest possible sum?

Follow Up 2:

How many different sets of 3 distinct, positive integers have a sum of 14?



2B. Follow Up 1:

A school choir is made up of an equal number of 10, 11 and 12 year old students.

Their director is 35 years old.

The sum of the ages of the choir members and the director is 530.

How many 10 year olds are in the choir?

Follow Up 2:

In the Marble Club, the average number of marbles the members have is 100.

When Person X joins the club with his 80 marbles, the average number of marbles per member becomes 96. How many people, including Person X, are in the Marble Club?



2C. Follow Up 1:

Boris and Natasha start at the same place and at the same time on a 200 metre circular track and run in opposite directions.

Boris runs at 5 metres per second and Natasha runs at 3 metres per second.

In how many seconds will they first meet after starting?

Follow Up 2:

Chloe regularly runs 200 metres at an average rate of 5 m/s.

James wants to run 15% further than Chloe.

If he runs his distance in the same amount of time Chloe runs hers, what must his average rate be?



2D. Follow Up 1:

A square contains two squares, A and B, with areas of 16 square metres and 9 square metres, as shown.

Find the number of metres in the perimeter of the shaded region.



Follow Up 2:

Find the area of the shaded region.





2E. Follow Up 1:

The positive whole numbers are written down in order.

This forms a string of digits:

123456789101112131415...

What is the sum of the first 200 digits in the string?

Follow Up 2:

What is the sum of the digits for all the whole numbers less than 100?