APSMO

2024 OLYMPIADS

Important

APSMO

2024 OLYMPIADS

Organisation and Procedures

For full details, see the Members' Area

To ensure the integrity of the competition, the Olympiads must be administered under examination conditions.

DO	DO NOT
• Supervise students at all times • Seat students apart • Maintain silence • Provide blank working paper • Give time warnings when 3 minutes remain, and again when 1 minute remains • Collect, mark and retain the papers	• Print the Olympiad papers prior to the Olympiad Date • Read the questions aloud to the students • Interpret the questions for students • Permit any discussion or movement around the room • Permit the use of calculators or other electronic devices

- Olympiad papers are scored by the PICO using the *Solutions and Answers* sheet provided.
- Results should be submitted in the Members' Area within 7 days of the Olympiad.
- Original student answer sheets should be retained by the PICO until the end of the year.
- *Solutions and Answers sheets* are not to be handed out to students. They are a teaching resource for use in class ***after*** completion of the Olympiad paper.

Timing of the Olympiad

- The *Total Time Allowed* for the Olympiad is **30 minutes**.
- The time for each individual question is a guide for the students.

Absent Student Policy

A student who is legitimately absent on the Olympiad date, may sit the Olympiad under examination conditions on their first day back at school (if that date is within 2 weeks of the original Olympiad date). If these conditions cannot be met, the student must be marked as absent on the submitted results.

The Absent Student Policy is available in the **Contest Administration** section of the Members' Area.

APSMO

MATHS OLYMPIAD

2024: DIVISION J
WEDNESDAY 8 MAY 2024

OLYMPIAD

1

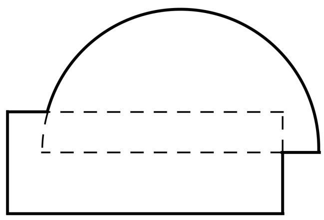
Total Time Allowed: ***30 Minutes***

Write your answers in the boxes on the back.

Keep your answers hidden by folding backwards on this line.

1A. There are 8 numbers.
The average of the first 3 numbers is 6.
The average of the last 5 numbers is 14.
What is the average of all 8 numbers?

1B. Mia designed a logo by overlapping a semicircle and a rectangle.
The area of the semicircle is 47 cm².
The area of the rectangle is 28 cm².
The area of the overlap is 9 cm².
What is the number of square centimetres in the area of the entire logo?



1C. Twenty identical cubes are stacked into piles.
Each pile has a different height.
If the tallest pile is as short as it can be, how many cubes are in that pile?

1D. In a cryptarithm, each letter represents the same digit every time it appears.

Different letters represent different digits.

What is the greatest possible value for the word LEARN in the given cryptarithm?

$$\begin{array}{r} I\ D\ E\ A \\ +\ I\ D\ E\ A \\ \hline L\ E\ A\ R\ N \end{array}$$

1E. The letters of the word KAYAK can be rearranged to form different five-letter arrangements, such as AYKKA.

How many different five-letter arrangements can be formed where no letter in the arrangement is in its original position?

APSMO

MATHS OLYMPIAD

APSMO

2024: DIVISION J
WEDNESDAY 8 MAY 2024

OLYMPIAD

1

Student Name:

1A.

1B.

1C.

1D.

1E.

Fold Here. Keep your answers hidden.

APSMO

2024: DIVISION J
WEDNESDAY 8 MAY 2024

OLYMPIAD 1

Solutions and Answers

For teacher use only. Not for Distribution.

1A: 11	**1B:** 66	**1C:** 6	**1D:** 18 570	**1E:** 4

1A. The question is:

There are 8 numbers. The average of the first 3 numbers is 6. The average of the last 5 numbers is 14.

What is the average of all 8 numbers?

METHOD 1 *Strategy*: *Use a number sentence to find the average.*

The sum of the first three numbers is 3 times their average. 3 x 6 = 18

The sum of the last 5 numbers is 5 times their average. 5 x 14 = 70

To calculate the average we divide the sum of all 8 numbers by 8.

(18 + 70) ÷ 8

= 88 ÷ 8 = 11

The average of all 8 numbers is **11**.

METHOD 2 *Strategy*: *Use an area model.*

We can represent the sum of the first three and the last 5 numbers with an area model. The model is not to scale.

The area of rectangle A represents the sum of the first three numbers. Its dimensions are 3 by 6.

The area of rectangle B represents the sum of the final 5 numbers. Its dimensions are 5 by 14.

Split B into C and D so that D has the same height as A. The area of rectangle A is 3 x 6 = 18 units^2.

The area of rectangle C is 5 x 8 = 40 units^2. The area of rectangle D is 5 x 6 = 30 units^2.

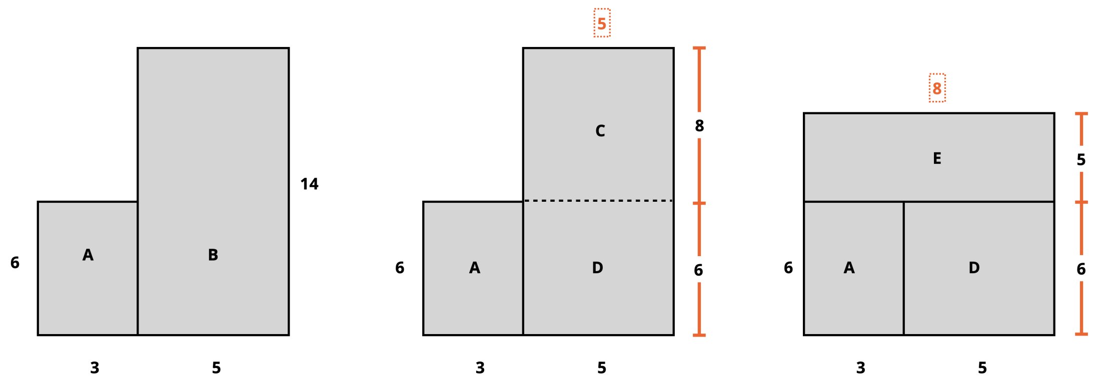
We can preserve the overall area of A, C and D by replacing C with a rectangle that is the same area.

We can create rectangle E with the same width as rectangles A and D combined (3 + 5) and height 5 and this has the same area as rectangle C.

To find the average of all 8 numbers in the problem, we can combine the area of rectangles A, D and E and divide the total by 8.

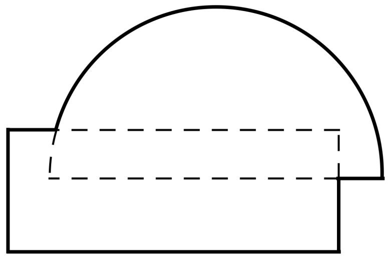
[(3 x 6) + (5 x 6) + (8 x 5)] ÷ 8

The average of all 8 numbers is 88 ÷ 8 = **11.**



1B. The question is:

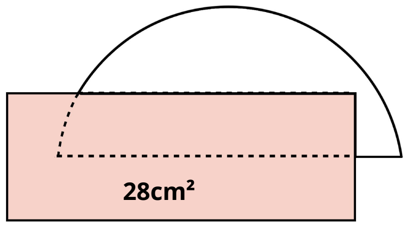
Mia designed a logo by overlapping a semicircle and a rectangle.
The area of the semicircle is 47 cm².
The area of the rectangle is 28 cm².
The area of the overlap is 9 cm².
What is the number of square centimetres in the area of the entire logo?



METHOD 1 ***Strategy***: *Separate the shapes into non-overlapping regions.*

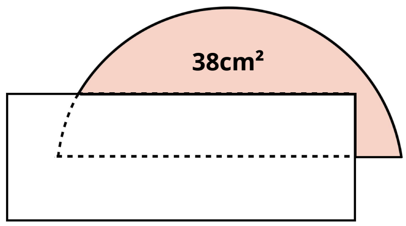
We can simplify the problem by breaking the logo into two non-overlapping shapes, the rectangle and the semicircle with the overlapped region removed.

The area of the rectangle is 28cm².



To find the area of the semicircle with the overlap removed, we then subtract the area of the overlap (9cm²) from the area of the semicircle (47cm²) .

47cm² − 9cm² = 38cm²



The sum of the rectangle and the semicircle with the overlap removed gives the area of the entire logo.

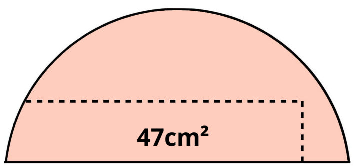
28cm²+ 38cm² = 66cm²

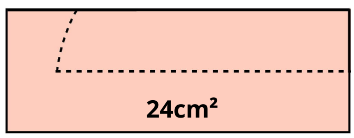
The number of square centimetres in the entire logo is **66.**

METHOD 2 ***Strategy***: *Combine the area of the shapes and subtract the overlap.*

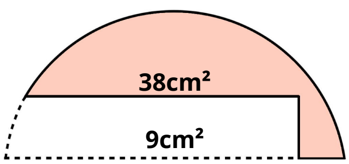
The sum of the areas of the rectangle and the semicircle if they didn't overlap is 47cm² + 28cm² = 75cm².

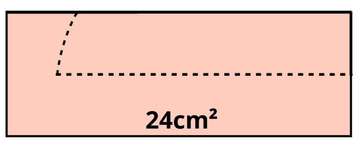
This sum counts the overlapped area twice, once within the rectangle and again within the semicircle.





We can subtract the overlapped area of 9cm² from the 75cm² to find the area of the entire logo.





75cm² - 9cm² = 66cm².

The number of square centimetres in the entire logo is **66.**

1C. The question is:

Twenty identical cubes are stacked into piles.

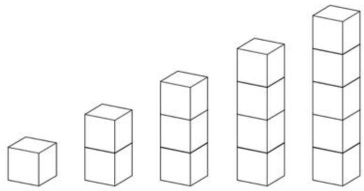
Each pile has a different height.

If the tallest pile is as short as it can be, how many cubes are in that pile?

METHOD 1 *Strategy*: *Start low and increase the height.*

We know that we have 20 cubes to distribute into piles, and that each pile of cubes must be a different height.

As we want our **tallest** pile to be as **short** as it can be, let's start with the shortest pile possible and increase the size of each subsequent pile by a single cube.



1 + 2 + 3 + 4 + 5 = 15

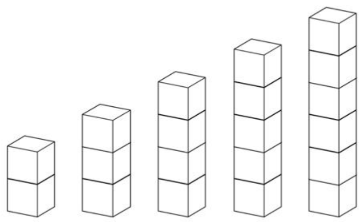
1 + 2 + 3 + 4 + 5 = 15

The total number of cubes used so far in these piles is 15, and we must include 20 cubes in our solution.

If we simply add another pile that is 6 cubes high, the total number cubes included will be 21.

We cannot make another pile that is 5 cubes high, as each pile must be a different height.

Therefore, if we add one more pile of 6 cubes and take away the lowest pile containing only one cube, we will have met the criteria of the problem.



2 + 3 + 4 + 5 + 6 = 20

2 + 3 + 4 + 5 + 6 = 20

There are **6** cubes in the tallest pile.

METHOD 2 *Strategy*: *Use a table and guess, check and refine.*

We know that we have 20 cubes to distribute into piles, and that each pile of cubes must be a different height.

We can build a table to record possible results and search for the solution that has piles of different heights that include 20 cubes.

Let's start with 8 as the height of our tallest pile.

If our tallest pile has 8 cubes, we have 12 cubes left to build a pile of 7 and a pile of 5 cubes.

If our tallest pile has 7 cubes, we have 13 cubes left to build piles of 6, 5 and 2 cubes.

Tallest Pile	**8**	**7**	**6**	**5**	**4**
	7	6	5	4	3
	5	5	4	3	2
		2	3	2	1
			2	1	
Number of Cubes	**20**	**20**	**20**	**15**	**10**

If our tallest pile has 6 cubes, we have 13 cubes left build piles of 5, 4, 3 and 2 cubes.

If our tallest pile has 5 or fewer cubes, we are not able to reach 20 cubes, which the problem requires.

The solution with 20 cubes that has the taller pile as short as it can be is 6.

There are **6** cubes in the tallest pile.

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MATHS OLYMPIAD

2024: DIVISION J
WEDNESDAY 8 MAY 2024

OLYMPIAD 1

1D. The question is:

In a cryptarithm, each letter represents the same digit every time it appears.
Different letters represent different digits.
What is the greatest possible value for the word LEARN in the given cryptarithm?

		I	D	E	A
+		I	D	E	A
	L	E	A	R	N

METHOD 1 *Strategy*: *Use number properties to reduce search.*

Step 1 We know that each letter in a cryptarithm represents the same digit every time, and that different letters represent different digits. Note that E and A appear in both words.

		I	D	E	A
+		I	D	E	A
	L	E	A	R	N

Step 2 The only possible digit that **L** can be is **one**, as doubling even the greatest 4 digit number 9999 results in 19 998. Is it possible for **E** to be our largest digit, **9**? No. For this to be possible, the letter **I** would have to be 9, doubling to make 18 and using a carry forward from the hundreds to make 19. If we were to do this, then **9** would be assigned to both the letter **I** and **E**.

		1			
		9	D	9	A
+		9	D	9	A
	1	9	A	R	N

Step 3 We can continue our search with letter **I** as **9**, assigning **8** to **E**, as long as there is no carry from the hundreds column to the thousands. There are 3 places where there is an E, and so let's assign 8 to them and see if we can continue to work towards a solution.

Assigning 8 to E must result in a carry forward from the tens column into the hundreds. This is marked on our working.

			1		
		9	D	8	A
+		9	D	8	A
	1	8	A	R	N

Step 4 Looking at the hundreds column, we see that D + D = A. As we have determined that there cannot be a carry forward (see Step 3) D cannot have a value higher than 4.

Can we assign **4** to **D**? No, as that would result in **A** being **8**, and we have already used **8** for **E**.

			1	1	
		9	4	8	9
+		9	4	8	9
	1	8	9	7	4

Can we assign **3** to **D**? If we do, **A** becomes **7**. Looking at the units column, we see that this would result in **N** being **4**, with a carry to the tens column, resulting in **R** also having the value of **7**. This is not possible as we know each letter represents a different digit.

			1	1	
		9	3	8	7
+		9	3	8	7
	1	8	7	7	4

Can we assign **2** to **D**? If we do, then **A** must be **5**. This is recorded where A appears in both words. If A is 5, then **N** must be **zero**, and **R** becomes **7**. Neither of these digits have been assigned previously.

By finding the greatest possible value for **A,** we have found that the greatest possible value for the word LEARN is **18 570.**

			1		
		9	2	8	5
+		9	2	8	5
	1	8	5	7	0

1E. The question is:

The letters of the word KAYAK can be rearranged to form different five-letter arrangements, such as AYKKA.

How many different five-letter arrangements can be formed where no letter in the arrangement is in its original position?

METHOD 1 *Strategy*: *Create an organised list.*

The original word starts with the letter K.

Therefore any new letter arrangements must start with either **A** or **Y**.

Arrangements that start with A cannot have the letter A as the second or fourth letter as no letter can be in the same spot as where it started.

This also means that the letter Y can only be second, fourth or fifth and the letter K can only be the second, third or fourth letter.

	K	A	Y	A	K
1	A	K	A	K	Y
2	A	K	K	Y	A
3	A	Y	K	K	A

With these constraints in place, we can create an organised list to find how many arrangements are possible starting with the letter A.

There are 3 possible arrangements that start with **A**.

Y is the only other letter that can begin a new arrangement.

As there is only one letter Y, if it is the first letter, the only possible way to arrange K and A so they are not in the same spot as where they started is to swap their positions.

	K	A	Y	A	K
1	A	K	A	K	Y
2	A	K	K	Y	A
3	A	Y	K	K	A
4	Y	K	A	K	A

There are **4 different letter arrangements** that can be formed.

METHOD 2 *Strategy*: *Draw a diagram.*

The original word starts with the letter K.

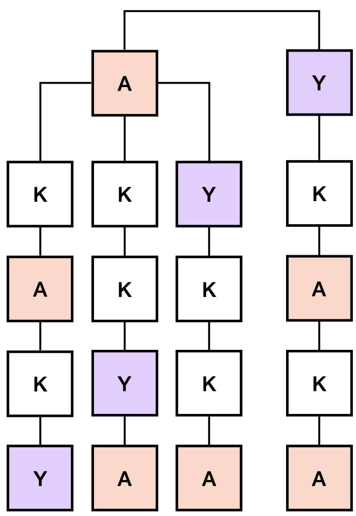
Therefore any new letter arrangements must start with either **A** or **Y**.

Draw a diagram showing all possible arrangements that begin with the letter A or Y.

No arrangement can have a letter in the same spot as it is in the original arrangement, KAYAK.

There are 3 possible arrangements starting with A if no letter is in the same spot as it started, and only one arrangement beginning with the letter Y.

Altogether, there are **4 different letter arrangements** that can be formed.



APSMO

MATHS OLYMPIAD

2024: DIVISION J
FOLLOW UP QUESTIONS

OLYMPIAD 1

Questions and Answers
For teacher use only. Not for distribution.

1A. **Follow Up 1:**

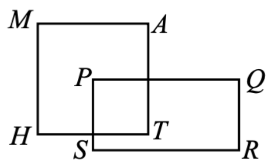
The average of 10 numbers is 54. One of the numbers was incorrectly recorded. When the correction was made, the average increased to 56. By how much was the incorrect value changed? **20**

Follow Up 2:

How many x and y pairs of whole numbers satisfy $5x + 3y = 100$? **7**

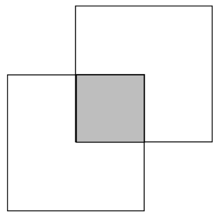
1B. **Follow Up 1:**

Find the area of the intersection of square *MATH* and the rectangle *PQRS*, if vertex P is the "centre" of square *MATH*, and *MA* = 12, *PQ* = 16, *QR* = 7. **36**



Follow Up 2:

The figure shows two congruent squares that overlap to form a non- convex octagon.
Each square has a vertex at the other's centre, and the sides of one square are parallel to to other's sides.
What fraction of the octagon lies in both squares (shaded region)? **1/7**



1C. **Follow Up 1:**

Twenty identical cubes are stacked into at least 2 piles each of which has a different height.
If the tallest pile is as tall as it can be, how many cubes are in that pile? **19**

Follow Up 2:

A 3-D solid is created from 20 identical cubes. What is the least possible surface area of the solid? **48 units²**

1D. **Follow Up 1 & 2:**

In the cryptarithm shown, each letter represents a unique digit, and no "crypto-word" can begin with the digit '0'.
1) Determine the greatest value that the sum IDEAL can have given that E = 1. **92105**
2) What is the greatest value of the sum IDEAL if E = 8? **76841**

$$\begin{array}{r} \mathrm{TEACH} \\ +\,\mathrm{LEARN} \\ \hline \mathrm{IDEAL} \end{array}$$

1E. **Follow Up 1:**
How many ways can the letters in the word PEPPER be arranged so that no two P's are next to each other and the two E's are not next to each other. **10**

Follow Up 2:
The letters of the word "CANCAN" can be rearranged to form different six-letter arrangements such as "ACANNC". How many different six-letter arrangements can be formed such that no letter in the arrangement is in its original position? **10**

1A. Follow Up 1:

The average of 10 numbers is 54. One of the numbers was incorrectly recorded.

When the correction was made, the average increased to 56.

By how much was the incorrect value changed?

Follow Up 2:

How many x and y pairs of whole numbers satisfy $5x + 3y = 100$?

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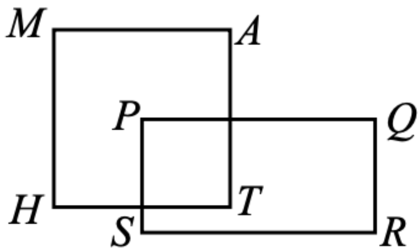

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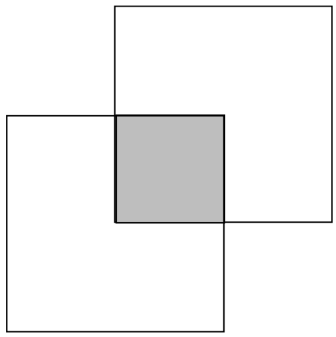

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Determine the greatest value that the sum IDEAL can have given that E = 1.

$$\begin{array}{r} \mathrm{T\,E\,A\,C\,H} \\ +\,\mathrm{L\,E\,A\,R\,N} \\ \hline \mathrm{I\,D\,E\,A\,L} \end{array}$$

Follow Up 2:

What is the greatest value of the sum IDEAL if E = 8?

$$\begin{array}{r} \mathrm{T\,E\,A\,C\,H} \\ +\,\mathrm{L\,E\,A\,R\,N} \\ \hline \mathrm{I\,D\,E\,A\,L} \end{array}$$

1E. Follow Up 1:

How many ways can the letters in the word PEPPER be arranged so that no two P's are next to each other and the two E's are not next to each other.

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How many different six-letter arrangements can be formed such that no letter in the arrangement is in its original position?