2024 Maths Games Senior - Years 7 & 8
Resource Kit 4
Teaching Problem Solving

MATHS GAMES

Problem Solving Strategies

This resource kit focuses on the following problem solving strategies:

1. Divide a Complex Shape

Sometimes you can divide an unusual shape into two or more common shapes that are easier to work with.

2. Convert to a More Convenient Form

There are times when changing some of the conditions of a problem makes a solution clearer or more convenient.

It follows on from strategies introduced in the Preparation Resource Kit and Resource Kits 1, 2 and 3:

Guess, Check and Refine
Draw a Diagram

Find a Pattern
Build a Table

Work Backwards
Make an Organised List

Solve a Simpler Related Problem
Eliminate All But One Possibility

Resource Kit 4 focuses on:

Divide a Complex Shape

Convert to a More Convenient Form

Set Yellow

Example problems for which full worked solutions are included.

Set Green

Problems that are designed to be similar to Set Yellow, but with fewer difficult elements.

Set Orange

Problems that are similar in mathematical structure to the corresponding Yellow problems.

Further questions and solution methods can be found in the APSMO resource book "Building Confidence in Maths Problem Solving", available from www.apsmo.edu.au.

How to use these problems

At the start of the lesson, present the problem and ask the students to think about it. Encourage students to try to solve it in any way they like. When the students have had enough time to consider their solutions, ask them to describe or present their methods, taking particular note of different ways of arriving at the same solution.

Each question includes at least one solution method that the majority of students should be able to follow. By participating in lessons that demonstrate achievable problem solving techniques, students may gain increased confidence in their own ability to address unfamiliar problems.

Finally, the consideration of different solution methods is fundamental to the students' development as effective and sophisticated problem solvers. Even when students have solved a problem to their own satisfaction, it is important to expose them to other methods and encourage them to judge whether or not the other methods are more efficient.

Preparation Kit

Guess, Check and Refine

This involves making a reasonable guess of the answer, and checking it against the conditions of the problem. An incorrect guess may provide more information that may lead to the answer.

Draw a Diagram

A diagram may reveal information that may not be obvious just by reading the problem.

It is also useful for keeping track of where the student is up to in a multi-step problem.

Resource Kit 1

Find a Pattern

A frequently used problem solving strategy is that of recognising and extending a pattern.

Students can often simplify a difficult problem by identifying a pattern in the problem.

Build a Table

A table displays information so that it is easily located and understood.

A table is an excellent way to record data so the student doesn't have to repeat their efforts.

Resource Kit 2

Work Backwards

If a problem describes a procedure and then specifies the final result, this method usually makes the problem much easier to solve.

Make an Organised List

Listing every possibility in an organised way is an important tool.

How students organise the data often reveals additional information.

Resource Kit 3

Solve a Simpler Related Problem

Many hard problems are actually simpler problems that have been extended to larger numbers.

Patterns can sometimes be identified by trying the problem with smaller numbers.

Eliminate All But One Possibility

Deciding what a quantity is not, can narrow the field to a very small number of possibilities.

These can then be tested against the conditions of the original problem.

Resource Kit 4

Convert to a More Convenient Form

There are times when changing some of the conditions of a problem makes a solution clearer or more convenient.

Divide a Complex Shape

Sometimes it is possible to divide an unusual shape into two or more common shapes that are easier to work with.

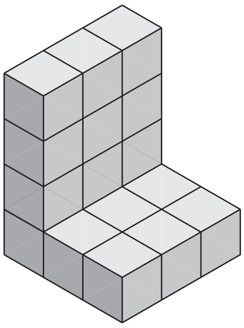
Set Yellow

4.1) A Super-Brick is formed by arranging nine 1 cm × 1 cm × 1 cm cubes into a 3 cm × 3 cm layer, 1 cm thick.

Two Super-Bricks are glued together into the L-shape shown.

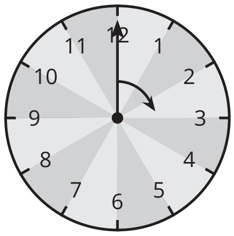
All faces of the L-shape are completely painted.

What is the area of the painted surface, in square centimetres?



4.2) An analog clock is showing the time 6:50.

What is the size of the smaller angle, in degrees, between the hour and the minute hands?



4.3) Zoe has 40 cards in a box: 10 blue, 10 red, 10 green, and 10 orange.

She reaches into the box without looking.

What is the smallest number of cards Zoe needs to pick up, to be certain she has at least three cards of the same colour?

4.4) A red beetle takes 30 seconds to walk across a table.

A green beetle takes 20 seconds to walk across the same table.

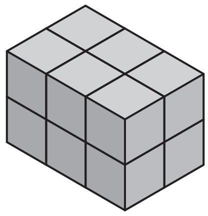
A red beetle and a green beetle both start at the same time from opposite ends of the table, and walk towards each other.

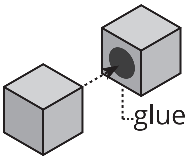
After how many seconds will they meet?

Set Yellow

4.5) Chloe is using 12 small cubes to make this rectangular prism, as shown at the right.

The prism is 2 cubes high, 2 cubes wide and 3 cubes long.





Wherever the faces of two cubes meet, she adds one dot of glue between them to stick them together, as shown in the diagram on the left.

How many dots of glue does she need to construct the rectangular prism?

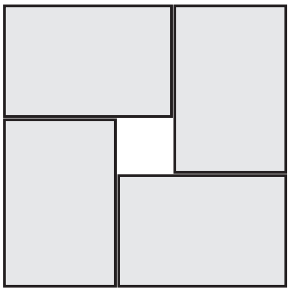
4.6) Four identical rectangular trays are arranged to form a square as shown.

One side-length of the large square is 50 cm.

The hole in the middle is also a square.

One side-length of the square hole is 10 cm.

What is the area of a single tray, in square centimetres?



4.7) A chime clock strikes

- 1 chime at one o’clock,
- 2 chimes at two o’clock,
- 3 chimes at three o’clock,

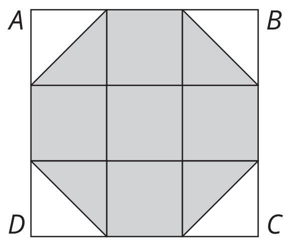
and so forth.

What is the total number of chimes the clock will strike in a twelve-hour period?

4.8) Square *ABCD* is composed of nine identical squares as shown.

The area of the shaded region is 14 square centimetres.

What is the area of square *ABCD*, in square centimetres?

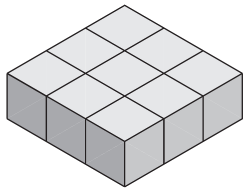


Set Green

4.1) Nine 1 cm × 1 cm × 1 cm cubes are glued together into a 3 cm × 3 cm layer, as shown.

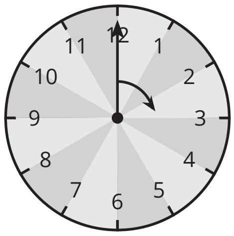
All faces of the object are completely painted.

What is the area of the painted surface, in square centimetres?



4.2) An analog clock is showing the time 3:30.

What is the size of the smaller angle, in degrees, between the hour and the minute hands?



4.3) Fabian has 12 marbles in a jar: 4 blue, 4 red and 4 green.

He reaches into the jar without looking.

What is the smallest number of marbles Fabian needs to pick up, to be certain he has at least two of the same colour?

4.4) A blue beetle takes 3 seconds to walk across a table.

A yellow beetle takes 6 seconds to walk across the same table.

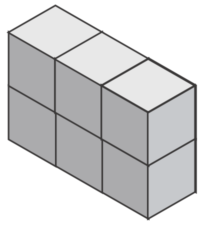
A blue beetle and a yellow beetle both start at the same time from opposite ends of the table, and walk towards each other.

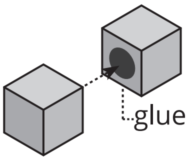
After how many seconds will they meet?

Set Green

4.5) Chloe is using 6 small cubes to make this rectangular prism, as shown at the right.

The prism is 2 cubes high, 1 cube wide and 3 cubes long.





Wherever the faces of two cubes meet, she adds one dot of glue between them to stick them together, as shown in the diagram on the left.

How many dots of glue does she need to construct the rectangular prism?

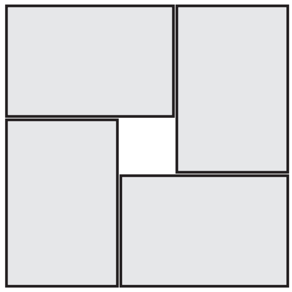
4.6) Four identical rectangular trays are arranged to form a square as shown.

One side-length of the large square is 10 cm.

The hole in the middle is also a square.

One side-length of the square hole is 2 cm.

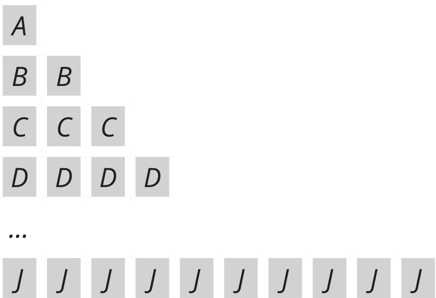
What is the area of a single tray, in square centimetres?



4.7) I have a set of letter cards.

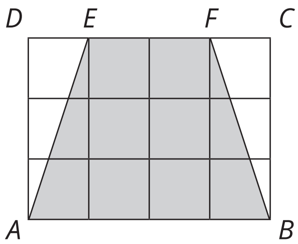
There is one *A*, two *B*s, three *C*s, four *D*s, and so on, up to ten *J*s.

How many letter cards are there in the set?



4.8) *ABCD* is a rectangle whose area is 12 square centimetres.

How many square centimetres are contained in the area of trapezium *EFBA*?

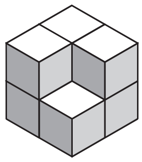


Set Orange

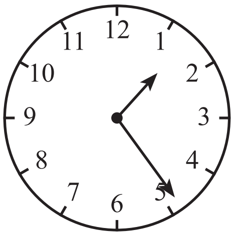
4.1) Jason has a 2 cm × 2 cm × 2 cm wooden block.

He removes a 1 cm × 1 cm × 1 cm wooden cube from one corner, as shown in the diagram.

What is the surface area, in square centimetres, of the resulting object that has a volume of 7 cubic centimetres?



4.2) At 1:24pm (as shown), what is the size of the smaller angle formed by the hands of a clock, in degrees?



4.3) A bag contains 18 lollies.

4 are red, 6 are white, and 8 are blue.

Amanda takes them out one at a time without looking.

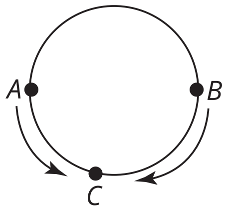
What is the smallest number of lollies she must take out to be certain that at least 2 of the lollies she takes out are blue?

4.4) Andy and Brett begin at opposite points on a circular track 200 metres in length, and run towards one another in opposite directions about the track as shown.

Andy runs at 2 metres per second, and Brett runs at 3 metres per second.

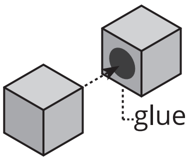
They first pass one another at point C after running for 20 seconds.

How many times did they pass one another during the first 8 minutes?

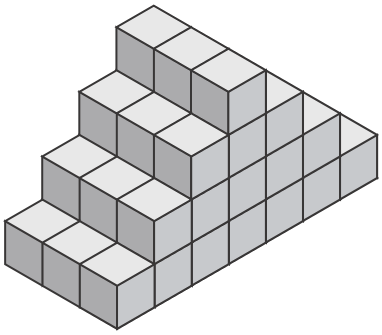


Set Orange

4.5) Chloe is using 48 small cubes to make this object, as shown at the right.



Wherever the faces of two cubes meet, she adds one dot of glue between them to stick them together, as shown in the diagram on the left.



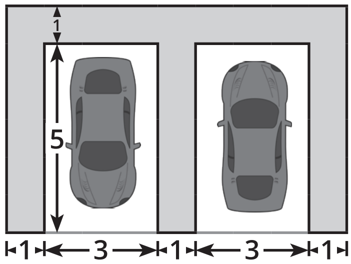
How many dots of glue does she need to construct this object?

4.6) A store has two parking spaces.

Each parking space is 3 metres wide and 5 metres long.

There is a 1 metre wide footpath around the sides and the back of the spaces. There is also a shared 1 metre wide section of footpath between the spaces, as shown in the diagram.

What is the area of the entire footpath, in square metres?



4.7) Each row of *s has two more *s than the row immediately above it, as shown.

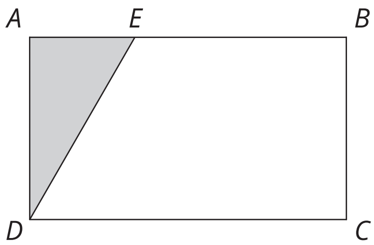
Altogether, how many *s are contained in the first ten rows?

*
* * *
* * * * *
* * * * * * *
and so on.

4.8) *ABCD* is a rectangle whose area is 360 square centimetres.

Point *E* is one-third of the way from *A* to *B*.

What is the area of the triangle *AED* in square centimetres?



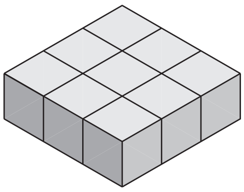
Maths Games – Example Problem 4.1

Example Problem 4.1 - Green

Nine 1 cm × 1 cm × 1 cm cubes are glued together into a 3 cm × 3 cm layer, as shown.

All faces of the object are completely painted.

What is the area of the painted surface, in square centimetres?



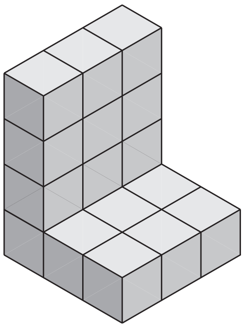
Example Problem 4.1 - Yellow

A Super-Brick is formed by arranging nine 1 cm × 1 cm × 1 cm cubes into a 3 cm × 3 cm layer, 1 cm thick.

Two Super-Bricks are glued together into the L-shape shown.

All faces of the L-shape are completely painted.

What is the area of the painted surface, in square centimetres?

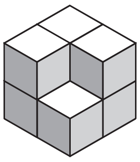


Example Problem 4.1 - Orange

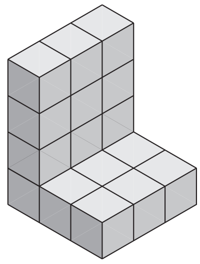
Jason has a 2 cm × 2 cm × 2 cm wooden block.

He removes a 1 cm × 1 cm × 1 cm wooden cube from one corner, as shown in the diagram.

What is the surface area, in square centimetres, of the resulting object that has a volume of 7 cubic centimetres?



Maths Games Example Solution 4.1 - Yellow



A Super-Brick is formed by arranging nine 1 cm × 1 cm × 1 cm cubes into a 3 cm × 3 cm layer.

Two Super-Bricks are glued together into the L-shape shown.

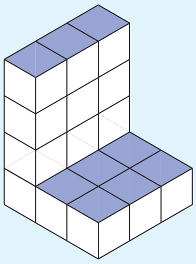
All faces of the L-shape are completely painted.

What is the area of the painted surface, in square centimetres?

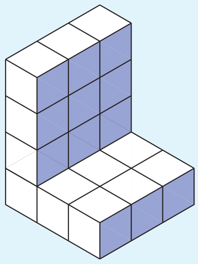
Strategy: Divide a Complex Shape (1)

We can consider the faces of the L-shape from six different directions.

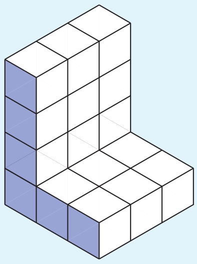
Top: 3 + 6 = 9 faces.



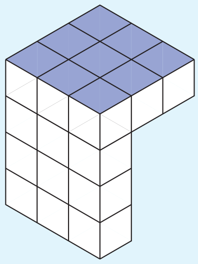
Front: 9 + 3 = 12 faces.



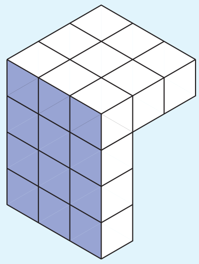
Left: 4 + 2 = 6 faces.



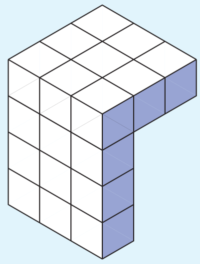
Bottom: 9 faces.



Back: 12 faces.



Right: 4 + 2 = 6 faces.

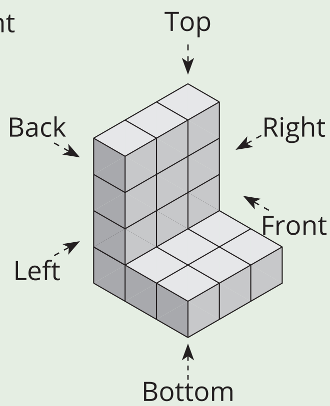


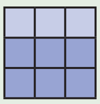
There are 9 + 12 + 6 + 9 + 12 + 6 = 54 faces.

Since each face is **1** square centimetre, the area of the painted surface is **54** square centimetres.

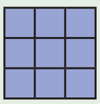
Strategy: Convert to a More Convenient Form

We can represent the surfaces of the L-shape by showing the different viewpoints in two dimensions.



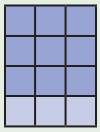


Top view

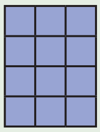


Bottom view

9 cm^2 each

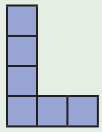


Front view

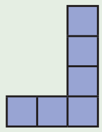


Back view

12 cm^2 each



Left view

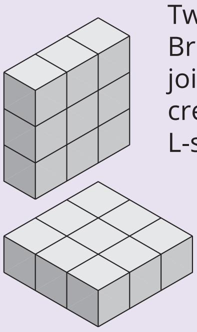


Right view

6 cm^2 each

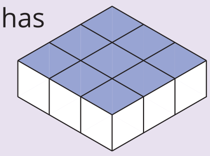
The surface area is $2 \times 9 + 2 \times 12 + 2 \times 6$ $= \mathbf{54}\,cm^2$.

Strategy: Divide a Complex Shape (2)

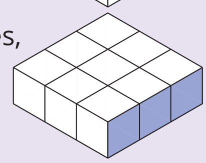


Two Super-Bricks were joined to create the L-shape.

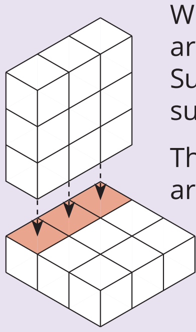
Each Super-Brick has $2 \times 9\,cm^2$ faces,



and $4 \times 3\,cm^2$ faces,



for a surface area of $18 + 12 = 30\,cm^2$.



When the two Super-Bricks are glued together, each Super-Brick loses $3\,cm^2$ of surface area.

The total amount of surface area lost is $2 \times 3\,cm^2 = 6\,cm^2$.

Therefore the surface area of the L-shape is $30 + 30 - 6 = \mathbf{54}\,cm^2$.

Answers

4.1 - Green: 30 (cm^2)

4.1 - Orange: 24 (cm^2)

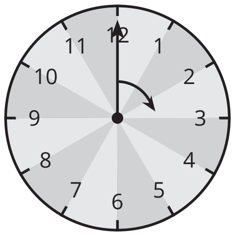
4.1 - Yellow: 54 (cm^2)

Maths Games – Example Problem 4.2

Example Problem 4.2 - Green

An analog clock is showing the time 3:30.

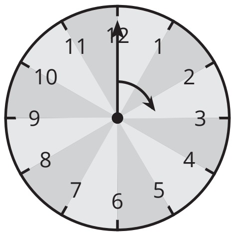
What is the size of the smaller angle, in degrees, between the hour and the minute hands?



Example Problem 4.2 - Yellow

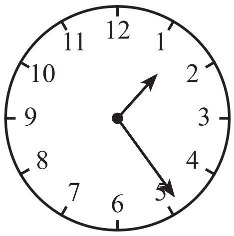
An analog clock is showing the time 6:50.

What is the size of the smaller angle, in degrees, between the hour and the minute hands?



Example Problem 4.2 - Orange

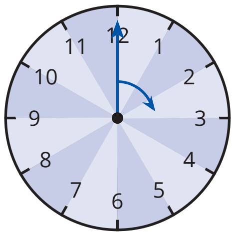
At 1:24pm (as shown), what is the size of the smaller angle formed by the hands of a clock, in degrees?



Maths Games Example Solution 4.2 - Yellow

An analog clock is showing the time 6:50.

What is the size of the smaller angle, in degrees, between the hour and the minute hands?



Strategy: Divide a Complex Shape

We can begin by dividing the clock face into sectors as shown.

There are **12** equal sectors, each occupying an angle of **360** ÷ **12** = **30** degrees.

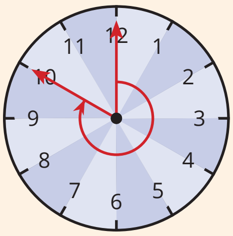
Method 1: Consider the angle between each hand and the 12 o'clock position.

We will define an angle of rotation for a clock hand as the size of the clockwise angle from **12** o'clock.

The minute hand takes **60** minutes to rotate **360** degrees.

With **12** numbers equally spaced, it takes **60** ÷ **12** = **5** minutes for the minute hand to rotate through **30** degrees.

At **50** minutes past the hour, the minute hand will have rotated **10** × **30** = **300** degrees past the **12** o'clock position.

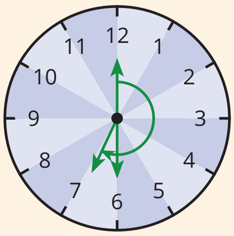


The hour hand takes **60** minutes to rotate **30** degrees.

In **1** minute, the hour hand would rotate **30** ÷ **60** = **0.5** degrees.

At **6**:**00**, the hour hand is pointing to the number **6**. This is **6** × **30** = **180** degrees past the **12** o'clock position.

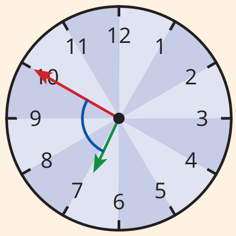
By **6**:**50**, the hour hand will have rotated another **50** × **0.5** = **25** degrees.



The minute hand has rotated **300** degrees past the **12** o'clock position.

The hour hand has rotated **180** + **25** = **205** degrees past the **12** o'clock position.

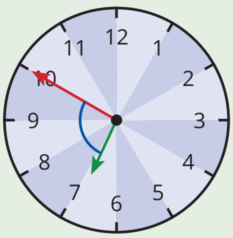
The smaller angle between the hands is **300** – **205** = **95** degrees.



Method 2: Consider the amount that one hand must turn, before it reaches the position of the other.

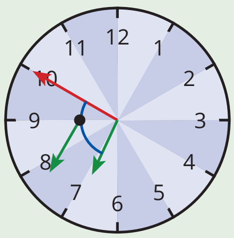
The minute hand is pointing to the **10**.

At **7** o'clock, the hour hand will be pointing to the **7**, so at **10** minutes to **7** o'clock, the hour hand will be nearly to the **7**.



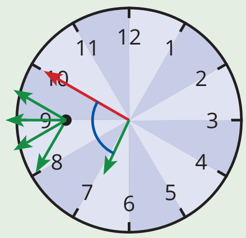
Since the hour hand rotates **0.5** degrees every minute, in the **10** minutes before **7** o'clock, it will rotate **10** × **0.5** = **5** degrees.

After rotating through **5** degrees, it will be pointing directly at the **7**.



The hour hand now needs to rotate through three **30** degree sectors before it is pointing to the **10**.

The angle between the two hands is **5** + **3** × **30** = **95** degrees.



Answers

4.2 - Green: 75°

4.2 - Orange: 102°

4.2 - Yellow: 95°

Maths Games – Example Problem 4.3

Example Problem 4.3 - Green

Fabian has 12 marbles in a jar: 4 blue, 4 red and 4 green.

He reaches into the jar without looking.

What is the smallest number of marbles Fabian needs to pick up, to be certain he has at least two of the same colour?

Example Problem 4.3 - Yellow

Zoe has 40 cards in a box: 10 blue, 10 red, 10 green, and 10 orange.

She reaches into the box without looking.

What is the smallest number of cards Zoe needs to pick up, to be certain she has at least three cards of the same colour?

Example Problem 4.3 - Orange

A bag contains 18 lollies.

4 are red, 6 are white, and 8 are blue.

Amanda takes them out one at a time without looking.

What is the smallest number of lollies she must take out to be certain that at least 2 of the lollies she takes out are blue?

Maths Games Example Solution 4.3 - Yellow

Zoe has 40 cards in a box: 10 blue, 10 red, 10 green, and 10 orange.

She reaches into the box without looking.

What is the smallest number of cards Zoe needs to pick up, to be certain she has at least three cards of the same colour?

Strategy: Convert to a More Convenient Form

Instead of thinking about what might happen if Zoe reaches into the box without looking, it may be easier to think about what might happen if Zoe can see the cards that she is choosing.

To make this work, we would have to change the question.

Instead of this question:

> *What is the smallest number of cards Zoe needs to pick up, to be certain she has at least three cards of the same colour?*

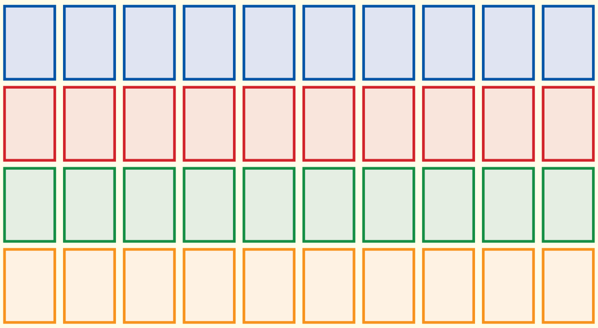
we might ask:

> *What is the largest number of cards Zoe can pick up, if she must stop when she has three cards of the same colour?*

In both cases, we are aiming to work out a selection of cards that it is possible for Zoe to choose.

Zoe's box contains **10** blue, **10** red, **10** green, and **10** orange cards.

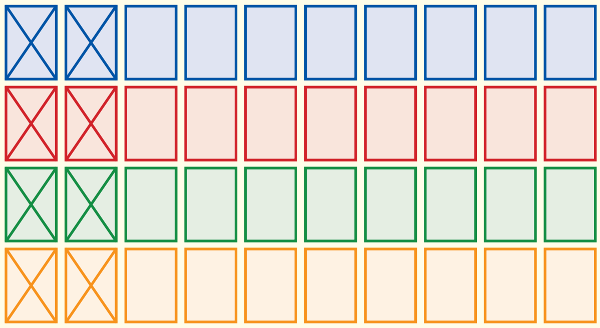
With the original question, she wants to pick up just enough cards to be certain that she has three cards of the same colour.



With our alternative question, Zoe is trying to pick up as many cards as she can.

She must stop as soon as she has picked up **3** cards of any colour.

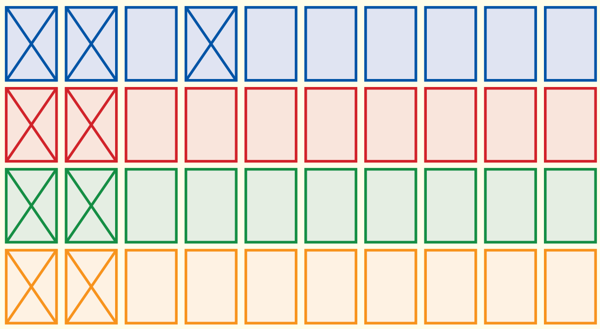
This means that she can keep going as long as she has only picked up **2** cards of each colour.



We can see that the result from working with the alternative question is possible even if Zoe had been picking cards without looking.

Zoe still doesn't have **3** of one colour. However, picking one more card, of any colour, would mean that she will have **3** cards of the same colour.

Zoe must pick **9** cards to be certain that she has three cards of the same colour.



Answers

4.3 - Green: 4

4.3 - Orange: 12

4.3 - Yellow: 9

Maths Games – Example Problem 4.4

Example Problem 4.4 - Green

A blue beetle takes 3 seconds to walk across a table.

A yellow beetle takes 6 seconds to walk across the same table.

A blue beetle and a yellow beetle both start at the same time from opposite ends of the table, and walk towards each other.

After how many seconds will they meet?

Example Problem 4.4 - Yellow

A red beetle takes 30 seconds to walk across a table.

A green beetle takes 20 seconds to walk across the same table.

A red beetle and a green beetle both start at the same time from opposite ends of the table, and walk towards each other.

After how many seconds will they meet?

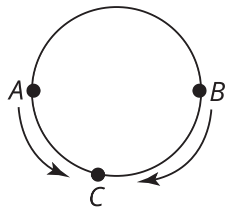
Example Problem 4.4 - Orange

Andy and Brett begin at opposite points on a circular track 200 metres in length, and run towards one another in opposite directions about the track as shown.

Andy runs at 2 metres per second, and Brett runs at 3 metres per second.

They first pass one another at point *C* after running for 20 seconds.

How many times did they pass one another during the first 8 minutes?

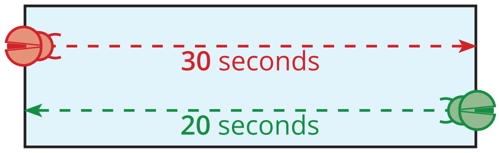


Maths Games Example Solution 4.4 - Yellow

A red beetle takes 30 seconds to walk across a table. A green beetle takes 20 seconds to walk across the same table. A red beetle and a green beetle both start at the same time from opposite ends of the table, and walk towards each other.

After how many seconds will they meet?

Strategy 1: Convert to a More Convenient Form



A red beetle takes 30 seconds to walk across a table.

A green beetle takes 20 seconds to walk across the same table.

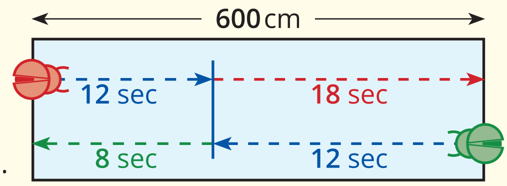
To make the problem easier to think about, let's make up a value for the width of the table.

The aim is to find a convenient value for the distance travelled by both beetles in **1** second.

- Since the red beetle takes 30 seconds to walk across the table, the width should be divisible by **30**.
- Since the green beetle takes 20 seconds, the width should also be divisible by **20**.

Suppose the table is **30** × **20** = **600** cm wide. In **1** second:

- The red beetle will travel **600** ÷ **30** = **20** cm.
- The green beetle will travel **600** ÷ **20** = **30** cm.
- Together, they will close the gap between them by **20** + **30** = **50** cm.



Since the table is **600** cm wide, the beetles will meet after **600** ÷ **50** = **12** seconds.

We can likewise use any common multiple of **30** and **20**. For example, the LCM of **30** and **20** is **60**.

Let's suppose the table is **60** cm wide.

In **1** second, the red beetle will travel **60** ÷ **30** = **2** cm, and the green beetle will travel **60** ÷ **20** = **3** cm.

Together, they will close the gap between them by **2** + **3** = **5** cm.

Since the table is **60** cm wide, the beetles will meet after **60** ÷ **5** = **12** seconds.

Strategy 2: Build a Table

When the beetles meet, they will both have been walking for the same length of time.

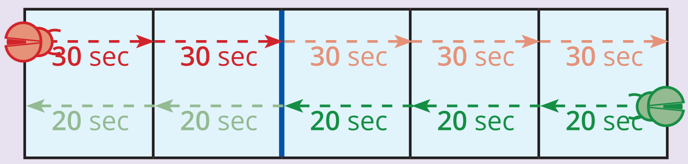
Let's build a table to work out how far they will both travel, given the same amount of time.

Table widths	1	2	3	4
Red Beetle (seconds)	30	60	90	120
Green Beetle (seconds)	20	40	60	80

In **60** seconds:

- the red beetle walks across **2** tables, and
- the green beetle walks across **3** tables.

If we have **5** tables placed end to end, and the two beetles start at opposite ends, then they will meet after walking for **60** seconds.



In our case, the beetles are traversing just **1** table, or one-fifth of this distance.

This means that they will complete the distance in one-fifth of this time.

The beetles will meet after **60** ÷ **5** = **12** seconds.

Answers

4.4 - Green: 2

4.4 - Orange: 12

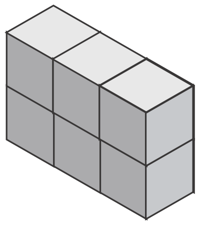
4.4 - Yellow: 12 (seconds)

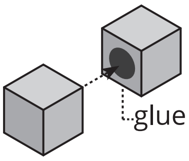
Maths Games – Example Problem 4.5

Example Problem 4.5 - Green

Chloe is using 6 small cubes to make this rectangular prism, as shown at the right.

The prism is 2 cubes high, 1 cube wide and 3 cubes long.





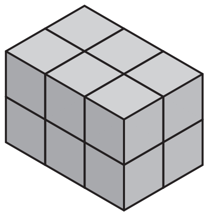
Wherever the faces of two cubes meet, she adds one dot of glue between them to stick them together, as shown in the diagram on the left.

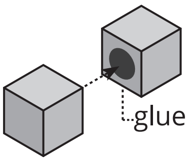
How many dots of glue does she need to construct the rectangular prism?

Example Problem 4.5 - Yellow

Chloe is using 12 small cubes to make this rectangular prism, as shown at the right.

The prism is 2 cubes high, 2 cubes wide and 3 cubes long.



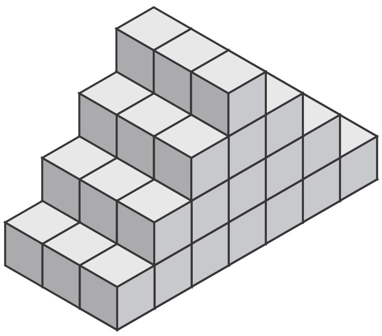


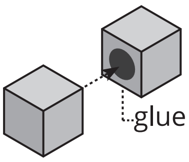
Wherever the faces of two cubes meet, she adds one dot of glue between them to stick them together, as shown in the diagram on the left.

How many dots of glue does she need to construct the rectangular prism?

Example Problem 4.5 - Orange

Chloe is using 48 small cubes to make this object, as shown at the right.





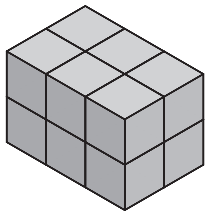
Wherever the faces of two cubes meet, she adds one dot of glue between them to stick them together, as shown in the diagram on the left.

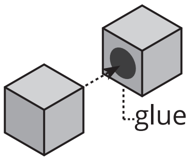
How many dots of glue does she need to construct this object?

Maths Games Example Solution 4.5 - Yellow

Chloe is using 12 small cubes to make this rectangular prism, as shown at the right.

The prism is 2 cubes high, 2 cubes wide and 3 cubes long.





Wherever the faces of two cubes meet, she adds one dot of glue between them to stick them together, as shown in the diagram on the left.

How many dots of glue does she need to construct the rectangular prism?

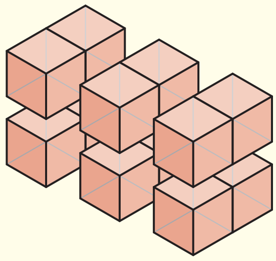
Strategy: Divide a Complex Shape

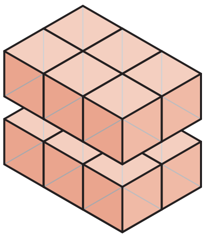
Let's start by constructing two-cube pieces.

Each two-cube piece would be put together with a single dot of glue.

The prism has **12** cubes, and so there are **12 ÷ 2 = 6** two-cube pieces.

To make six two-cube pieces, Chloe will need **6** dots of glue.

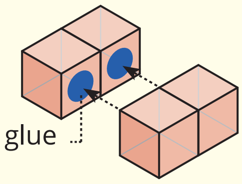




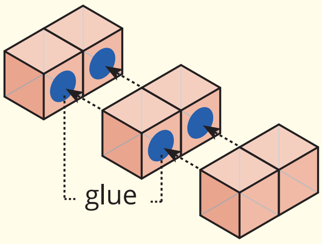
So far, Chloe has used **6** dots of glue.

Next, let's stick the two-cube pieces together to make one-cube-thick slices, like this.

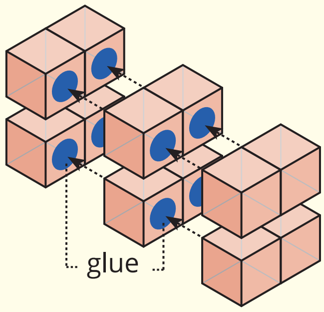
To join two 2-cube pieces together, Chloe needs **2** dots of glue.



She would need **2 + 2 = 4** dots of glue to complete one slice.

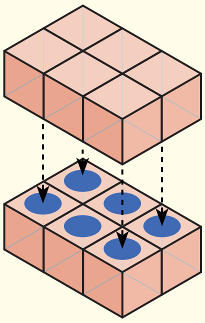


We can see that, to make two slices, she would need another **2 × 4 = 8** dots of glue.



So far, Chloe has used **6 + 8** dots of glue.

Finally, to join two 6-cube slices together, Chloe needs another **6** dots of glue.



The rectangular prism is complete.

Chloe will need **6 + 8 + 6 = 20** dots of glue to construct the rectangular prism.

Answers

4.5 - Green: 7

4.5 - Orange: 95

4.5 - Yellow: 20

Maths Games – Example Problem 4.6

Example Problem 4.6 - Green

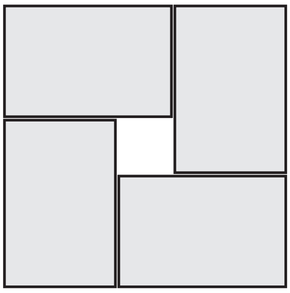
Four identical rectangular trays are arranged to form a square as shown.

One side-length of the large square is 10 cm.

The hole in the middle is also a square.

One side-length of the square hole is 2 cm.

What is the area of a single tray, in square centimetres?



Example Problem 4.6 - Yellow

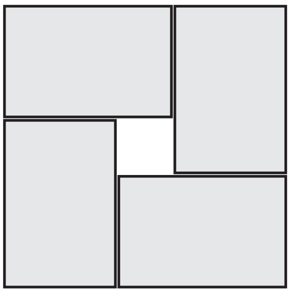
Four identical rectangular trays are arranged to form a square as shown.

One side-length of the large square is 50 cm.

The hole in the middle is also a square.

One side-length of the square hole is 10 cm.

What is the area of a single tray, in square centimetres?



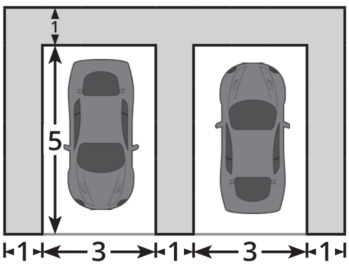
Example Problem 4.6 - Orange

A store has two parking spaces.

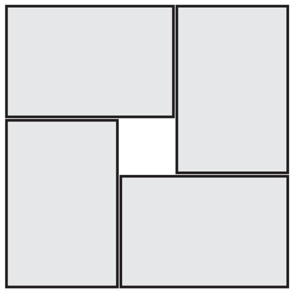
Each parking space is 3 metres wide and 5 metres long.

There is a 1 metre wide footpath around the sides and the back of the spaces. There is also a shared 1 metre wide section of footpath between the spaces, as shown in the diagram.

What is the area of the entire footpath, in square metres?



Maths Games Example Solution 4.6 - Yellow



Four identical rectangular trays are arranged to form a square as shown.

One side-length of the large square is 50 cm.

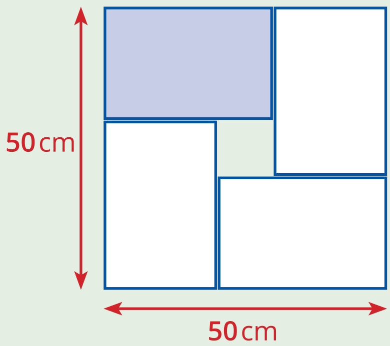
The hole in the middle is also a square.

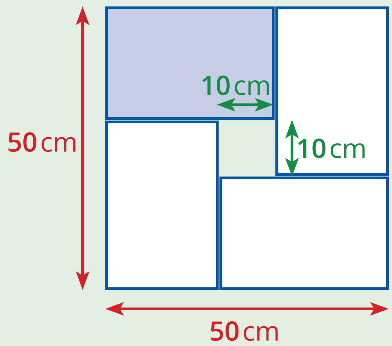
One side-length of the square hole is 10 cm.

What is the area of a single tray, in square centimetres?

Strategy 1: Divide a Complex Shape (1)

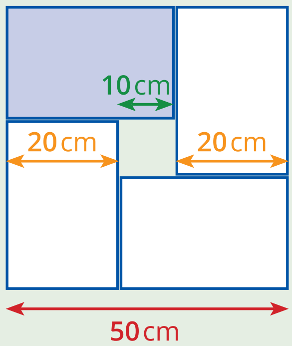
One side-length of the large square is 50 cm. One side-length of the square hole is 10 cm.





We can see that the width of the large square is the same as the width of the square hole, plus two of the short sides of a tray.

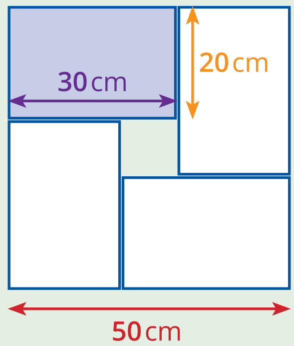
Two short sides is 50 cm – 10 cm = 40 cm, so one short side is 40 cm ÷ 2 = 20 cm long.



The long side of a tray is 20 cm + 10 cm = 30 cm long.

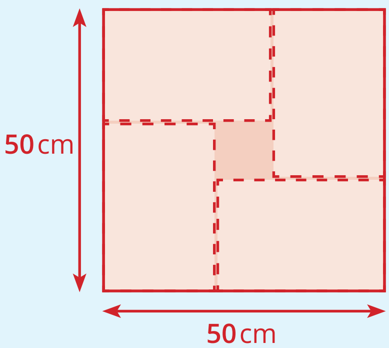
The short side is 20 cm long.

Therefore, the area of one tray is 30 cm × 20 cm = **600 cm²**.

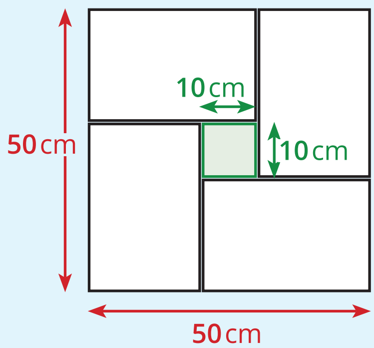


Strategy 2: Divide a Complex Shape (2)

If one side-length of the large square is 50 cm, then the area of the large square is 50 cm × 50 cm = 2500 cm².



If one side-length of the square hole is 10 cm, then the area of the square hole is 10 cm × 10 cm = 100 cm².



We can subtract the area of the square hole from the area of the large square, to find the area of the 4 trays:

2500 cm² – 100 cm² = 2400 cm².

4 trays have a combined area of 2400 cm².

Therefore, 1 tray will have an area of 2400 cm² ÷ 4 = **600 cm²**.

Answers

4.6 - Green: 24 (cm²)

4.6 - Orange: 24 (m²)

4.6 - Yellow: 600 (cm²)

Maths Games – Example Problem 4.7

Example Problem 4.7 - Green

I have a set of letter cards.

There is one *A*, two *B*s, three *C*s, four *D*s, and so on, up to ten *J*s.

How many letter cards are there in the set?

A
B B
C C C
D D D D
…
J J J J J J J J J J

Example Problem 4.7 - Yellow

A chime clock strikes

- 1 chime at one o’clock,
- 2 chimes at two o’clock,
- 3 chimes at three o’clock,

and so forth.

What is the total number of chimes the clock will strike in a twelve-hour period?

Example Problem 4.7 - Orange

Each row of *s has two more *s than the row immediately above it, as shown.

Altogether, how many *s are contained in the first ten rows?

```
*
* * *
* * * * *
* * * * * * *
```
and so on.

Maths Games Example Solution 4.7 - Yellow

A chime clock strikes 1 chime at one o'clock, 2 chimes at two o'clock, 3 chimes at three o'clock, and so forth.
What is the total number of chimes the clock will strike in a twelve-hour period?

Strategy 1: Convert to a More Convenient Form (1)

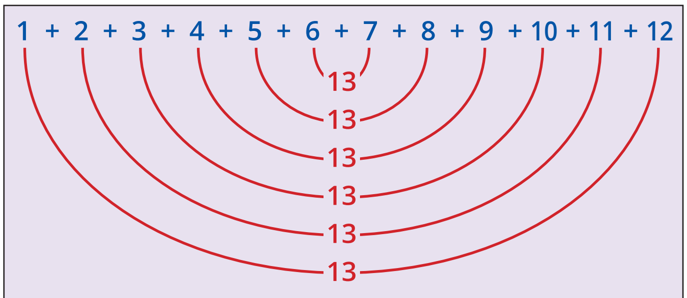
Let's start our twelve-hour period at 12:30 am. It will end at 12:30 pm.
By selecting these start and end times, we will definitely include:

- all of the chimes for one o'clock (at the start),
- twelve o'clock (near the end of the time period), and
- all of the chimes for the hours in-between.

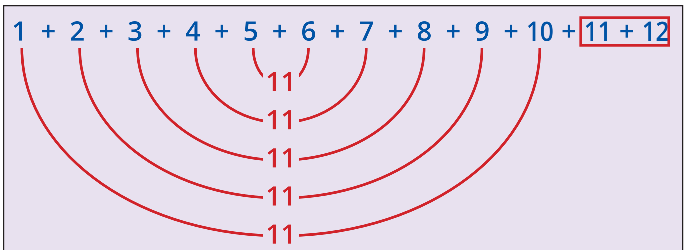
So the total number of chimes will be 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12.

To find the sum more easily, we can look for combinations of numbers that are easier to add.

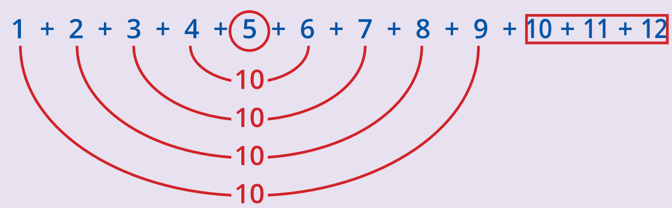
Here are some possibilities:



13 + 13 + 13 + 13 + 13 + 13 = 6 × 13 = 78.



11 + 11 + 11 + 11 + 11 + 11 + 12 = 6 × 11 + 12
= 66 + 12 = 78.



5 + 10 + 10 + 10 + 10 + 10 + 11 + 12
= 5 + (5 × 10) + 11 + 12
= 5 + 50 + 11 + 12
= 78.

Therefore the total number of chimes will be **78**.

Strategy 2: Convert to a More Convenient Form (2)

We know that the total number of chimes will be	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12
Let's pretend that we have a second clock that runs backwards. It will chime like this:	12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1
If we have both of these clocks sitting side by side, we'll hear this many chimes in a 12 hour period:	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1
Let's add all these chimes together. We can see that the total is equal to 12 × 13 = 156.	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 13 + 13 + 13 + 13 + 13 + 13 + 13 + 13 + 13 + 13 + 13 + 13

Since two clocks will chime 156 times in a twelve hour period, one clock must chime 156 ÷ 2 = **78** times.

Answers

4.7 - Green: 55

4.7 - Orange: 100

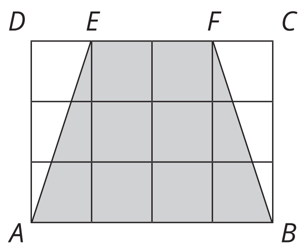
4.7 - Yellow: 78

Maths Games – Example Problem 4.8

Example Problem 4.8 - Green

ABCD is a rectangle whose area is 12 square centimetres.

How many square centimetres are contained in the area of trapezium *EFBA*?

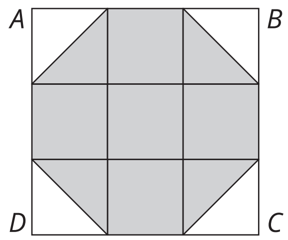


Example Problem 4.8 - Yellow

Square *ABCD* is composed of nine identical squares as shown.

The area of the shaded region is 14 square centimetres.

What is the area of square *ABCD*, in square centimetres?

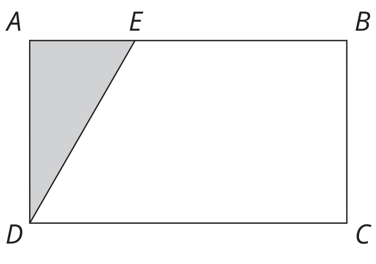


Example Problem 4.8 - Orange

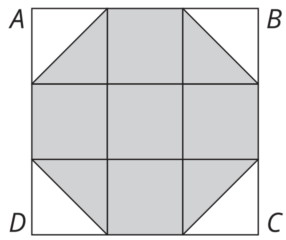
ABCD is a rectangle whose area is 360 square centimetres.

Point *E* is one-third of the way from *A* to *B*.

What is the area of the triangle *AED* in square centimetres?



Maths Games Example Solution 4.8 - Yellow



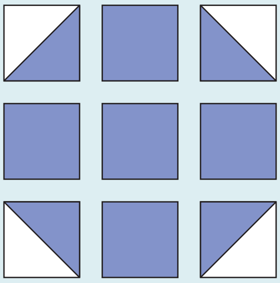
Square *ABCD* is composed of nine identical squares as shown.

The area of the shaded region is 14 square centimetres.

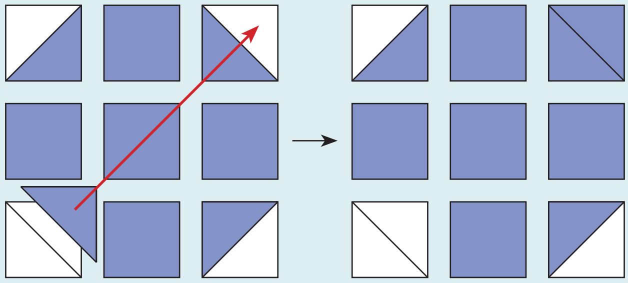
What is the area of square *ABCD*, in square centimetres?

Strategy 1: Divide a Complex Shape

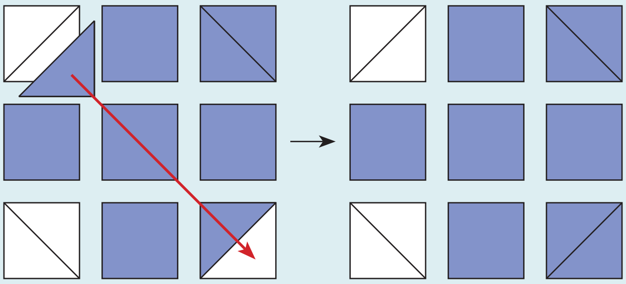
Let's break the shape up into the square sections.



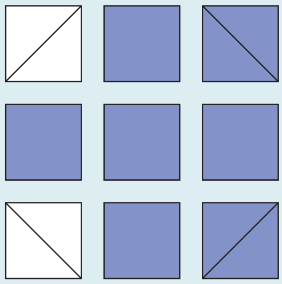
We can move the shaded area from the bottom-left square to fill the remaining space in the top-right square.



We can also move the shaded area from the top-left square to fill the remaining space in the bottom-right square.



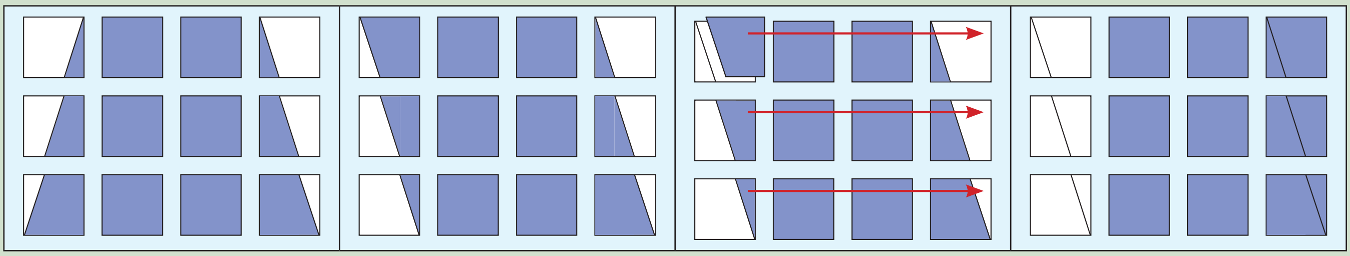
All together, there would be 7 identical shaded squares.



The combined area of the 7 identical squares is 14 square centimetres, so each square must have an area of $14 \div 7 = 2$ square centimetres.

Since *ABCD* is composed of 9 of these squares, the area of *ABCD* is $9 \times 2 = 18$ square centimetres.

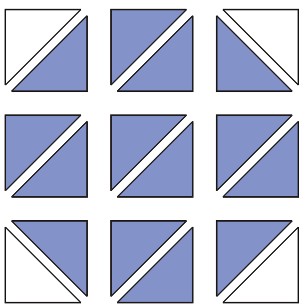
The Set Green question can also be solved in this way. In this case, each square in *ABCD* has an area of **1** cm^2. The shaded regions can be rearranged to form **9** shaded squares, for a total area of $9 \times 1 = 9\,\text{cm}^2$.



Strategy 2: Divide a Complex Shape (Alternative Method)

Let's break the shape up into triangular sections that are all the same size.

Each square can be broken into two triangles.



There are 14 shaded triangles.

Since the area of the shaded region is $14\,\text{cm}^2$, each triangle must have an area of $14 \div 14 = 1\,\text{cm}^2$.

ABCD is composed of 18 triangles of this size.

Therefore the area of *ABCD* is $18 \times 1\,\text{cm}^2 = 18\,\text{cm}^2$.

Answers

4.8 - Green: 9 (cm^2)

4.8 - Orange: 60 (cm^2)

4.8 - Yellow: 18 (cm^2)

Answers

Set Green

4.1	30 (cm^2)
4.2	75°
4.3	4
4.4	2
4.5	7
4.6	24 (cm^2)
4.7	55
4.8	9 (cm^2)

Set Yellow

4.1	54 (cm^2)
4.2	95°
4.3	9
4.4	12 (seconds)
4.5	20
4.6	600 (cm^2)
4.7	78
4.8	18 (cm^2)

Set Orange

4.1	24 (cm^2)
4.2	102°
4.3	12
4.4	12
4.5	95
4.6	24 (m^2)
4.7	100
4.8	60 (cm^2)