# 2024 Maths Games Senior - Years 7 & 8
# Resource Kit 3
# Teaching Problem Solving

MATHS GAMES

## Problem Solving Strategies

This resource kit focuses on the following problem solving strategies:

### 1. Solve a Simpler Related Problem

Many hard problems are actually simpler problems that have been extended to larger numbers.

Patterns can sometimes be identified by trying the problem with smaller numbers.

### 2. Eliminate All But One Possibility

Deciding what a quantity is not, can narrow the field to a very small number of possibilities.

These can then be tested against the conditions of the original problem.

It follows on from strategies introduced in the Preparation Resource Kit and Resource Kits 1 and 2:

Guess, Check and Refine

Draw a Diagram

Find a Pattern

Build a Table

Work Backwards

Make an Organised List

### Resource Kit 3 focuses on:

Solve a Simpler Related Problem

Eliminate All But One Possibility

#### Set Yellow

Example problems for which full worked solutions are included.

#### Set Green

Problems that are designed to be similar to Set Yellow, but with fewer difficult elements.

#### Set Orange

Problems that are similar in mathematical structure to the corresponding Yellow problems.

Further questions and solution methods can be found in the APSMO resource book "Building Confidence in Maths Problem Solving", available from www.apsmo.edu.au.

## How to use these problems

At the start of the lesson, present the problem and ask the students to think about it. Encourage students to try to solve it in any way they like. When the students have had enough time to consider their solutions, ask them to describe or present their methods, taking particular note of different ways of arriving at the same solution.

Each question includes at least one solution method that the majority of students should be able to follow. By participating in lessons that demonstrate achievable problem solving techniques, students may gain increased confidence in their own ability to address unfamiliar problems.

Finally, the consideration of different solution methods is fundamental to the students' development as effective and sophisticated problem solvers. Even when students have solved a problem to their own satisfaction, it is important to expose them to other methods and encourage them to judge whether or not the other methods are more efficient.

## Preparation Kit

### Guess, Check and Refine

This involves making a reasonable guess of the answer, and checking it against the conditions of the problem. An incorrect guess may provide more information that may lead to the answer.

### Draw a Diagram

A diagram may reveal information that may not be obvious just by reading the problem.

It is also useful for keeping track of where the student is up to in a multi-step problem.

## Resource Kit 1

### Find a Pattern

A frequently used problem solving strategy is that of recognising and extending a pattern.

Students can often simplify a difficult problem by identifying a pattern in the problem.

### Build a Table

A table displays information so that it is easily located and understood.

A table is an excellent way to record data so the student doesn't have to repeat their efforts.

## Resource Kit 2

### Work Backwards

If a problem describes a procedure and then specifies the final result, this method usually makes the problem much easier to solve.

### Make an Organised List

Listing every possibility in an organised way is an important tool.

How students organise the data often reveals additional information.

## Resource Kit 3

### Solve a Simpler Related Problem

Many hard problems are actually simpler problems that have been extended to larger numbers.

Patterns can sometimes be identified by trying the problem with smaller numbers.

### Eliminate All But One Possibility

Deciding what a quantity is not, can narrow the field to a very small number of possibilities.

These can then be tested against the conditions of the original problem.

## Resource Kit 4

### Convert to a More Convenient Form

There are times when changing some of the conditions of a problem makes a solution clearer or more convenient.

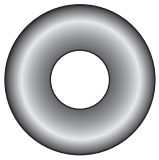
### Divide a Complex Shape

Sometimes it is possible to divide an unusual shape into two or more common shapes that are easier to work with.

# Set Yellow

3.1) Three machines can make ninety doughnuts in two minutes.

How many doughnuts will four machines make in three minutes?



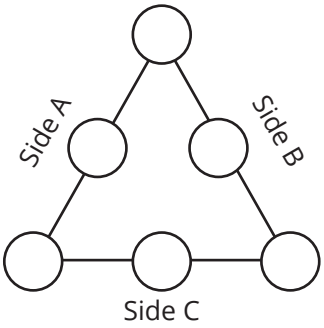
3.2) What is the value of the following?

$(8 \times 4) + (8 \times 3) + (8 \times 2) + (8 \times 1)$

3.3) The numbers 1, 2, 3, 4, 5, and 6 are placed in the diagram, one in each circle.

The sum of the three numbers along Side A is 13, along Side B is 13, and along Side C is 6.

What number is in the circle at the top of the diagram?

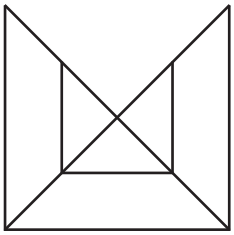



3.4) Find the whole number which is:

- less than 100,
- a multiple of 3,
- a multiple of 5,
- odd, and
- the sum of its digits is odd.

# Set Yellow

3.5) How many triangles, of any size, can be drawn by tracing over lines in this diagram?



3.6) In a trivia game, each player is asked 10 questions.
You get 10 points for each correct answer.
If you don't answer a question correctly, you lose 5 points.
At the end of the game, Clint's total was 55 points.
How many questions did Clint answer correctly?

3.7) In the following cryptarithm, different letters represent different digits.
What is the greatest possible value represented by *MEOW*?

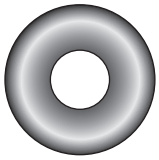
$$\begin{array}{r} C\ A\ T \\ +\ C\ A\ T \\ \hline M\ E\ O\ W \end{array}$$

3.8) Adam, Bree, Chloe, Duncan, Elena and Fahim are holding hands in a circle, facing the centre.
Adam is opposite Bree.
Duncan is holding Bree's left hand.
Elena is holding hands with both Adam and Fahim.
Who is holding Chloe's left hand?

# Set Green

3.1) Two machines can make 45 doughnuts in one minute.

How many doughnuts will four machines make in two minutes?



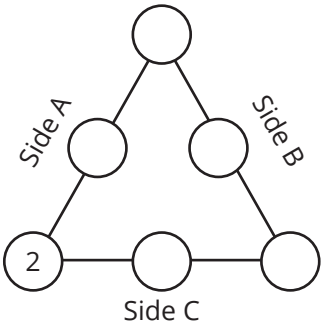
3.2) What is the value of the following?

$(3 \times 4) + (3 \times 3) + (3 \times 2) + (3 \times 1)$

3.3) The numbers 1, 2, 3, 4, 5, and 6 are placed in the diagram, one in each circle.

The sum of the three numbers along Side A is 13, along Side B is 13, and along Side C is 6.

What number is in the circle at the top of the diagram?

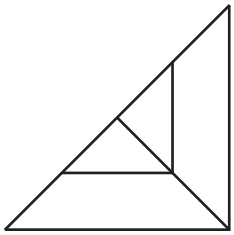



3.4) Find the whole number which is:

- less than 50,
- a multiple of 3,
- a multiple of 5,
- odd, and
- the sum of its digits is odd.

# Set Green

3.5) How many triangles, of any size, can be drawn by tracing over lines in this diagram?

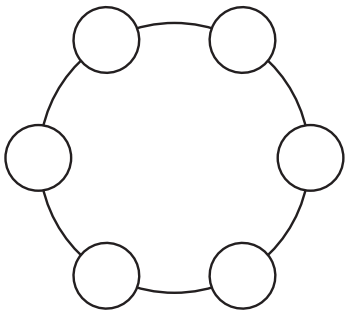


3.6) In a trivia game, each player is asked 5 questions.
You get 2 points for each correct answer.
If you don't answer a question correctly, you lose 1 point.
At the end of the game, Jennifer's total was 4 points.
How many questions did Jennifer answer correctly?

3.7) In the following cryptarithm, different letters represent different digits.
What is the value represented by *HA*?

$$\begin{array}{rr} & A \\ & A \\ + & A \\ \hline H & A \end{array}$$

3.8) Adam, Bree, Chloe, Duncan, Elena and Fahim are holding hands in a circle, facing the centre.
Adam is opposite Bree.
Duncan is holding Bree's left hand.
Elena is holding hands with both Adam and Fahim.
Who is holding Chloe's left hand?
Hint: You can rotate the paper.



# Set Orange

3.1) A car takes 1 minute 30 seconds to travel a distance of 1 kilometre.

At this rate, how many kilometres will the car travel in 1 hour?

3.2) What is the value of this expression?

(1234 × 9) + 1234

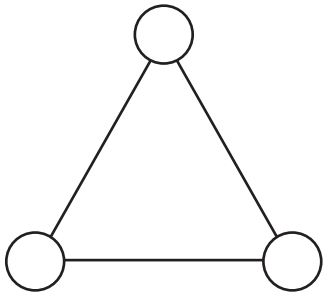
3.3) Anna drew three circles joined by three lines.

She wrote a number in each circle.

Then, she added the numbers from each pair of circles, and wrote the sum on the line joining them.

She found that she had all of the numbers 1, 2, 3, 4, 5 and 6 somewhere on her diagram.

What were the numbers in the circles, from smallest to largest?



3.4) A four-digit number is written on a piece of paper.

Ashley spills whiteout on it. Now the last two digits are no longer visible:

8 6 ? ?

The four-digit number is divisible by three, by four, and by five.

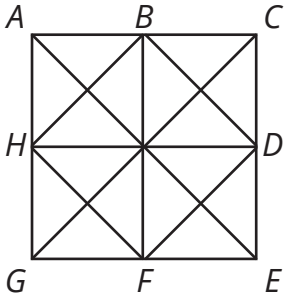
Find the four-digit number.

# Set Orange

3.5) Square *ACEG* is drawn at the right.

Points *B*, *D*, *F*, and *H* are halfway along the sides of the square.

What is the total number of squares of all sizes which can be traced using only the lines drawn?




3.6) I am building a 50-metre-long wire fence along one side of a straight road.

The wires will be attached to posts, which are set into the ground at 5 metre intervals.

To begin with there are no posts along this stretch of road.

How many posts do I need to construct this fence?

3.7) In this subtraction, the squares ( □ ) contain the digits 3, 4, 6, and 9 in some order and the hexagons ( ⬡ ) contain the digits 4, 5, 8, and 9 in some order.

What four-digit number is represented by the squares?

□ □ □ □
– ⬡ ⬡ ⬡ ⬡
3 4 9 7

3.8) Amy, Dawn and Soumiya sit at a round table facing the centre.

Each girl wears a ring on her finger.

Each ring is a different colour. One ring is red, one is green, and one is blue.

The girl with the green ring is to the right of Amy.

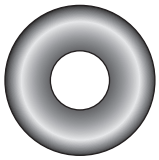
The girl with the red ring is to the left of Soumiya.

State the colour of each girl's ring.

# Maths Games – Example Problem 3.1

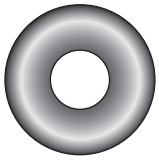
### Example Problem 3.1 - Green

Two machines can make 45 doughnuts in one minute.
How many doughnuts will four machines make in two minutes?



### Example Problem 3.1 - Yellow

Three machines can make ninety doughnuts in two minutes.
How many doughnuts will four machines make in three minutes?



### Example Problem 3.1 - Orange

A car takes 1 minute 30 seconds to travel a distance of 1 kilometre.
At this rate, how many kilometres will the car travel in 1 hour?

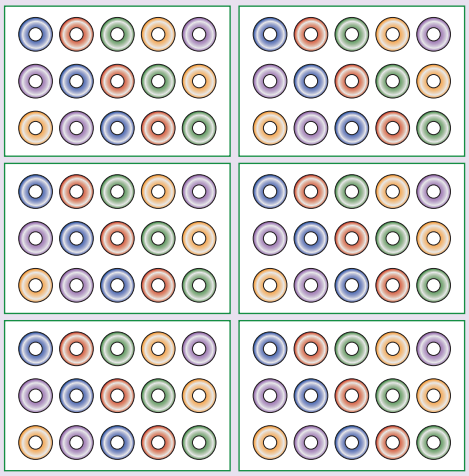
# Maths Games Example Solution 3.1 - Yellow

Three machines can make ninety doughnuts in two minutes.

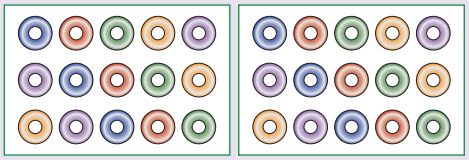
How many doughnuts will four machines make in three minutes?

## Strategy: Solve a Simpler Related Problem

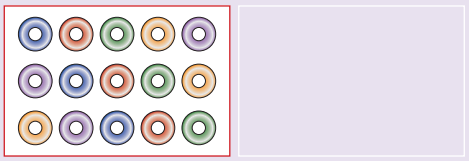
We know that 3 machines will make 90 doughnuts in 2 minutes.



So, in 2 minutes, 1 of the machines will make 90 ÷ 3 = 30 doughnuts.

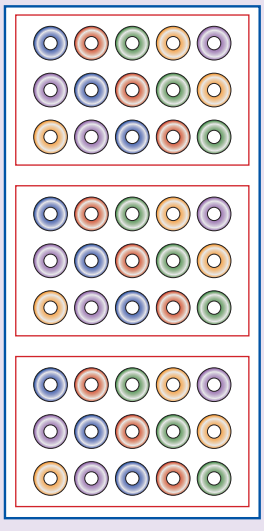


In 1 minute, 1 of the machines will make 30 ÷ 2 = 15 doughnuts.

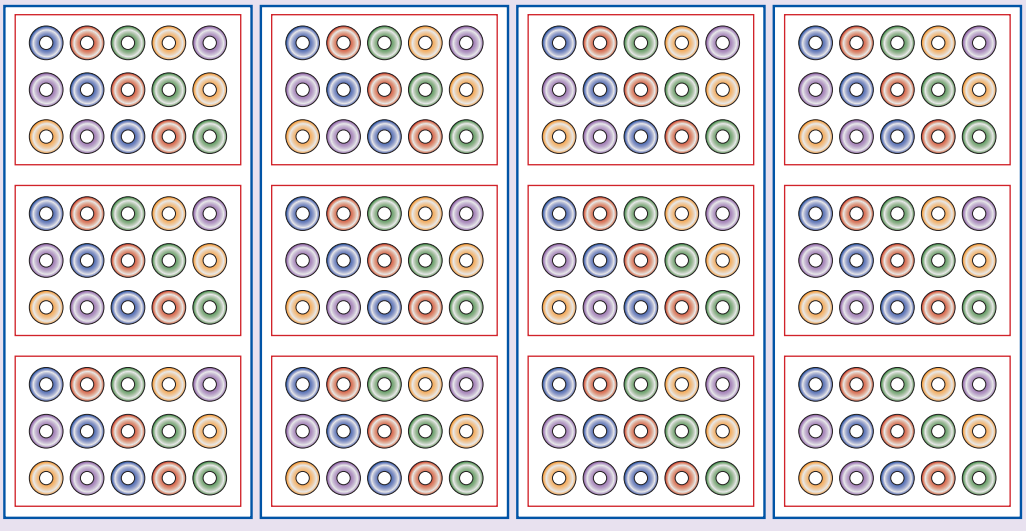


We now know that it takes 1 minute for 1 machine to make 15 doughnuts.

In 3 minutes, 1 machine will make 3 × 15 = 45 doughnuts.



If we run 4 of the machines for 3 minutes, together they will make 4 × 45 = 180 doughnuts.



In 3 minutes, 4 machines will make **180** doughnuts.

**Answers**

**3.1 - Green:** 180

**3.1 - Orange:** 40

**3.1 - Yellow:** 180

# Maths Games – Example Problem 3.2

### Example Problem 3.2 - Green

What is the value of the following?

$(3 \times 4) + (3 \times 3) + (3 \times 2) + (3 \times 1)$

### Example Problem 3.2 - Yellow

What is the value of the following?

$(8 \times 4) + (8 \times 3) + (8 \times 2) + (8 \times 1)$

### Example Problem 3.2 - Orange

What is the value of this expression?

$(1234 \times 9) + 1234$

# Maths Games Example Solution 3.2 - Yellow

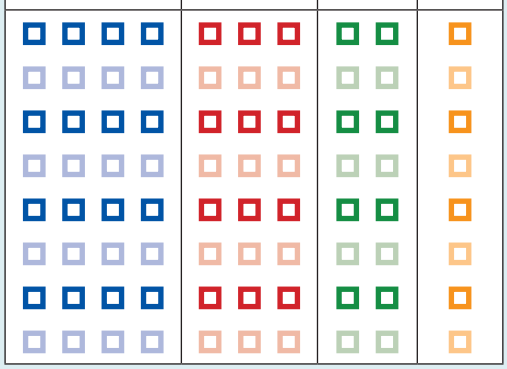
What is the value of the following?

$(8 \times 4) + (8 \times 3) + (8 \times 2) + (8 \times 1)$

## Strategy 1: Solve a Simpler Related Problem

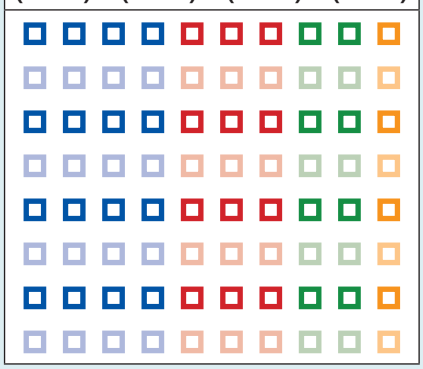
We can represent each multiplication as an array, like this:

| 8 × 4 | 8 × 3 | 8 × 2 | 8 × 1 |
|---|---|---|---|



The following amount results from adding the values together.

(8 × 4) + (8 × 3) + (8 × 2) + (8 × 1)



We can see that the diagram for

$(8 \times 4) + (8 \times 3) + (8 \times 2) + (8 \times 1)$

is the same as the array we would draw to represent

$8 \times (4 + 3 + 2 + 1)$

$= 8 \times 10.$

Therefore the value is equal to $8 \times 10 = \mathbf{80}$.

## Strategy 2: Perform the Calculation

$(8 \times 4) + (8 \times 3) + (8 \times 2) + (8 \times 1)$

$= 32 + 24 + 16 + 8$

### Method: Jump Strategy

$32 + 24 + 16 + 8$

$= 56 + 16 + 8$

$= 72 + 8$

$= 80.$

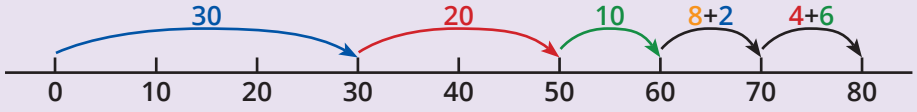

### Method: Split Strategy

$32 + 24 + 16 + 8$

$= 30 + 2 + 20 + 4 + 10 + 6 + 8$

$= 30 + 20 + 10 + 8 + 2 + 4 + 6$

$= 80.$



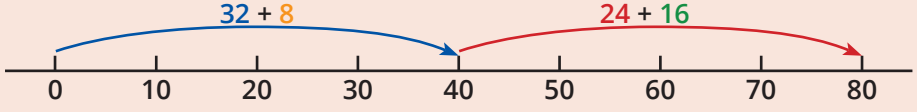

### Method: Friends of 40

$32 + 24 + 16 + 8$

$= 32 + 8 + 24 + 16$

$= 40 + 40$

$= 80.$




The value of $32 + 24 + 16 + 8$ is **80**.

**Answers**

**3.2 - Green:** 30

**3.2 - Orange:** 12340

**3.2 - Yellow:** 80

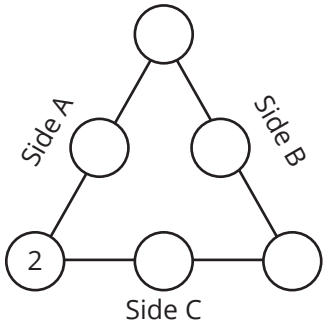
# Maths Games – Example Problem 3.3

## Example Problem 3.3 - Green

The numbers 1, 2, 3, 4, 5, and 6 are placed in the diagram, one in each circle.

The sum of the three numbers along Side A is 13, along Side B is 13, and along Side C is 6.

What number is in the circle at the top of the diagram?

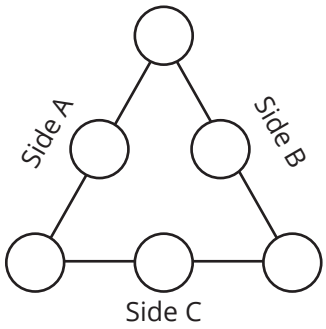



## Example Problem 3.3 - Yellow

The numbers 1, 2, 3, 4, 5, and 6 are placed in the diagram, one in each circle.

The sum of the three numbers along Side A is 13, along Side B is 13, and along Side C is 6.

What number is in the circle at the top of the diagram?




## Example Problem 3.3 - Orange

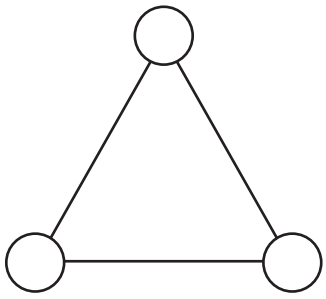
Anna drew three circles joined by three lines.

She wrote a number in each circle.

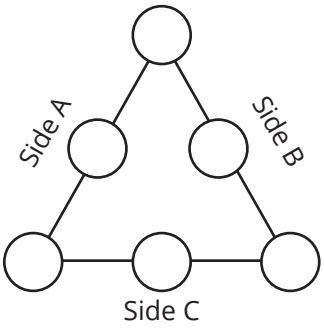
Then, she added the numbers from each pair of circles, and wrote the sum on the line joining them.

She found that she had all of the numbers 1, 2, 3, 4, 5 and 6 somewhere on her diagram.

What were the numbers in the circles, from smallest to largest?



# Maths Games Example Solution 3.3 - Yellow




The numbers 1, 2, 3, 4, 5, and 6 are placed in the diagram, one in each circle.

The sum of the three numbers along Side A is 13, along Side B is 13, and along Side C is 6.

What number is in the circle at the top of the diagram?

## Strategy: Eliminate All But One Possibility

Using the numbers 1, 2, 3, 4, 5 and 6, we need to find:

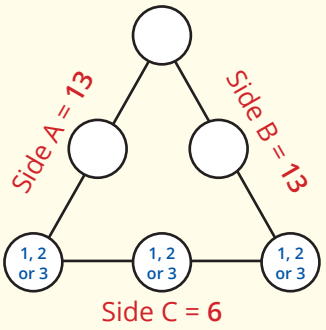
- Three numbers that add to 13,
- Another three numbers that add to 13, and
- Three numbers that add to 6.

Let's start by considering how to make 6.

It's a smaller total, so we'll try smaller numbers.

The three smallest numbers are 1, 2, and 3.

Since 1 + 2 + 3 = 6, we know that this is the only possible combination that will work for Side C.

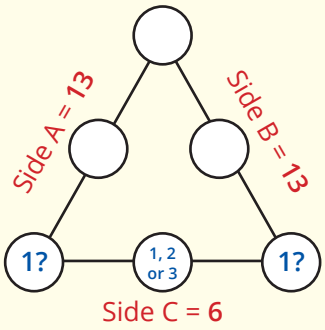



Suppose the value in the bottom left circle is 1.

Then Side A's total of 13 would include a 1.

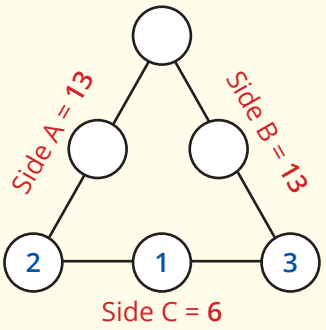
However, since 1 + 5 + 6 = 12 we can see that it's not possible to include the 1 and still make 13.

The same thing would happen if we had 1 in the bottom right circle.



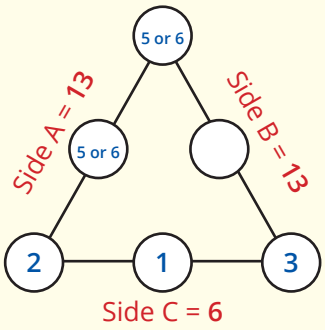

The only position left for 1 is the middle of Side C.

Since both Side A and Side B add up to 13, it doesn't matter which way around we position the 2 and the 3.



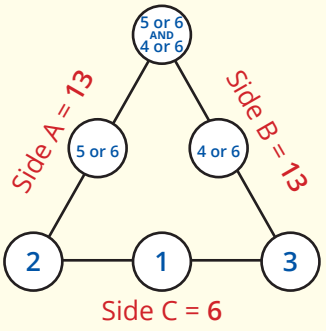

Since Side A's total of 13 includes a 2, we need to find a combination of two of the remaining numbers that add up to 13 - 2 = 11.

Since the remaining numbers are 4, 5 and 6, the only way to make 11 will be 5 + 6.




Since Side B's total of 13 includes a 3, we need to find a combination of two of the remaining numbers that add up to 13 - 3 = 10.

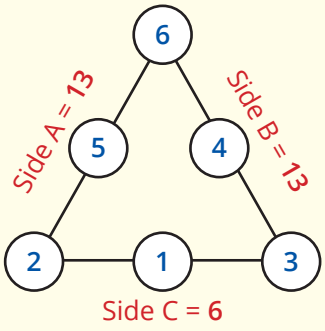
Since the remaining numbers are 4, 5 and 6, the only way to make 10 will be 4 + 6.




We can see that the topmost circle must satisfy the following:

- Either 5 or 6, AND
- Either 4 or 6.

The only way this could possibly work is for the value to actually be 6.




The number in the circle at the top of the diagram must be **6**.

## Answers

**3.3 - Green:** 6

**3.3 - Orange:** 1, 2, 4

**3.3 - Yellow:** 6

# Maths Games – Example Problem 3.4

## Example Problem 3.4 - Green

Find the whole number which is:

- less than 50,
- a multiple of 3,
- a multiple of 5,
- odd, and
- the sum of its digits is odd.

## Example Problem 3.4 - Yellow

Find the whole number which is:

- less than 100,
- a multiple of 3,
- a multiple of 5,
- odd, and
- the sum of its digits is odd.

## Example Problem 3.4 - Orange

A four-digit number is written on a piece of paper.

Ashley spills whiteout on it. Now the last two digits are no longer visible:

8 6 ? ?

The four-digit number is divisible by three, by four, and by five.

Find the four-digit number.

# Maths Games Example Solution 3.4 - Yellow

Find the whole number which is less than 100, a multiple of 3, a multiple of 5, odd, and the sum of its digits is odd.

## Strategy: Eliminate All But One Possibility

Since the number is less than **100**, we know that it will be a value on a **100** chart.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

The number is a multiple of 3, so we will eliminate all of the numbers that are ***not*** multiples of 3.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

The number is a multiple of 5, so we will eliminate all of the numbers that are ***not*** multiples of 5.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

The number is **odd**, so we will eliminate all of the numbers that are ***not*** **odd**.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

There are only three numbers remaining: **15**, **45** and **75**.

Of these, the only number that results in an odd digit sum is **45**.

**Answers**

**3.4 - Green:** 45

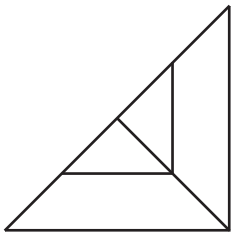
**3.4 - Orange:** 8640

**3.4 - Yellow:** 45

# Maths Games – Example Problem 3.5

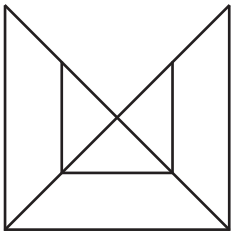
### Example Problem 3.5 - Green

How many triangles, of any size, can be drawn by tracing over lines in this diagram?



### Example Problem 3.5 - Yellow

How many triangles, of any size, can be drawn by tracing over lines in this diagram?

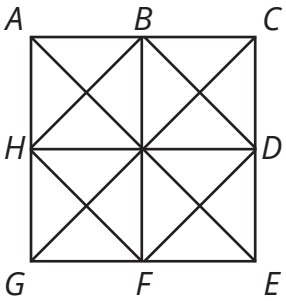


### Example Problem 3.5 - Orange

Square *ACEG* is drawn at the right.

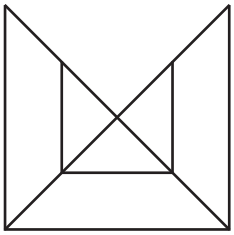
Points *B*, *D*, *F*, and *H* are halfway along the sides of the square.

What is the total number of squares of all sizes which can be traced using only the lines drawn?



# Maths Games Example Solution 3.5 - Yellow

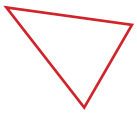
How many triangles, of any size, can be drawn by tracing over lines in this diagram?



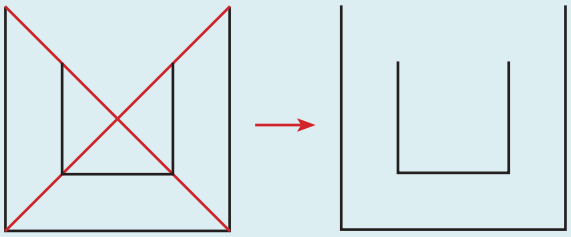
## Strategy: Solve a Simpler Related Problem

A triangle has **3** sides.

This means that each side of the triangle must be connected to both of the other sides.



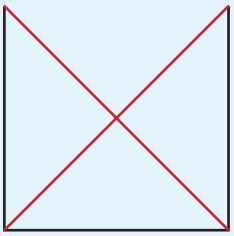
In our diagram, if we remove the "X" in the middle, none of the inner lines are connected to any of the outer lines.



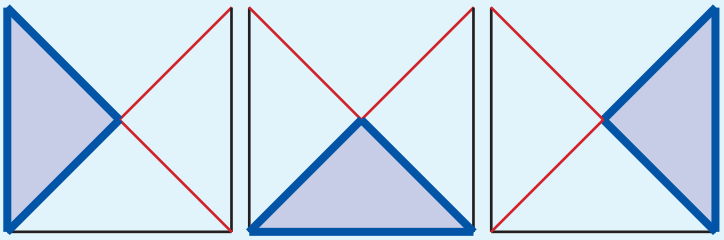
So there can't be a triangle that includes a line from the inner U-shape, and also includes a line from the outer U-shape.

Let's simplify the problem by breaking it into two parts.

We'll start by finding the triangles that are connected to the outer U-shape.

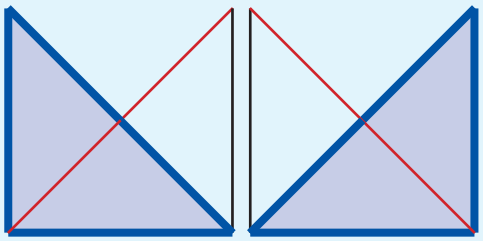


There are 3 triangles which look like this:



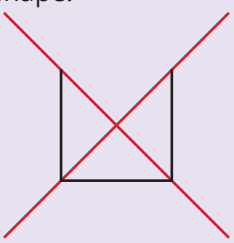
There are also some larger triangles, made up of 2 of these smaller triangles.

We can't make a bigger triangle here - it is not possible to combine all three of the small triangles to make a single triangle.

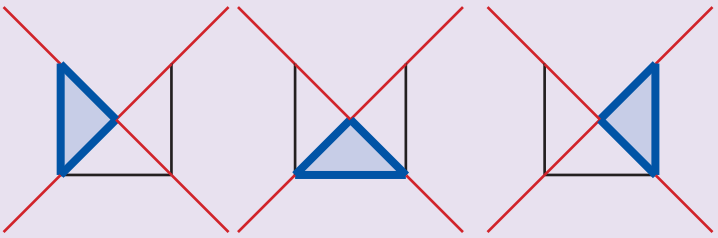


Next, let's find the triangles that are connected to the inner U-shape.

This question is very similar to the one we solved for the outer U-shape.

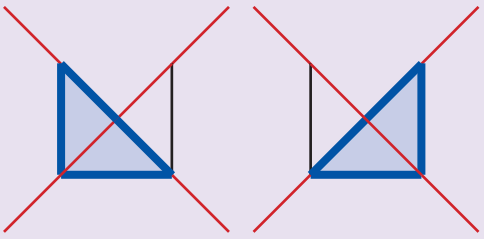


There are 3 triangles which look like this.



Similarly, there are 2 larger triangles.

Again, we can't make a bigger triangle here.



In total, **3 + 2 + 3 + 2 = 10** triangles can be drawn on these lines.

**Answers**

**3.5 - Green:** 6

**3.5 - Orange:** 10

**3.5 - Yellow:** 10

# Maths Games – Example Problem 3.6

### Example Problem 3.6 - Green

In a trivia game, each player is asked 5 questions.
You get 2 points for each correct answer.
If you don't answer a question correctly, you lose 1 point.
At the end of the game, Jennifer's total was 4 points.
How many questions did Jennifer answer correctly?

### Example Problem 3.6 - Yellow

In a trivia game, each player is asked 10 questions.
You get 10 points for each correct answer.
If you don't answer a question correctly, you lose 5 points.
At the end of the game, Clint's total was 55 points.
How many questions did Clint answer correctly?

### Example Problem 3.6 - Orange

I am building a 50-metre-long wire fence along one side of a straight road.
The wires will be attached to posts, which are set into the ground at 5 metre intervals.
To begin with there are no posts along this stretch of road.
How many posts do I need to construct this fence?

# Maths Games Example Solution 3.6 - Yellow

In a trivia game, each player is asked 10 questions. You get 10 points for each correct answer.

If you don't answer a question correctly, you lose 5 points. At the end of the game, Clint's total was 55 points.

How many questions did Clint answer correctly?

## Strategy 1: Build a Table

In this trivia game, each player is asked **10** questions.

You get 10 points for each correct answer, and you lose 5 points if you don't answer correctly.

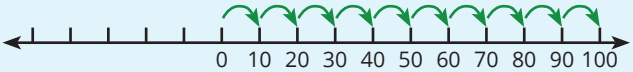
| No. questions answered correctly | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| No. questions not answered correctly | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| Score | -50 | -35 | -20 | -5 | 10 | 25 | 40 | 55 | 70 | 85 | 100 |

Clint's total was **55** points, so Clint must have answered **7** questions correctly.

## Strategy 2: Draw a Diagram, and Find a Pattern

Suppose Clint answered all **10** questions correctly.

He would score **10 × 10 = 100** points.

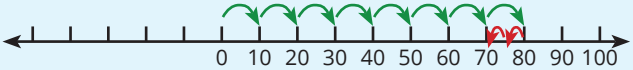



If Clint answered **9** questions correctly, he would score **9 × 10 – 1 × 5 = 85** points.

That's **100 – 85 = 15** less than the highest score.


If Clint answered **8** questions correctly, he would score **8 × 10 – 2 × 5 = 70** points.

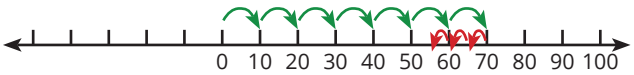
That's **100 – 70 = 30** less than the highest score.




For every incorrect answer, Clint loses **15** points from the high score of **100**. Why does this occur?

A score of **55** means that Clint lost **100 – 55 = 45** points.

Therefore, Clint answered **45 ÷ 15 = 3** questions incorrectly, and **10 – 3 = 7** questions correctly.




## Strategy 3: Solve a Simpler Related Problem

Suppose we change the scoring for the trivia game, as follows.

- Every time a player is asked a question, they automatically score 5 points.
- If they answer correctly, they pick up another 10 points.

  This means that they will score a total of **5 + 10 = 15** points for a correct response.
- If they do not answer correctly, they lose the automatic 5 points.

  This means that they will score a total of **5 – 5 = 0** points for an incorrect response.

Using this new scoring method, every participant would receive an extra **10 × 5 = 50** points.

Under the original scoring method, Clint scored **55** points, so using the new method, Clint would have scored **55 + 50 = 105** points.

Using the new method, each correct answer scores 15 points.

To reach a total of **105** points, Clint would have answered **105 ÷ 15 = 7** questions.

**Answers**

**3.6 - Green:** 3

**3.6 - Orange:** 11

**3.6 - Yellow:** 7

# Maths Games – Example Problem 3.7

## Example Problem 3.7 - Green

In the following cryptarithm, different letters represent different digits.
What is the value represented by *HA*?

$$\begin{array}{r} A \\ A \\ +\ A \\ \hline H\ A \end{array}$$

## Example Problem 3.7 - Yellow

In the following cryptarithm, different letters represent different digits.
What is the greatest possible value represented by *MEOW*?

$$\begin{array}{r} C\ A\ T \\ +\ C\ A\ T \\ \hline M\ E\ O\ W \end{array}$$

## Example Problem 3.7 - Orange

In this subtraction, the squares ( $\square$ ) contain the digits 3, 4, 6, and 9 in some order and the hexagons ( $\hexagon$ ) contain the digits 4, 5, 8, and 9 in some order.
What four-digit number is represented by the squares?

$$\begin{array}{r} \square\square\square\square \\ -\ \hexagon\hexagon\hexagon\hexagon \\ \hline 3\ 4\ 9\ 7 \end{array}$$

# Maths Games Example Solution 3.7 - Yellow

In the following cryptarithm, different letters represent different digits.

What is the greatest possible value represented by *MEOW*?

$$\begin{array}{r} C\ A\ T \\ +\ C\ A\ T \\ \hline M\ E\ O\ W \end{array}$$

## Strategy: Eliminate All But One Possibility

To find the greatest possible value for ***MEOW***, we might begin by considering the greatest possible value for ***M***.

Suppose ***M*** represented 9. Then, the greatest possible value for ***C*** would be 8.

$$\begin{array}{r} 8\ A\ T \\ +\ 8\ A\ T \\ \hline 9\ E\ O\ W \end{array}$$

Clearly, having ***C*** = 8 makes it impossible for ***M*** to be 9.

The greatest possible value for ***M*** is 1.

$$\begin{array}{r} 8\ A\ T \\ +\ 8\ A\ T \\ \hline 1\ 7\ O\ W \end{array}$$

Using the preceding argument, we can see that the greatest possible value for ***MEOW*** requires us to maximise the value for ***C***.

Let's set ***C*** to 9.

Since 9 is taken, the greatest possible value for ***E*** would then be 8.

$$\begin{array}{r} 9\ A\ T \\ +\ 9\ A\ T \\ \hline 1\ 8\ O\ W \end{array}$$

Since 9 and 8 are taken, the greatest possible value for ***O*** would be 7.

$$\begin{array}{r} 9\ A\ T \\ +\ 9\ A\ T \\ \hline 1\ 8\ 7\ W \end{array}$$

It's not possible for ***A*** + ***A*** = 7, but with trading from the ones place, we can have ***A*** + ***A*** + 1 = 7.

$$\begin{array}{r} {\scriptstyle 1}\ \ \ \\ 9\ 3\ T \\ +\ 9\ 3\ T \\ \hline 1\ 8\ 7\ W \end{array}$$

9, 8, and 7 are now taken.

The greatest possible value for ***W*** is 6.

$$\begin{array}{r} {\scriptstyle 1}\ \ \ \\ 9\ 3\ T \\ +\ 9\ 3\ T \\ \hline 1\ 8\ 7\ 6 \end{array}$$

If ***W*** is 6, then with the trading requirement we will have ***T*** + ***T*** = 16.

That cannot be right, because ***E*** = 8. So ***T*** cannot be 8.

$$\begin{array}{r} {\scriptstyle 1}\ \ \ \\ 9\ 3\ (8) \\ +\ 9\ 3\ (8) \\ \hline 1\ \boxed{8}\ 7\ 6 \end{array}$$

The next greatest possible value for ***W*** is 5. If so, we will have ***T*** + ***T*** = 15.

This is not possible, since ***T*** cannot be 7.5.

$$\begin{array}{r} {\scriptstyle 1}\ \ \ \\ 9\ 3\ T \\ +\ 9\ 3\ T \\ \hline 1\ 8\ 7\ 5 \end{array}$$

The following values are likewise not possible:

- ***T*** + ***T*** = 14. ***T*** cannot be 7.
- ***T*** + ***T*** = 13. ***T*** cannot be 6.5.

$$\begin{array}{r} {\scriptstyle 1}\ \ \ \\ 9\ 3\ T \\ +\ 9\ 3\ T \\ \hline 1\ 8\ 7\ W \end{array}$$

The next greatest possible value for ***W*** is 2.

If so, we will have ***T*** + ***T*** = 12, and so ***T*** = 6.

$$\begin{array}{r} {\scriptstyle 1}\ \ \ \\ 9\ 3\ 6 \\ +\ 9\ 3\ 6 \\ \hline 1\ 8\ 7\ 2 \end{array}$$

The greatest possible value for ***MEOW*** is **1872**.

**Answers** **3.7 - Green:** 15 **3.7 - Orange:** 9346

**3.7 - Yellow:** 1872

# Maths Games – Example Problem 3.8

## Example Problem 3.8 - Green

Adam, Bree, Chloe, Duncan, Elena and Fahim are holding hands in a circle, facing the centre.

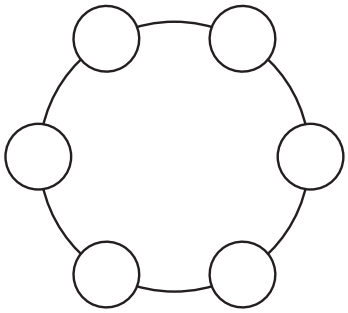
Adam is opposite Bree.

Duncan is holding Bree's left hand.

Elena is holding hands with both Adam and Fahim.

Who is holding Chloe's left hand?

Hint: You can rotate the paper.



## Example Problem 3.8 - Yellow

Adam, Bree, Chloe, Duncan, Elena and Fahim are holding hands in a circle, facing the centre.

Adam is opposite Bree.

Duncan is holding Bree's left hand.

Elena is holding hands with both Adam and Fahim.

Who is holding Chloe's left hand?

## Example Problem 3.8 - Orange

Amy, Dawn and Soumiya sit at a round table facing the centre.

Each girl wears a ring on her finger.

Each ring is a different colour. One ring is red, one is green, and one is blue.

The girl with the green ring is to the right of Amy.

The girl with the red ring is to the left of Soumiya.

State the colour of each girl's ring.

# Maths Games Example Solution 3.8 - Yellow

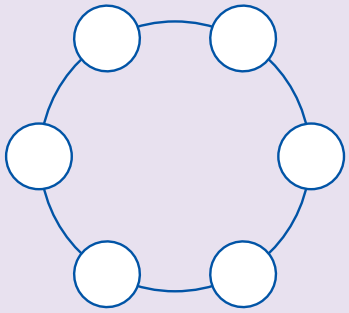
Adam, Bree, Chloe, Duncan, Elena and Fahim are holding hands in a circle, facing the centre.

Adam is opposite Bree. Duncan is holding Bree's left hand. Elena is holding hands with both Adam and Fahim.

Who is holding Chloe's left hand?

## Strategy: Eliminate All But One Possibility

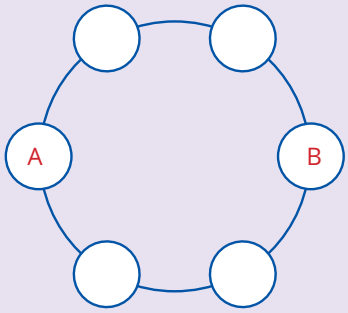
Let's draw a diagram showing how the children are standing.



Adam is opposite Bree.

Let's choose a position for Adam.

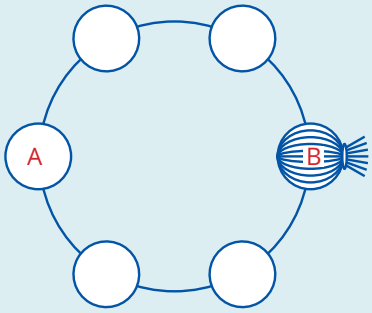
We will put Bree directly opposite.



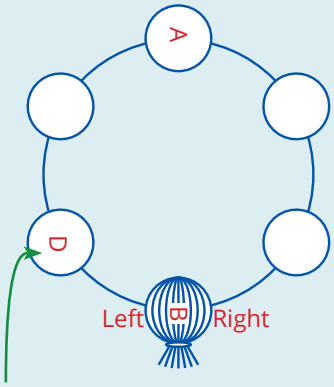

Duncan is holding Bree's left hand - but which side is Bree's left?

The children are facing the centre of the circle.

Let's draw Bree with a ponytail, so that we can imagine her facing the centre.



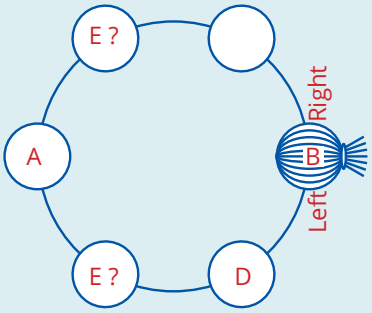

It is easier to find Bree's left hand if we rotate the picture so that we are in Bree's position.



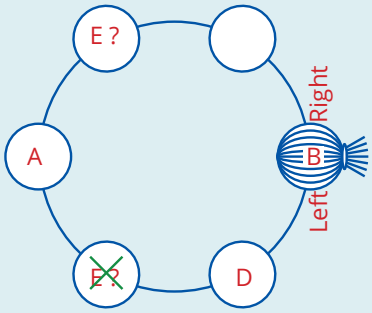

Now we can place Duncan, who is holding Bree's left hand.

Elena is holding hands with both Adam and Fahim.

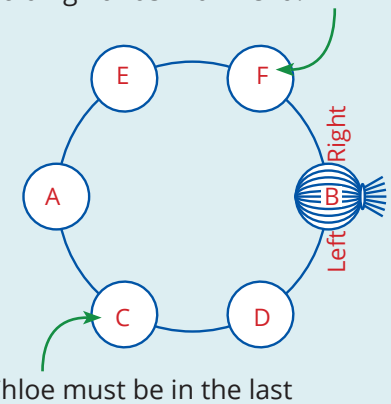
So Elena must be in one of the positions next to Adam.



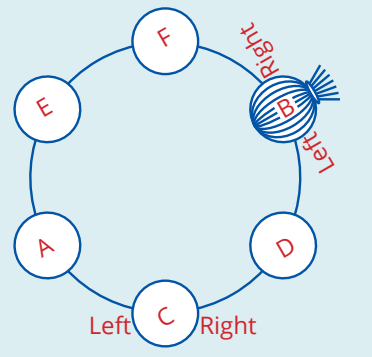

Since Fahim is on Elena's other side, Elena can't be next to Duncan.




Now we can place Fahim, who is holding hands with Elena.




Chloe must be in the last remaining position.




**Adam** is holding Chloe's left hand.

## Answers

**3.8 - Green:** Adam

**3.8 - Yellow:** Adam

**3.8 - Orange:** Amy: red, Dawn: blue, Soumiya: green

## Answers

| Set Green | |
|---|---|
| **3.1** | 180 |
| **3.2** | 30 |
| **3.3** | 6 |
| **3.4** | 45 |
| **3.5** | 6 |
| **3.6** | 3 |
| **3.7** | 15 |
| **3.8** | Adam |

| Set Yellow | |
|---|---|
| **3.1** | 180 |
| **3.2** | 80 |
| **3.3** | 6 |
| **3.4** | 45 |
| **3.5** | 10 |
| **3.6** | 7 |
| **3.7** | 1872 |
| **3.8** | Adam |

| Set Orange | |
|---|---|
| **3.1** | 40 |
| **3.2** | 12340 |
| **3.3** | 1, 2, 4 |
| **3.4** | 8640 |
| **3.5** | 10 |
| **3.6** | 11 |
| **3.7** | 9346 |
| **3.8** | Amy: red, Dawn: blue, Soumiya: green |