2024 Maths Games Senior - Years 7 & 8
Resource Kit 2
Teaching Problem Solving

MATHS GAMES

Problem Solving Strategies

This resource kit focuses on the following problem solving strategies:

1. Work Backwards

If a problem describes a procedure and then specifies the final result, this method usually makes the problem much easier to solve.

2. Make an Organised List

Listing every possibility in an organised way is an important tool.

How students organise the data often reveals additional information.

It follows on from strategies introduced in the preparation resource kit and resource kit 1:

Guess, Check and Refine

Draw a Diagram

Find a Pattern

Build a Table

Resource Kit 2 focuses on:

Work Backwards

Make an Organised List

Set Yellow

Example problems for which full worked solutions are included.

Set Green

Problems that are designed to be similar to Set Yellow, but with fewer difficult elements.

Set Orange

Problems that are similar in mathematical structure to the corresponding Yellow problems.

Further questions and solution methods can be found in the APSMO resource book "Building Confidence in Maths Problem Solving", available from www.apsmo.edu.au.

How to use these problems

At the start of the lesson, present the problem and ask the students to think about it. Encourage students to try to solve it in any way they like. When the students have had enough time to consider their solutions, ask them to describe or present their methods, taking particular note of different ways of arriving at the same solution.

Each question includes at least one solution method that the majority of students should be able to follow. By participating in lessons that demonstrate achievable problem solving techniques, students may gain increased confidence in their own ability to address unfamiliar problems.

Finally, the consideration of different solution methods is fundamental to the students' development as effective and sophisticated problem solvers. Even when students have solved a problem to their own satisfaction, it is important to expose them to other methods and encourage them to judge whether or not the other methods are more efficient.

Preparation Kit

Guess, Check and Refine

This involves making a reasonable guess of the answer, and checking it against the conditions of the problem. An incorrect guess may provide more information that may lead to the answer.

Draw a Diagram

A diagram may reveal information that may not be obvious just by reading the problem.

It is also useful for keeping track of where the student is up to in a multi-step problem.

Resource Kit 1

Find a Pattern

A frequently used problem solving strategy is that of recognising and extending a pattern.

Students can often simplify a difficult problem by identifying a pattern in the problem.

Build a Table

A table displays information so that it is easily located and understood.

A table is an excellent way to record data so the student doesn't have to repeat their efforts.

Resource Kit 2

Work Backwards

If a problem describes a procedure and then specifies the final result, this method usually makes the problem much easier to solve.

Make an Organised List

Listing every possibility in an organised way is an important tool.

How students organise the data often reveals additional information.

Resource Kit 3

Solve a Simpler Related Problem

Many hard problems are actually simpler problems that have been extended to larger numbers.

Patterns can sometimes be identified by trying the problem with smaller numbers.

Eliminate All But One Possibility

Deciding what a quantity is not, can narrow the field to a very small number of possibilities.

These can then be tested against the conditions of the original problem.

Resource Kit 4

Convert to a More Convenient Form

There are times when changing some of the conditions of a problem makes a solution clearer or more convenient.

Divide a Complex Shape

Sometimes it is possible to divide an unusual shape into two or more common shapes that are easier to work with.

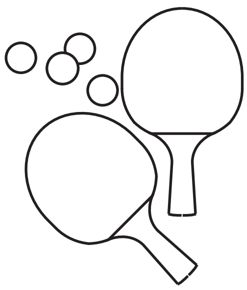
Set Yellow

2.1) The school band has woodwind, brass, and percussion instruments only.
20 students play woodwind instruments.
28 students play brass instruments.
One fifth of the students play percussion instruments.
How many students are there in the school band?

2.2) Find the largest factor of 100 that is not divisible by 5.

2.3) Each student's final mark was calculated by averaging marks for 6 tests.
Holly, who had an average of 12 for the six tests, had not been well for the last test.
The teacher decided to give her a final mark that was an average of the 5 tests she sat when she was well.
Holly ended up with a final mark of 14.
What was the mark that Holly had received for the last test?

2.4) Noah has some pairs of table tennis bats (racquets, or paddles).
Each pair came with 4 table tennis balls.
Noah lost 8 of the balls.
Then, he gave half of the remaining balls to his friend.
Noah has 6 balls left.
How many bats does he have?



Set Yellow

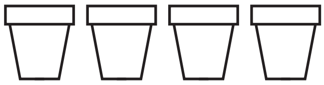
2.5) A loop of string was stretched to form a regular pentagon.

The same loop of string was then stretched to form a square.

The length of each side of the square was 20 cm longer than the length of each side of the regular pentagon.

What was the length of the loop of string, in centimetres?

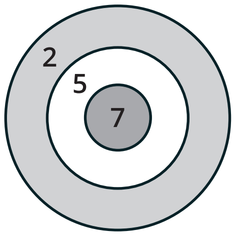
2.6) Don has four pot plants which he arranges in a straight line.



One pot has herbs, one pot has tomatoes, one pot has daisies, and one pot has bamboo.

In how many different ways can he arrange his four plants?

2.7) The three regions of a target are assigned point values of 7, 5, and 2.



Three darts are thrown and each lands somewhere on this target.

How many different totals are possible?

2.8) The sum of the digits of the number 789 is 24.

How many 3-digit numbers have the sum of their digits equal to 24 including 789?

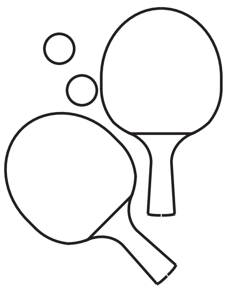
Set Green

2.1) The school band has woodwind and brass instruments only.
12 students play woodwind instruments.
One quarter of the students play brass instruments.
How many students are there in the school band?

2.2) Find the largest factor of 30 that is not divisible by 3.

2.3) Each student's final mark was calculated by averaging marks for 4 tests.
Holly, who had an average of 10 for the four tests, had not been well for the last test.
The teacher decided to give her a final mark that was an average of the 3 tests she sat when she was well.
Holly ended up with a final mark of 12.
What was the mark that Holly had received for the last test?

2.4) Noah has some pairs of table tennis bats (racquets, or paddles).
Each pair came with 2 table tennis balls.
Noah lost 5 of the balls.
Then, he gave two of the remaining balls to his friend.
Noah has 3 balls left.
How many bats does he have?



Set Green

2.5) A loop of string was stretched to form a regular triangle.

The same loop of string was then stretched to form a regular hexagon.

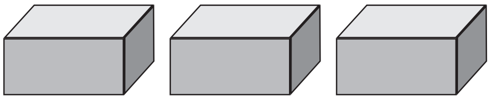
The length of each side of the triangle was one metre longer than the length of each side of the regular hexagon.

What was the length of the loop of string, in metres?

2.6) Lara has three coloured bricks which she arranges in a row.

One brick is green, one is blue, and one is red.

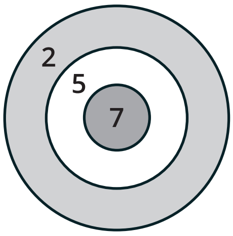
In how many different ways can she arrange her three bricks?



2.7) The three regions of a target are assigned point values of 7, 5, and 2.

Two darts are thrown and each lands somewhere on this target.

How many different totals are possible?



2.8) The sum of the digits of the number 210 is 3.

How many 3-digit numbers have the sum of their digits equal to 3 including 210?

Note that a number must start with a non-zero digit, so for example 012 is not a 3-digit number.

Set Orange

2.1) At the Ocean Club, everybody likes windsurfing or parasailing or both.
Three-fifths of those who like windsurfing also like parasailing.
One-quarter of those who like parasailing also like windsurfing.
Find, in simplest form, the fraction of Ocean Club members who like only windsurfing.

2.2) How many whole numbers are less than 3^4 but greater than 4^3 ?

2.3) In a bowling competition, Jason's average score was 223 per match for his first 12 matches.
How much must Jason average for his next 2 matches in order to achieve an average score of 225 for the entire 14 match competition?

2.4) Ms Williams spent $\frac{3}{5}$ of her money on clothing.
She then spent $\frac{2}{5}$ of her remaining money at the gas station.
Lastly, she spent $\frac{1}{5}$ of the remaining money on hamburgers.
She ended the day with $48.
How much money did Ms Williams have at the start of the day?

Set Orange

2.5) A given cube has a volume of $125\,cm^3$.

A rectangular prism is constructed such that, when compared to the original cube, the height is doubled, the width is reduced by 3 cm, and the depth is increased by 1 cm.

Determine the number of cubic centimetres in the volume of the newly constructed prism.

2.6) Mark wants to create a password for his computer.

He wants it to consist of one letter chosen from the word *MARK*, one even-numbered digit, and one odd-numbered digit.

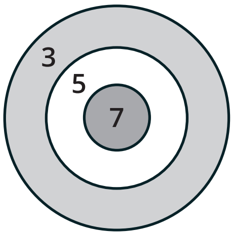
These three characters can appear in any order.

How many different passwords could Mark create?

2.7) The three regions of a target are assigned point values of 7, 5, and 3.

Ten darts are thrown and each lands somewhere on this target.

In how many different ways can the ten darts score a total of 45 points?



2.8) Lee, Mel, Nate and Olivia have each saved exactly $25 in $5, $10 and/or $20 notes.

They find that each of them has a different number of notes.

All together, how many $5 notes do they have?

Maths Games – Example Problem 2.1

Example Problem 2.1 - Green

The school band has woodwind and brass instruments only.

12 students play woodwind instruments.

One quarter of the students play brass instruments.

How many students are there in the school band?

Example Problem 2.1 - Yellow

The school band has woodwind, brass, and percussion instruments only.

20 students play woodwind instruments.

28 students play brass instruments.

One fifth of the students play percussion instruments.

How many students are there in the school band?

Example Problem 2.1 - Orange

At the Ocean Club, everybody likes windsurfing or parasailing or both.

Three-fifths of those who like windsurfing also like parasailing.

One-quarter of those who like parasailing also like windsurfing.

Find, in simplest form, the fraction of Ocean Club members who like only windsurfing.

Maths Games Example Solution 2.1 - Yellow

The school band has woodwind, brass, and percussion instruments only. 20 students play woodwind instruments. 28 students play brass instruments. One fifth of the students play percussion instruments.

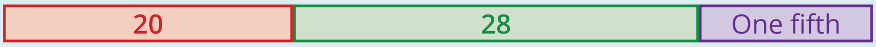
How many students are there in the school band?

Strategy 1: Work Backwards (1)

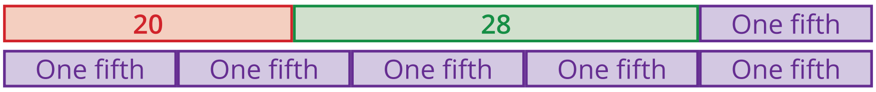
In the school band, 20 students play woodwind instruments.

28 students play brass instruments.

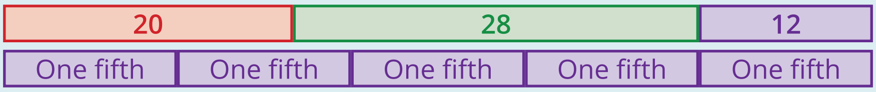
One-fifth of the students play percussion instruments.



If one-fifth of the students play percussion instruments, then four-fifths of the students must play woodwind or brass.



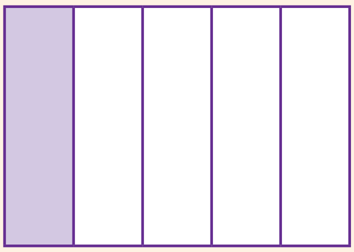
Four-fifths of the students is equivalent to 20 + 28 = 48 students, so one-fifth must be 48 ÷ 4 = 12 students.



There are 20 + 28 + 12 = **60** students in the band.

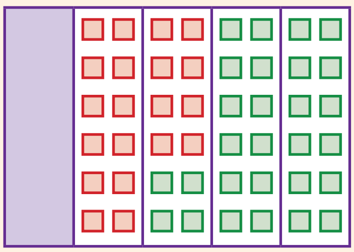
Strategy 2: Work Backwards (2)

One-fifth of the students play percussion instruments.

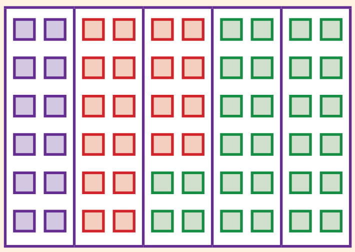


With **20** woodwind and **28** brass, there are 20 + 28 = **48** students who do not play percussion instruments.

These **48** students comprise four-fifths of the band.



With **48** students in four-fifths of the band, one-fifth of the band would be **48** ÷ **4** = **12** students.



One-fifth of the band is **12** students.

Five-fifths of the band (the whole band) is **5** × **12** = **60** students.

There are **60** students in the band.

Answers

2.1 - Green: 16

2.1 - Orange: $\frac{1}{7}$

2.1 - Yellow: 60

Maths Games – Example Problem 2.2

Example Problem 2.2 - Green

Find the largest factor of 30 that is not divisible by 3.

Example Problem 2.2 - Yellow

Find the largest factor of 100 that is not divisible by 5.

Example Problem 2.2 - Orange

How many whole numbers are less than 3^4 but greater than 4^3 ?

Maths Games Example Solution 2.2 - Yellow

Find the largest factor of 100 that is not divisible by 5.

Strategy 1: Make an Organised List

To list the factors of **100** in an organised way, we can consider every whole number, beginning with **1**.

If the number divides evenly into **100** with no remainder, then both the number, and its factor pair, will be factors of **100**.

We can stop as soon as we find a factor pair where the second number is less than or equal to the number we are testing.

(Why is this?)

Number	Factor pair for 100	Factors found so far
1	100 ÷ 1 = 100	1, 100
2	100 ÷ 2 = 50	1, 2, 50, 100
3	100 ÷ 3 = 33 r. 1	*3 is not a factor*
4	100 ÷ 4 = 25	1, 2, 4, 25, 50, 100
5	100 ÷ 5 = 20	1, 2, 4, 5, 20, 25, 50, 100
6	100 ÷ 6 = 16 r. 4	*6 is not a factor*
7	100 ÷ 7 = 14 r. 2	*7 is not a factor*
8	100 ÷ 8 = 12 r. 4	*8 is not a factor*
9	100 ÷ 9 = 11 r. 1	*9 is not a factor*
10	100 ÷ 10 = 10	1, 2, 4, 5, 10, 20, 25, 50, 100

We can see that the factors of **100** are **1**, **2**, **4**, **5**, **10**, **20**, **25**, **50**, and **100**.

We know that numbers that end in a **5** or a **0**, will be divisible by **5**. (Why is this?)

Therefore, the largest factor of **100** that is not divisible by **5**, is **4**.

Strategy 2: Use Number Sense

To find factors of **100** that are not divisible by **5**, we can just keep dividing by **5**, as long as the result has no remainder.

100 ÷ 5 = 20

20 ÷ 5 = 4

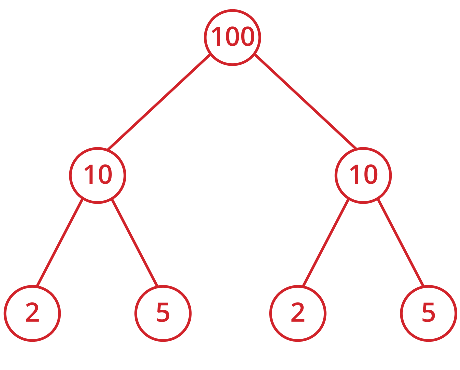
Since **4** is not divisible by **5**, we can see that **4** must be the largest factor of **100** that is not divisible by **5**.

Strategy 3: Consider Prime Factors

Apart from **1**, all of the factors of a number can be expressed as products of its prime factors.

We can use a factor tree to find the prime factors of **100**.

It involves breaking **100** into factor pairs, and then breaking each of those factors into factor pairs, until every resulting factor is prime.



We can see that **100** can be expressed as a product of the following prime factors:

100 = 2 × 5 × 2 × 5

= 2 × 2 × 5 × 5

Therefore, the largest factor of **100** that is not divisible by **5** is **2 × 2 = 4**.

Answers

2.2 - Green: 10

2.2 - Orange: 16

2.2 - Yellow: 4

Maths Games – Example Problem 2.3

Example Problem 2.3 - Green

Each student's final mark was calculated by averaging marks for 4 tests.

Holly, who had an average of 10 for the four tests, had not been well for the last test.

The teacher decided to give her a final mark that was an average of the 3 tests she sat when she was well.

Holly ended up with a final mark of 12.

What was the mark that Holly had received for the last test?

Example Problem 2.3 - Yellow

Each student's final mark was calculated by averaging marks for 6 tests.

Holly, who had an average of 12 for the six tests, had not been well for the last test.

The teacher decided to give her a final mark that was an average of the 5 tests she sat when she was well.

Holly ended up with a final mark of 14.

What was the mark that Holly had received for the last test?

Example Problem 2.3 - Orange

In a bowling competition, Jason's average score was 223 per match for his first 12 matches.

How much must Jason average for his next 2 matches in order to achieve an average score of 225 for the entire 14 match competition?

Maths Games Example Solution 2.3 - Yellow

Each student's final mark was calculated by averaging marks for 6 tests. Holly, who had an average of 12 for the six tests, had not been well for the last test. The teacher decided to give her a final mark that was an average of the 5 tests she sat when she was well. Holly ended up with a final mark of 14.

What was the mark that Holly had received for the last test?

Strategy 1: Work Backwards (1)

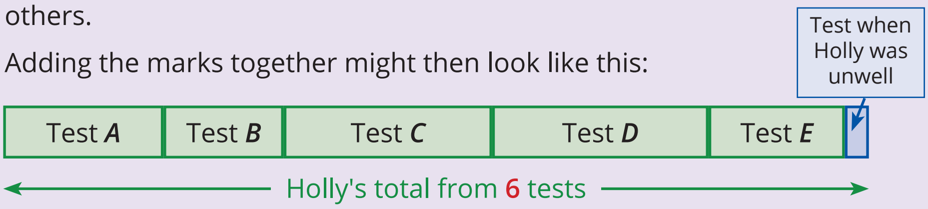
Holly had an average mark of 12 from 6 tests.

To work out Holly's average, her teacher would have:

- Added together all of Holly's marks, and
- Divided the total by 6.

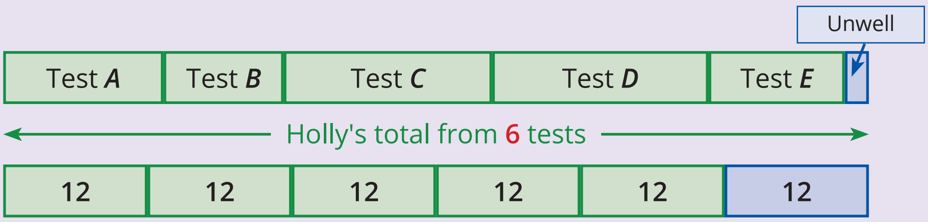
Without knowing exactly what Holly's marks were, we can imagine that Holly might have achieved higher marks in some tests, and lower marks in others.

Adding the marks together might then look like this:



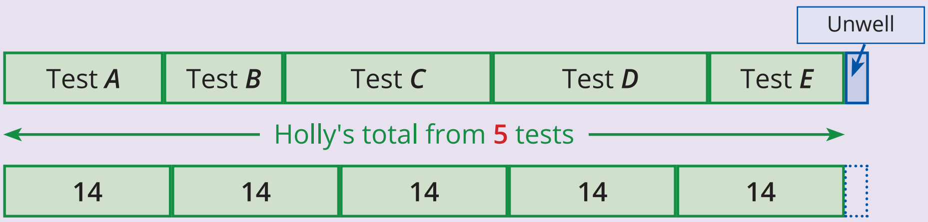
Her average mark of 12 was then calculated by dividing this total by the number of tests.

Holly's total from 6 tests must have been 6 × 12 = 72.



If the teacher does not count the test Holly sat when she was unwell, Holly's average becomes 14.

Holly's total from 5 tests must have been 5 × 14 = 70.

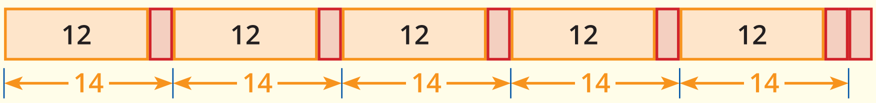


The mark that was not included must have been **72 – 70 = 2**.

Strategy 2: Work Backwards (2)

With an average mark of 12 from 6 tests, we can imagine redistributing Holly's marks for the tests so that each test ends up with 12 marks.

After deciding not to count one test, Holly's average increased by 2, to 14.



We can imagine breaking up the 12 marks we had distributed to that sixth test, and reallocating 2 marks to each of the other 5 tests.

We have reallocated 5 × 2 = 10 marks out of the 12 marks from that sixth test.

The sixth test, which Holly's teacher is not counting, must have contributed **12 – 10 = 2** marks.

Answers

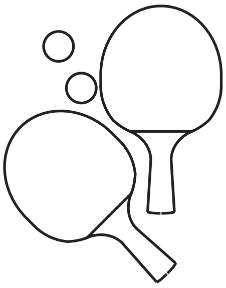
2.3 - Green: 4

2.3 - Orange: 237

2.3 - Yellow: 2

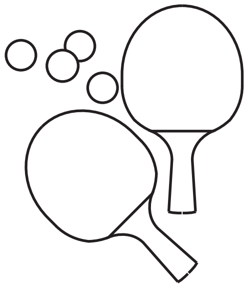
Maths Games – Example Problem 2.4

Example Problem 2.4 - Green



Noah has some pairs of table tennis bats (racquets, or paddles).
Each pair came with 2 table tennis balls.
Noah lost 5 of the balls.
Then, he gave two of the remaining balls to his friend.
Noah has 3 balls left.
How many bats does he have?

Example Problem 2.4 - Yellow

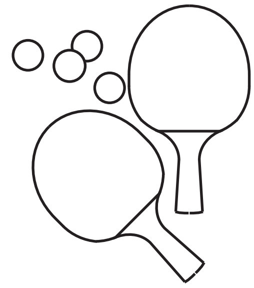


Noah has some pairs of table tennis bats (racquets, or paddles).
Each pair came with 4 table tennis balls.
Noah lost 8 of the balls.
Then, he gave half of the remaining balls to his friend.
Noah has 6 balls left.
How many bats does he have?

Example Problem 2.4 - Orange

Ms Williams spent $\frac{3}{5}$ of her money on clothing.
She then spent $\frac{2}{5}$ of her remaining money at the gas station.
Lastly, she spent $\frac{1}{5}$ of the remaining money on hamburgers.
She ended the day with $48.
How much money did Ms Williams have at the start of the day?

Maths Games Example Solution 2.4 - Yellow



Noah has some pairs of table tennis bats (racquets, or paddles). Each pair came with 4 table tennis balls. Noah lost 8 of the balls. Then, he gave half of the remaining balls to his friend. Noah has 6 balls left.

How many bats does he have?

Strategy 1: Work Backwards

Let's use a bar to represent the number of bats.

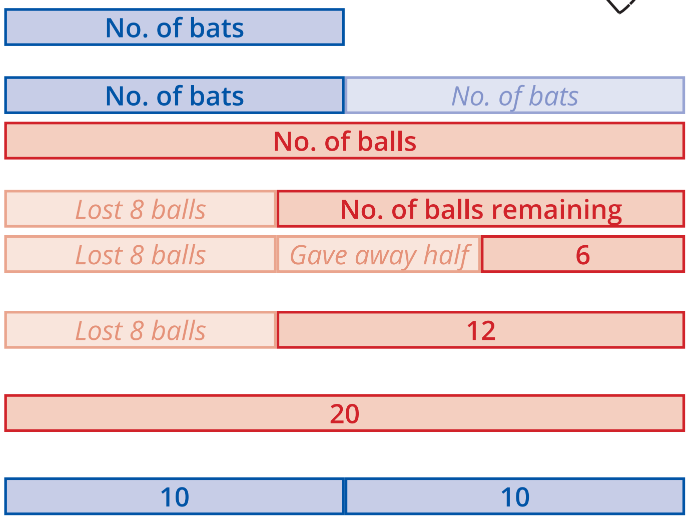
If each pair of bats comes with 4 balls, then it's the same as each bat coming with 2 balls.

After Noah lost 8 balls,
and gave away half of the remaining balls,
he had 6 balls left.

Before giving away half of the remaining balls, Noah must have had 2 × 6 = 12 balls.

Before losing 8 balls, Noah must have had 12 + 8 = 20 balls.

Therefore, since each bat comes with 2 balls, Noah must have 20 ÷ 2 = **10** bats.



Strategy 2: Build a Table, and Draw a Diagram

	No. of Bats	No. of Balls	After losing 8 balls	After giving away half
If Noah had just 1 pair of bats, he would have 4 table tennis balls. That's not enough for him to have lost 8 balls.				
With 3 pairs of bats, he'd have 3 × 4 = 12 balls. After losing 8 balls he would have 12 – 8 = 4 balls. After giving away half of the remaining balls, he would have 4 ÷ 2 = 2 balls left.				
With 4 pairs of bats, he'd have 4 × 4 = 16 balls. After losing 8 balls he would have 16 – 8 = 8 balls. After giving away half of the remaining balls, he would have 8 ÷ 2 = 4 balls left. This is closer to the target of having 6 balls left.				
With 5 pairs of bats, he'd have 5 × 4 = 20 balls. After losing 8 balls he would have 20 – 8 = 12 balls. After giving away half of the remaining balls, he would have 12 ÷ 2 = 6 balls left. That matches the question.				

So from the diagram in the table, we can see that Noah has **10** table tennis bats.

Answers

2.4 - Green: 10

2.4 - Orange: $250

2.4 - Yellow: 10

Maths Games – Example Problem 2.5

Example Problem 2.5 - Green

A loop of string was stretched to form a regular triangle.

The same loop of string was then stretched to form a regular hexagon.

The length of each side of the triangle was one metre longer than the length of each side of the regular hexagon.

What was the length of the loop of string, in metres?

Example Problem 2.5 - Yellow

A loop of string was stretched to form a regular pentagon.

The same loop of string was then stretched to form a square.

The length of each side of the square was 20 cm longer than the length of each side of the regular pentagon.

What was the length of the loop of string, in centimetres?

Example Problem 2.5 - Orange

A given cube has a volume of 125 cm^3.

A rectangular prism is constructed such that, when compared to the original cube, the height is doubled, the width is reduced by 3 cm, and the depth is increased by 1 cm.

Determine the number of cubic centimetres in the volume of the newly constructed prism.

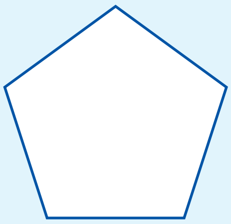
Maths Games Example Solution 2.5 - Yellow

A loop of string was stretched to form a regular pentagon. The same loop of string was then stretched to form a square. The length of each side of the square was 20 cm longer than the length of each side of the regular pentagon.

What was the length of the loop of string, in centimetres?

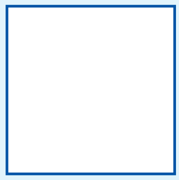
Strategy 1: Work Backwards

The loop of string was initially stretched to form a regular pentagon.

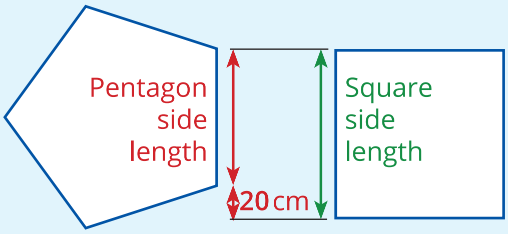


The same loop was then stretched to form a square.

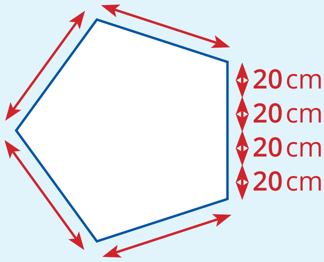
The perimeter of the square must therefore be the same as the perimeter of the pentagon.



The length of each side of the square was **20** cm longer than the length of each side of the regular pentagon.



Since each side of the pentagon is **20** cm shorter than each side of the square, the extra **20** cm for each side of the square must have come from the **5**th side of the pentagon.



One side of the pentagon would have been **4** × **20** cm = **80** cm long.

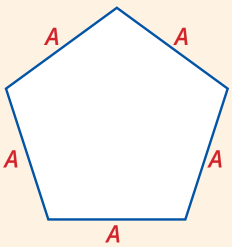
This means that the perimeter of the pentagon must have been **5** × **80** cm = **400** cm.

Therefore, the loop of string was **400** cm long.

Strategy 1: Work Backwards

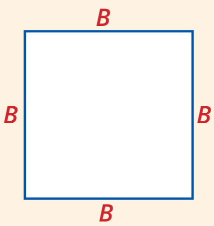
Let *A* represent the side length of the pentagon.

The perimeter of the pentagon would be **5***A*.



Let *B* represent the side length of the square.

The perimeter of the square would be **4***B*.



Since the pentagon and the square are both made from the same loop of string, we know that **4***B* = **5***A*.

The length of each side of the square was **20** cm longer than each side of the regular pentagon, so *B* = *A* + **20**.

We now have:	$4B = 5A$
Since $B = A + 20$,	$4(A + 20) = 5A$
	$(A + 20) + (A + 20) + (A + 20) + (A + 20) = 5A$
	$4A + 80 = 5A$
Subtracting **4***A* from both sides:	$80 = A$

Since the side length of the pentagon *A* = **80**, the perimeter of the pentagon would be **5***A* = **5** × **80** = **400**.

Therefore the loop of string was **400** cm long.

Answers

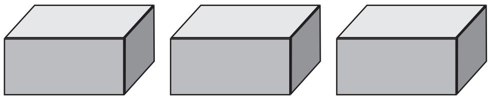
2.5 - Green: 6 m

2.5 - Orange: 120 cm^3

2.5 - Yellow: 400 cm

Maths Games – Example Problem 2.6

Example Problem 2.6 - Green

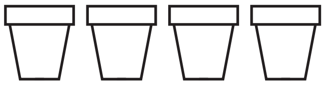


Lara has three coloured bricks which she arranges in a row.

One brick is green, one is blue, and one is red.

In how many different ways can she arrange her three bricks?

Example Problem 2.6 - Yellow



Don has four pot plants which he arranges in a straight line.

One pot has herbs, one pot has tomatoes, one pot has daisies, and one pot has bamboo.

In how many different ways can he arrange his four plants?

Example Problem 2.6 - Orange

Mark wants to create a password for his computer.

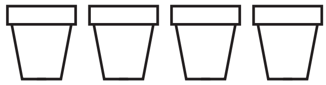
He wants it to consist of one letter chosen from the word *MARK*, one even-numbered digit, and one odd-numbered digit.

These three characters can appear in any order.

How many different passwords could Mark create?

Maths Games Example Solution 2.6 - Yellow

Don has four pot plants which he arranges in a straight line. One pot has herbs, one pot has tomatoes, one pot has daisies, and one pot has bamboo.



In how many different ways can he arrange his four plants?

Strategy 1: Make an Organised List

Don has four pot plants. There is one each of herbs, tomatoes, daisies and bamboo.

Let's begin by listing them alphabetically. →

We'll start with all of the options where the left-most pot is bamboo.

How many possibilities would there be if the daisies came next?

How many possibilities would there be if the herbs came next?

How many possibilities would there be if the tomatoes came next?

Every time we change the first pot plant, we'll get the same number of different possibilities.

There are **4** different ways to select the plant for the first (left-most) position.

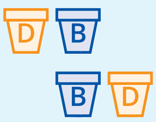
For each possible plant in the first position, there are **3** possible plants that can be placed in the second position, followed by **2** possible plants for the 3rd position, and **1** remaining plant for the fourth position.

All together, there are **4 × 3 × 2 × 1 = 24** ways to arrange Don's pot plants.

Strategy 2: Count in an Organised Way

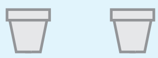
Let's put the bamboo down first.

We can now put the daisies either to the left or to the right of the bamboo.

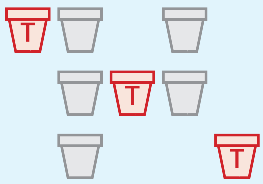


There are **2** possible positions for the daisies, relative to the bamboo.

So far, we have placed bamboo and daisies in some order.

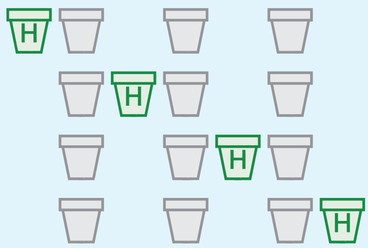


We can now put the tomatoes in one of **3** different positions relative to the plants that are already there.



With **2** ways to arrange the bamboo and daisies, and **3** ways to arrange the tomatoes around them, there are **2 × 3 = 6** different arrangements for these three plants.

The herbs can now be placed in one of **4** different positions relative to the other plants.



Therefore there are **2 × 3 × 4 = 24** ways to arrange Don's pot plants.

Answers

2.6 - Green: 6

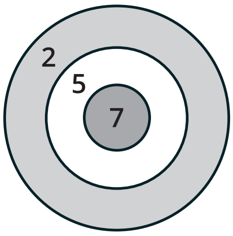
2.6 - Orange: 600

2.6 - Yellow: 24

Maths Games – Example Problem 2.7

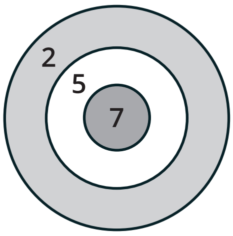
Example Problem 2.7 - Green

The three regions of a target are assigned point values of 7, 5, and 2.
Two darts are thrown and each lands somewhere on this target.
How many different totals are possible?



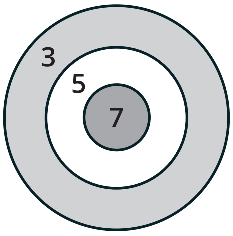
Example Problem 2.7 - Yellow

The three regions of a target are assigned point values of 7, 5, and 2.
Three darts are thrown and each lands somewhere on this target.
How many different totals are possible?

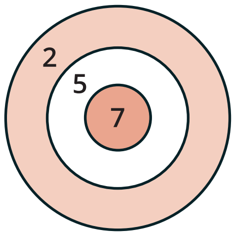


Example Problem 2.7 - Orange

The three regions of a target are assigned point values of 7, 5, and 3.
Ten darts are thrown and each lands somewhere on this target.
In how many different ways can the ten darts score a total of 45 points?



Maths Games Example Solution 2.7 - Yellow



The three regions of a target are assigned point values of 7, 5, and 2.

Three darts are thrown and each lands somewhere on this target.

How many different totals are possible?

Strategy 1: Make an Organised List (1)

Let's list the possible totals in an organised way.

Possible totals when the first score is **2**:

2+2+2 = 6	2+2+5 = 9	2+2+7 = 11
2+5+2 = 9	2+5+5 = 12	2+5+7 = 14
2+7+2 = 11	2+7+5 = 14	2+7+7 = 16

Possible totals when the first score is **5**:

5+2+2 = 9	5+2+5 = 12	5+2+7 = 14
5+5+2 = 12	5+5+5 = 15	5+5+7 = 17
5+7+2 = 14	5+7+5 = 17	5+7+7 = 19

Possible totals when the first score is **7**:

7+2+2 = 11	7+2+5 = 14	7+2+7 = 16
7+5+2 = 14	7+5+5 = 17	7+5+7 = 19
7+7+2 = 16	7+7+5 = 19	7+7+7 = 21

Now, we can find all of the different totals.

2+2+2 = (6)	2+2+5 = (9)	2+2+7 = (11)
2+5+2 = 9	2+5+5 = (12)	2+5+7 = (14)
2+7+2 = 11	2+7+5 = 14	2+7+7 = (16)

5+2+2 = 9	5+2+5 = 12	5+2+7 = 14
5+5+2 = 12	5+5+5 = (15)	5+5+7 = (17)
5+7+2 = 14	5+7+5 = 17	5+7+7 = (19)

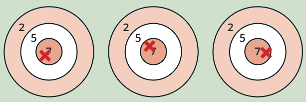
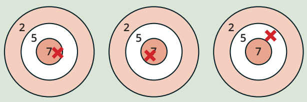
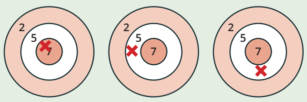
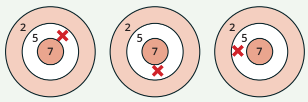
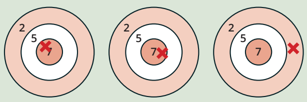
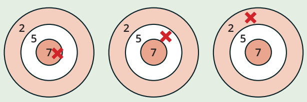
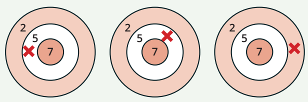
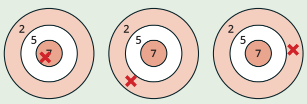
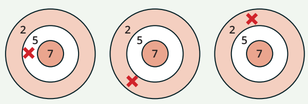
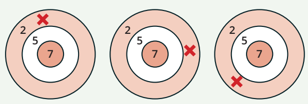
7+2+2 = 11	7+2+5 = 14	7+2+7 = 16
7+5+2 = 14	7+5+5 = 17	7+5+7 = 19
7+7+2 = 16	7+7+5 = 19	7+7+7 = (21)

Therefore there are **10** possible different totals.

Strategy 2: Make an Organised List (2)

We are only interested in the totals, not in the order of the darts hitting the target.

Let's list the totals in order from greatest to least.

3 darts hit bullseye	2 darts hit bullseye	1 dart hits bullseye	0 darts hit bullseye
 7+7+7 = 21	 7+7+5 = 19	 7+5+5 = 17	 5+5+5 = 15
	 7+7+2 = 16	 7+5+2 = 14	 5+5+2 = 12
		 7+2+2 = 11	 5+2+2 = 9
			 2+2+2 = 6

We can see that all of the totals are different.

Therefore there are **10** possible different totals.

Answers

2.7 - Green: 6

2.7 - Yellow: 10

2.7 - Orange: 0 (since the sum of 10 odd numbers will always be even)

Maths Games – Example Problem 2.8

Example Problem 2.8 - Green

The sum of the digits of the number 210 is 3.

How many 3-digit numbers have the sum of their digits equal to 3 including 210?

Note that a number must start with a non-zero digit, so for example 012 is not a 3-digit number.

Example Problem 2.8 - Yellow

The sum of the digits of the number 789 is 24.

How many 3-digit numbers have the sum of their digits equal to 24 including 789?

Example Problem 2.8 - Orange

Lee, Mel, Nate and Olivia have each saved exactly $25 in $5, $10 and/or $20 notes.

They find that each of them has a different number of notes.

All together, how many $5 notes do they have?

Maths Games Example Solution 2.8 - Yellow

The sum of the digits of the number 789 is 24.

How many 3-digit numbers have the sum of their digits equal to 24 including 789?

Strategy 1: Make an Organised List

Suppose we have 1 in the hundreds place.
Then the sum of the digits in the tens place and ones place must equal **24 – 1 = 23**.

This is impossible - the greatest possible sum for those two digits would be 9 + 9 = **18**.
Therefore the smallest possible value for the hundreds place would be **24 – 9 – 9 = 6**.

Let's construct numbers that have 6 or greater in the hundreds place, and a digit sum of **24**.

With 6 in the hundreds place, there is 1 way to combine two digits to total 24 – 6 = 18.

H	T	O
6	0	18
:	:	:
6	8	10
6	9	9

With 7 in the hundreds place, there are 2 ways to combine two digits to total 24 – 7 = 17.

H	T	O
7	0	17
:	:	:
7	7	10
7	8	9
7	9	8

With 8 in the hundreds place, there are 3 ways to combine two digits to total 24 – 8 = 16.

H	T	O
8	0	16
:	:	:
8	6	10
8	7	9
8	8	8
8	9	7

With 9 in the hundreds place, there are 4 ways to combine two digits to total 24 – 9 = 15.

H	T	O
9	0	15
:	:	:
9	5	10
9	6	9
9	7	8
9	8	7
9	9	6

Therefore there are **1 + 2 + 3 + 4 = 10** different three-digit numbers with a digit sum of **24**.

Strategy 2: Make an Organised List (Alternative Approach)

Since the maximum sum for two digits is **18**, the smallest digit this number can have is **24 – 18 = 6**.

So we only need to consider the digits **6**, **7**, **8**, and **9**.

There are **10** different three-digit numbers with a digit sum of **24**.

H	T	O
6	6	12
6	7	11
6	8	10
6	9	9

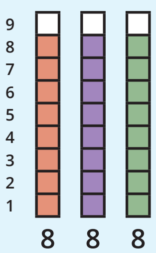
H	T	O
7	6	11
7	7	10
7	8	9
7	9	8

H	T	O
8	6	10
8	7	9
8	8	8
8	9	7

H	T	O
9	6	9
9	7	8
9	8	7
9	9	6

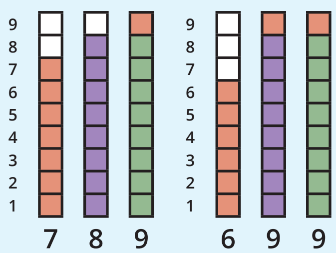
Strategy 3: Draw a Diagram

Since **24 ÷ 3 = 8**, one possible digit combination is **8**, **8**, **8**.

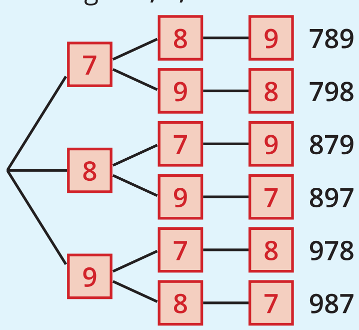


Any other combinations must include a **9**.

So the only other possible digit combinations are **7**, **8**, **9** and **6**, **9**, **9**.

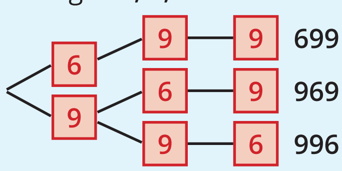


The digits **7**, **8**, **9** can create:



6 three-digit numbers can be constructed using the digits **7**, **8**, **9**.

The digits **6**, **9**, **9** can create:



3 three-digit numbers can be constructed using the digits **6**, **9**, **9**.

Finally, only 1 three-digit number (**888**) can be constructed using the digits **8**, **8**, **8**.

Therefore there are **6 + 3 + 1 = 10** different three-digit numbers with a digit sum of **24**.

Answers

2.8 - Green: 6

2.8 - Orange: 10

2.8 - Yellow: 10

Answers

Set Green

2.1	16
2.2	10
2.3	4
2.4	10
2.5	6 m
2.6	6
2.7	6
2.8	6

Set Yellow

2.1	60
2.2	4
2.3	2
2.4	10
2.5	400 cm
2.6	24
2.7	10
2.8	10

Set Orange

2.1	$\frac{1}{7}$
2.2	16
2.3	237
2.4	$250
2.5	120 cm^3
2.6	600
2.7	0
2.8	10