2024 Maths Games Senior - Years 7 & 8
Resource Kit 1
Teaching Problem Solving

MATHS GAMES

Problem Solving Strategies

This resource kit follows on from the Preparation Kit and its emphasis on:

Guess, Check and Refine

Draw a Diagram

The problems are sourced from previous Junior (Division J) Maths Olympiads and Maths Games papers.

They introduce two new problem solving strategies:

1. Find a Pattern

One of the most frequently used problem solving strategies is that of recognising and extending a pattern.

Students can often simplify a difficult problem by identifying a pattern in it, and then applying that pattern to the problem situation.

2. Build a Table

A table displays information so that it is easily located and understood, and missing information becomes obvious.

If students are not given the data for a problem, and must generate it themselves, a table is an excellent way to record what they have done so they don't have to repeat their efforts.

A table can also be invaluable for detecting significant patterns.

Resource Kit 1 focuses on:

Find a Pattern

Build a Table

Set Yellow

Example problems for which full worked solutions are included.

Set Green

Problems that are designed to be similar to Set Yellow, but with fewer difficult elements.

Set Orange

Problems that are similar in mathematical structure to the corresponding Yellow problems.

Further questions and solution methods can be found in the APSMO resource book "Building Confidence in Maths Problem Solving", available from www.apsmo.edu.au.

How to use these problems

At the start of the lesson, present the problem and ask the students to think about it. Encourage students to try to solve it in any way they like. When the students have had enough time to consider their solutions, ask them to describe or present their methods, taking particular note of different ways of arriving at the same solution.

Each question includes at least one solution method that the majority of students should be able to follow. By participating in lessons that demonstrate achievable problem solving techniques, students may gain increased confidence in their own ability to address unfamiliar problems.

Finally, the consideration of different solution methods is fundamental to the students' development as effective and sophisticated problem solvers. Even when students have solved a problem to their own satisfaction, it is important to expose them to other methods and encourage them to judge whether or not the other methods are more efficient.

Preparation Kit

Guess, Check and Refine

This involves making a reasonable guess of the answer, and checking it against the conditions of the problem. An incorrect guess may provide more information that may lead to the answer.

Draw a Diagram

A diagram may reveal information that may not be obvious just by reading the problem.

It is also useful for keeping track of where the student is up to in a multi-step problem.

Resource Kit 1

Find a Pattern

A frequently used problem solving strategy is that of recognising and extending a pattern.

Students can often simplify a difficult problem by identifying a pattern in the problem.

Build a Table

A table displays information so that it is easily located and understood.

A table is an excellent way to record data so the student doesn't have to repeat their efforts.

Resource Kit 2

Work Backwards

If a problem describes a procedure and then specifies the final result, this method usually makes the problem much easier to solve.

Make an Organised List

Listing every possibility in an organised way is an important tool.

How students organise the data often reveals additional information.

Resource Kit 3

Solve a Simpler Related Problem

Many hard problems are actually simpler problems that have been extended to larger numbers.

Patterns can sometimes be identified by trying the problem with smaller numbers.

Eliminate All But One Possibility

Deciding what a quantity is not, can narrow the field to a very small number of possibilities.

These can then be tested against the conditions of the original problem.

Resource Kit 4

Convert to a More Convenient Form

There are times when changing some of the conditions of a problem makes a solution clearer or more convenient.

Divide a Complex Shape

Sometimes it is possible to divide an unusual shape into two or more common shapes that are easier to work with.

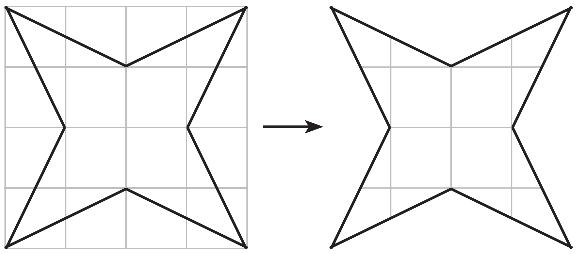
Set Yellow

1.1) Henry bought some 23c stamps and some 15c stamps for a total of exactly $2.50.

How many 15c stamps did he buy?

1.2) A four-pointed star is drawn on square grid paper and then cut out, as shown.

In how many ways can it be folded in half so that both halves overlap exactly?



1.3) A small suitcase has a capacity of 40 litres.

A large suitcase has a capacity of 45 litres.

10 people, each carrying either a small or a large suitcase, board a plane.

The total capacity of their suitcases is 410 litres.

How many of them are carrying a small suitcase?

1.4) Kayla has some marbles.

She groups them by threes and has one left over.

She groups them by sevens and has four left over.

Kayla has more than five marbles.

What is the smallest number of marbles Kayla could have?

Set Yellow

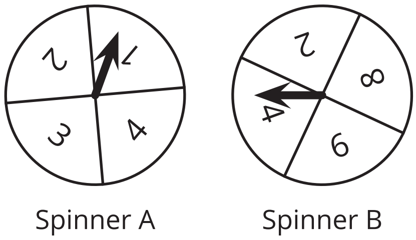
1.5) In a board game, a number is made by adding the number spun on Spinner A to the number spun on Spinner B.

Spinner A has the numbers 1, 2, 3 and 4 on it.

Spinner B has the numbers 2, 4, 6 and 8 on it.

In how many ways can you spin a total that is odd?

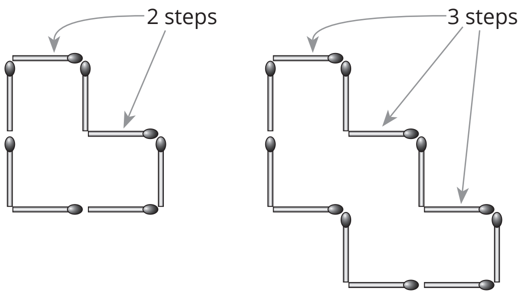
(We will consider 1+6 and 3+4 to be two different ways.)



1.6) I am using matchsticks to make a staircase pattern.

The diagram shows what the pattern looks like when I have 2 or 3 steps.

How many matchsticks do I need to make 5 steps?



1.7) Michelle's Number Recycling Machine obeys exactly two rules:

1. If an inserted number has exactly 1 digit, double the number.

2. If an inserted number has exactly 2 digits, compute the sum of the digits.

The first number Michelle inserts is 1.

Then every answer she gets is inserted back into the machine until fifty numbers are inserted.

What is the fiftieth number to be inserted?

1.8) Nat and Scott want to buy a particular jigsaw puzzle.

Nat needs another $18 before she can afford the puzzle on her own.

Scott needs another $10 before he can afford the puzzle on his own.

If they combine their money, they still cannot afford to buy the puzzle.

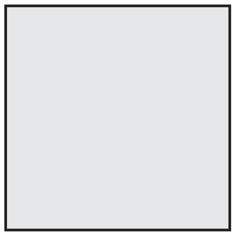
The puzzle costs a whole number of dollars.

What is the most that the puzzle could cost?

Set Green

1.1) Harry bought some 11c stamps and some 15c stamps for a total of exactly 85c.
How many 15c stamps did he buy?

1.2) In how many ways can a square be folded in half so that both halves overlap exactly?



1.3) A small suitcase has a capacity of 10 litres.
A large suitcase has a capacity of 15 litres.
5 people, each carrying either a small or a large suitcase, board a plane.
The total capacity of their suitcases is 65 litres.
How many of them are carrying a small suitcase?

1.4) Leila has some marbles.
She groups them by twos and has one left over.
She groups them by fives and has three left over.
Leila has more than three marbles.
What is the smallest number of marbles Leila could have?

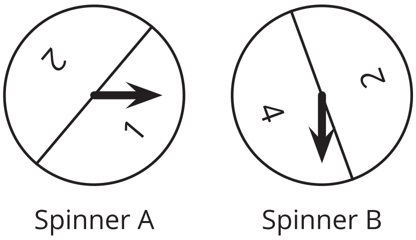
Set Green

1.5) In a board game, a number is made by adding the number spun on Spinner A to the number spun on Spinner B.

Spinner A has the numbers 1 and 2 on it.

Spinner B has the numbers 2 and 4 on it.

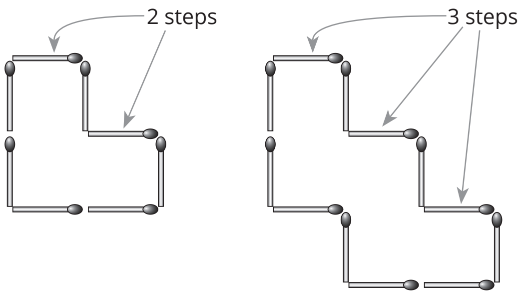
In how many ways can you spin a total that is odd?



1.6) I am using matchsticks to make a staircase pattern.

The diagram shows what the pattern looks like when I have 2 or 3 steps.

How many matchsticks do I need to make 4 steps?



1.7) Harry's Number Recycling Machine obeys exactly two rules:

If the inserted number has just 1 digit, double the number and print out the result.
Otherwise, if the inserted number has 2 digits, remove the tens digit and print out the result.

The first number Harry inserts is 2.

Then every answer he gets is inserted back into the machine until twenty numbers are inserted.

What is the twentieth number to be inserted?

1.8) Kat and Matt want to buy a particular board game.

Kat needs another $3 before she can afford the board game on her own.

Matt needs another $7 before he can afford the board game on his own.

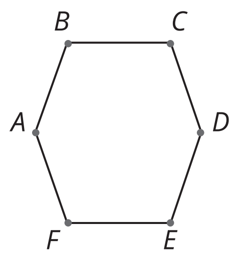
If they combine their money, they have just enough to afford the board game.

How much does the board game cost?

Set Orange

1.1) Jessie has $5.10 worth of stamps.
She has equal numbers of 50c, 20c, 10c and 5c stamps.
She has no other stamps.
How many 50c stamps does she have?

1.2) The figure shown is a hexagon with six sides of the same length.
Not all of the interior angles are the same size.
Angles *A* and *D* are the same size.
Angles *B*, *C*, *E*, and *F* are the same size.
How many lines of symmetry does the figure have?



1.3) Emily has 20 problems to solve.
She gets 3 marks for each one she solves and loses 5 marks for each one she does not solve.
She ended up with 28 marks.
How many of the 20 problems did Emily solve?

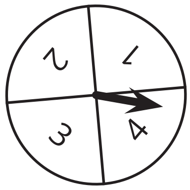
1.4) Taylor has some marbles.
She groups them by twos and has one left over.
She groups them by threes and has two left over.
She groups them by fives and has three left over.
What is the smallest number of marbles Taylor could have?

Set Orange

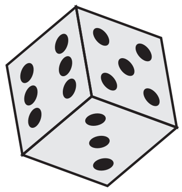
1.5) Laura spins Spinner A and rolls a standard die, and then adds the results together.

In how many ways can she get a total that is odd?

(We will consider spinning 1 on the spinner and rolling 4 on the die to be different from rolling 1 on the die and spinning 4 on the spinner.)



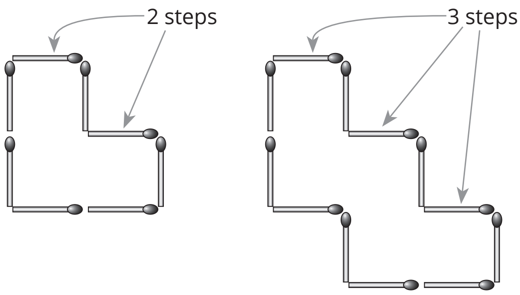
Spinner A



1.6) I am using matchsticks to make a staircase pattern.

The diagram shows what the pattern looks like when I have 2 or 3 steps.

How many matchsticks do I need to make 123 steps?



1.7) Harry is playing with Michelle's Number Recycling Machine, which obeys exactly two rules:

1. If an inserted number has exactly 1 digit, double the number.

2. If an inserted number has exactly 2 digits, compute the sum of the digits.

The first number Harry inserts is 3.

Then every answer he gets is inserted back into the machine until a hundred numbers are inserted.

What is the one-hundredth number to be inserted?

1.8) Mara and Tara each have some pencils.

If Mara gives four pencils to Tara, they will have the same number of pencils.

If Tara gives two pencils to Mara, Mara will have 3 times as many pencils as Tara.

How many pencils does Mara have?

Maths Games – Example Problem 1.1

Example Problem 1.1 - Green

Harry bought some 11c stamps and some 15c stamps for a total of exactly 85c.

How many 15c stamps did he buy?

Example Problem 1.1 - Yellow

Henry bought some 23c stamps and some 15c stamps for a total of exactly $2.50.

How many 15c stamps did he buy?

Example Problem 1.1 - Orange

Jessie has $5.10 worth of stamps.

She has equal numbers of 50c, 20c, 10c and 5c stamps.

She has no other stamps.

How many 50c stamps does she have?

Maths Games Example Solution 1.1 - Yellow

Henry was able to buy some 23c stamps and some 15c stamps for a total of exactly $2.50.

How many 15c stamps did he buy?

Strategy 1: Build a Table

Since Henry bought some 23c stamps and some 15c stamps, we might begin by listing multiples of 23c and 15c.

No. of 23c stamps	Cost, in cents	
1	23	
2	46	
3	69	
4	92	
5	115	
6	138	
7	161	
8	184	
9	207	
10	230	

No. of 15c stamps	Cost, in cents
1	15
2	30
3	45
4	60
5	75
6	90
7	105
8	120
9	135
10	150

Next, let's take each multiple of 23c, and find out how much we need to add to make it up to **$2.50**.

It might be easier here to think of the total as **250c**.

No. of 23c stamps	Cost, in cents	Difference to make up to 250c
1	23	227
2	46	204
3	69	181
4	92	158
5	115	135
6	138	112
7	161	89
8	184	66
9	207	43
10	230	20

No. of 15c stamps	Cost, in cents
1	15
2	30
3	45
4	60
5	75
6	90
7	105
8	120
9	135
10	150

We can see that **135c** is the only value present in both tables:

- as a multiple of 15c, and
- as a difference between **250c** and a multiple of 23c.

No. of 23c stamps	Cost, in cents	Difference to make up to 250c
:	:	:
5	115	135
:	:	:

No. of 15c stamps	Cost, in cents
:	:
9	135
:	:

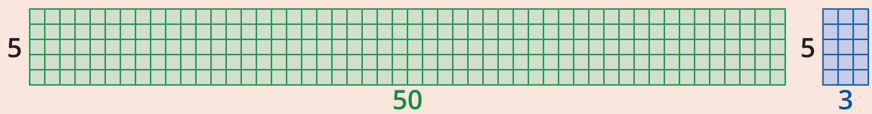
Henry must have bought **9** 15c stamps.

Strategy 2: Use Number Sense

Henry spent exactly 250c on stamps.

Some of the stamps cost 15c each.

Both 250 and 15 are multiples of **5**.



We can see that, when we subtract a multiple of 15 from 250, the result will always be a multiple of **5**.

This means that the total value of 23c stamps must be a multiple of **5**.

Since 23 and **5** are relatively prime, the total value of the 23c stamps must be a multiple of 23c × **5** = 115c.

250c – 115c = 135c.

135c = 9 × 15c.

250c – (2 × 115c) = 20c.

20c is not a multiple of 15c.

Therefore Henry must have bought **9 15c** stamps.

Answers

1.1 - Green: 2

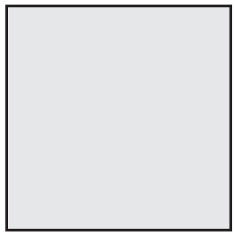
1.1 - Orange: 6

1.1 - Yellow: 9

Maths Games – Example Problem 1.2

Example Problem 1.2 - Green

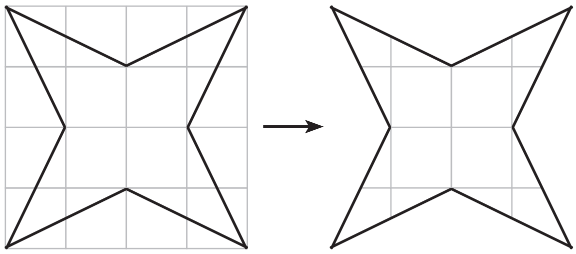
In how many ways can a square be folded in half so that both halves overlap exactly?



Example Problem 1.2 - Yellow

A four-pointed star is drawn on square grid paper and then cut out, as shown.

In how many ways can it be folded in half so that both halves overlap exactly?



Example Problem 1.2 - Orange

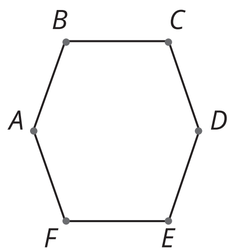
The figure shown is a hexagon with six sides of the same length.

Not all of the interior angles are the same size.

Angles *A* and *D* are the same size.

Angles *B*, *C*, *E*, and *F* are the same size.

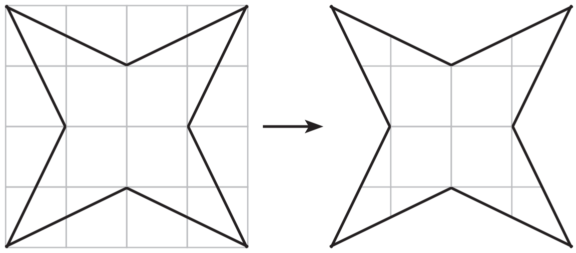
How many lines of symmetry does the figure have?



Maths Games Example Solution 1.2 - Yellow

A four-pointed star is drawn on square grid paper and then cut out, as shown.

In how many ways can it be folded in half so that both halves overlap exactly?



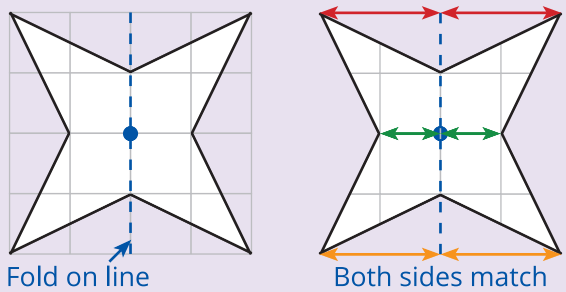
Strategy: Use Concrete Materials or Draw a Diagram, and Find a Pattern

Let's start by finding one way that we can fold the star in half, so that both halves overlap exactly.

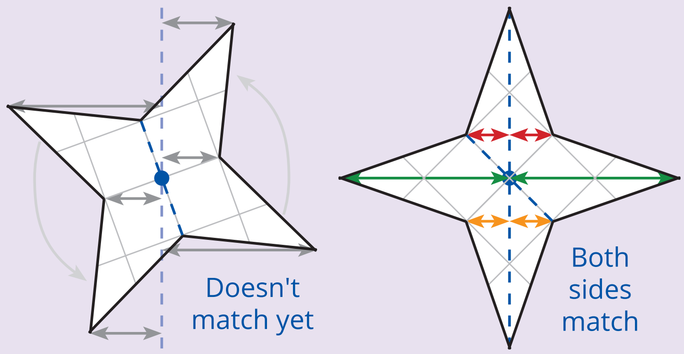
This may be more effectively done by folding a paper star.

We shall here attempt to use diagrams to demonstrate the symmetry of the shape.

All of the vertices on the left of the central grid line can be matched with a vertex on the right, so that both vertices are the same distance from this centre line.



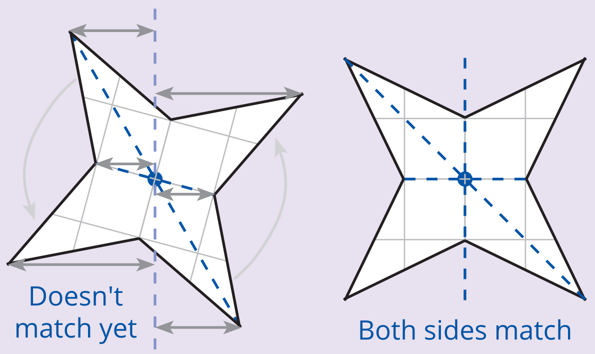
We can now rotate the star about its centre point until we can see this matching happening once again.



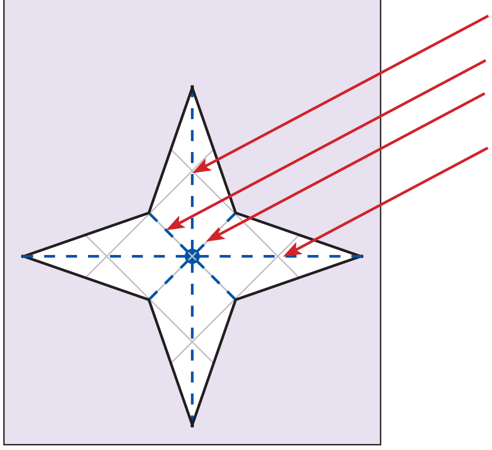
We can see a pattern occurring as we rotate about the centre point.

After another eighth-turn, the star will look similar to the original orientation.

Since it's the same, we know that it will also fold in half exactly along the grid line.



A further one-eighth rotation sees the star oriented in another familiar configuration.



We have created **4** fold lines.

If we rotate the star any further, we'll start duplicating fold lines that have already been created.

So there are **4** ways to fold the star so that both halves match exactly.

Answers

1.2 - Green: 4

1.2 - Orange: 2

1.2 - Yellow: 4

Maths Games – Example Problem 1.3

Example Problem 1.3 - Green

A small suitcase has a capacity of 10 litres.

A large suitcase has a capacity of 15 litres.

5 people, each carrying either a small or a large suitcase, board a plane.

The total capacity of their suitcases is 65 litres.

How many of them are carrying a small suitcase?

Example Problem 1.3 - Yellow

A small suitcase has a capacity of 40 litres.

A large suitcase has a capacity of 45 litres.

10 people, each carrying either a small or a large suitcase, board a plane.

The total capacity of their suitcases is 410 litres.

How many of them are carrying a small suitcase?

Example Problem 1.3 - Orange

Emily has 20 problems to solve.

She gets 3 marks for each one she solves and loses 5 marks for each one she does not solve.

She ended up with 28 marks.

How many of the 20 problems did Emily solve?

Maths Games Example Solution 1.3 - Yellow

A small suitcase has a capacity of 40 litres. A large suitcase has a capacity of 45 litres. 10 people, each carrying either a small or a large suitcase, board a plane. The total capacity of their suitcases is 410 litres.

How many of them are carrying a small suitcase?

Strategy 1: Build a Table

Suppose each person carried a small suitcase. The total capacity of the suitcases would be 10 × **40 L** = **400 L**.

Small Suitcases	10					
Large Suitcases	0					
Total Capacity (L)	400					

If just one person carried a large suitcase, the total capacity of the suitcases would be 9 × **40 L** + 1 × **45 L** = **405 L**.

Small Suitcases	10	9				
Large Suitcases	0	1				
Total Capacity (L)	400	405				

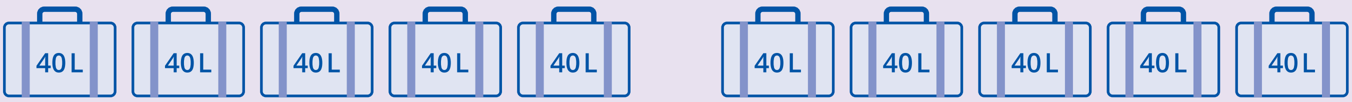
If one more person carried a large suitcase, the total capacity of the suitcases would be 8 × **40 L** + 2 × **45 L** = **410 L**.

Small Suitcases	10	9	8			
Large Suitcases	0	1	2			
Total Capacity (L)	400	405	410			

8 of the people are carrying a small suitcase.

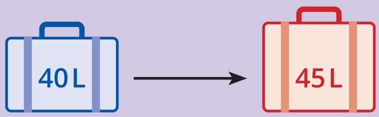
Strategy 2: Find a Pattern

We begin by noting that every person's suitcase has a capacity of at least **40 L**.



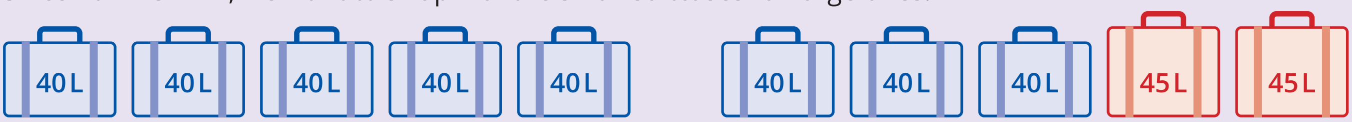
In total, the capacity of the suitcases must be no less than 10 × **40 L** = **400 L**.

If a single small suitcase is swapped for a large suitcase, the total capacity increases by 45 L – 40 L = **5 L**.



To increase the capacity to **410 L**, we need another **410 L** – **400 L** = **10 L** in capacity.

Since **10 L** ÷ **5 L** = **2**, we want to swap **2** of the small suitcases for large ones.



8 of the people are carrying a small suitcase.

Strategy 3: Reason Algebraically

Let there be x small suitcases.

With **10** suitcases in total, there must be $(10 - x)$ large suitcases. The total capacity, in litres, of all of the small suitcases is $40x$.

The total capacity of all of the large suitcases is $45(10 - x)$.

Therefore there are **8** small suitcases.

Since the total capacity is **410**:

$$40x + 45(10 - x) = 410$$
$$40x + 450 - 45x = 410$$

Subtracting **450** from both sides:

$$-5x = -40$$

Dividing both sides by **–5**:

$$x = 8$$

Answers

1.3 - Green: 2

1.3 - Orange: 16

1.3 - Yellow: 8

Maths Games – Example Problem 1.4

Example Problem 1.4 - Green

Leila has some marbles.

She groups them by twos and has one left over.

She groups them by fives and has three left over.

Leila has more than three marbles.

What is the smallest number of marbles Leila could have?

Example Problem 1.4 - Yellow

Kayla has some marbles.

She groups them by threes and has one left over.

She groups them by sevens and has four left over.

Kayla has more than five marbles.

What is the smallest number of marbles Kayla could have?

Example Problem 1.4 - Orange

Taylor has some marbles.

She groups them by twos and has one left over.

She groups them by threes and has two left over.

She groups them by fives and has three left over.

What is the smallest number of marbles Taylor could have?

Maths Games Example Solution 1.4 - Yellow

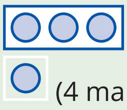
Kayla has some marbles. She groups them by threes and has one left over. She groups them by sevens and has four left over.

Kayla has more than five marbles. What is the smallest number of marbles Kayla could have?

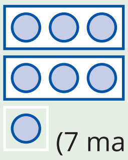
Strategy: Build a Table, and Find a Pattern

When Kayla groups her marbles in threes, she has one left over.

So she might group her marbles like this:


(4 marbles),

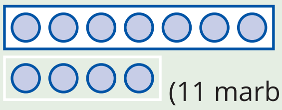
or


(7 marbles),

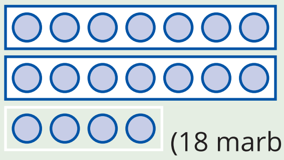
and so on, where there are a number of groups of **3**, and then **1** left over.

When Kayla groups her marbles in sevens, she has four left over.

So she might group her marbles like this:


(11 marbles),

or


(18 marbles),

and so on, where there are a number of groups of **7**, and then **4** left over.

Method: Build a Table

Let's list numbers that can make groups of **3**, with **1** left over.

No. of groups of 3	1	2	3	4	5	6	7	8	9	10
Marbles in groups	3	6	9	12	15	18	21	24	27	30
Total Marbles	4	7	10	13	16	19	22	25	28	31

Next we'll list numbers that can make groups of **7**, with **4** left over.

No. of groups of 7	1	2	3	4	5	6	7	8	9	10
Marbles in groups	7	14	21	28	35	42	49	56	63	70
Total Marbles	11	18	25	32	39	46	53	60	67	74

Looking at the Total Marbles for each grouping method, we can see that **25** occurs in both cases.

25 is the smallest number of marbles for which Kayla can make these groups.

Method: Build a Table, and Find a Pattern

Let's use a hundreds chart to indicate possible numbers of marbles.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

We'll put a line through numbers that work for the first case: groups of **3**, with **1** left over.

1	2	3	~~4~~	5	6	~~7~~	8	9	~~10~~
11	12	~~13~~	14	15	~~16~~	17	18	~~19~~	20
21	~~22~~	23	24	~~25~~	26	27	~~28~~	29	30
~~31~~	32	33	~~34~~	35	36	~~37~~	38	39	~~40~~
41	42	~~43~~	44	45	~~46~~	47	48	~~49~~	50
51	~~52~~	53	54	~~55~~	56	57	~~58~~	59	60
~~61~~	62	63	~~64~~	65	66	~~67~~	68	69	~~70~~
71	72	~~73~~	74	75	~~76~~	77	78	~~79~~	80
81	~~82~~	83	84	~~85~~	86	87	~~88~~	89	90
~~91~~	92	93	~~94~~	95	96	~~97~~	98	99	~~100~~

Next we'll circle numbers that can make groups of **7** with **4** left over, until we find a number that works in both situations.

1	2	3	~~4~~	5	6	~~7~~	8	9	~~10~~
(11)	12	~~13~~	14	15	~~16~~	17	(18)	~~19~~	20
21	~~22~~	23	24	(~~25~~)	26	27	~~28~~	29	30
~~31~~	32	33	~~34~~	35	36	~~37~~	38	39	~~40~~
41	42	~~43~~	44	45	~~46~~	47	48	~~49~~	50
51	~~52~~	53	54	~~55~~	56	57	~~58~~	59	60
~~61~~	62	63	~~64~~	65	66	~~67~~	68	69	~~70~~
71	72	~~73~~	74	75	~~76~~	77	78	~~79~~	80
81	~~82~~	83	84	~~85~~	86	87	~~88~~	89	90
~~91~~	92	93	~~94~~	95	96	~~97~~	98	99	~~100~~

The first number we come across that works in both situations is **25**.

So the smallest number of marbles Kayla could have is **25**.

Answers

1.4 - Green: 13

1.4 - Orange: 23

1.4 - Yellow: 25

Maths Games – Example Problem 1.5

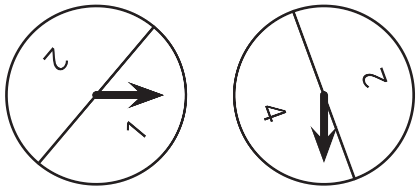
Example Problem 1.5 - Green

In a board game, a number is made by adding the number spun on Spinner A to the number spun on Spinner B.

Spinner A has the numbers 1 and 2 on it.

Spinner B has the numbers 2 and 4 on it.

In how many ways can you spin a total that is odd?



Spinner A Spinner B

Example Problem 1.5 - Yellow

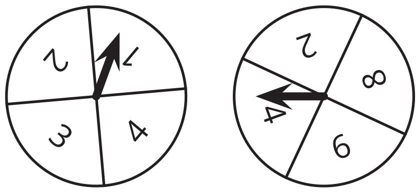
In a board game, a number is made by adding the number spun on Spinner A to the number spun on Spinner B.

Spinner A has the numbers 1, 2, 3 and 4 on it.

Spinner B has the numbers 2, 4, 6 and 8 on it.

In how many ways can you spin a total that is odd?

(We will consider 1+6 and 3+4 to be two different ways.)



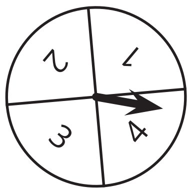
Spinner A Spinner B

Example Problem 1.5 - Orange

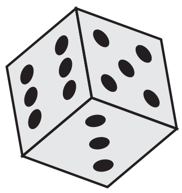
Laura spins Spinner A and rolls a standard die, and then adds the results together.

In how many ways can she get a total that is odd?

(We will consider spinning 1 on the spinner and rolling 4 on the die to be different from rolling 1 on the die and spinning 4 on the spinner.)



Spinner A



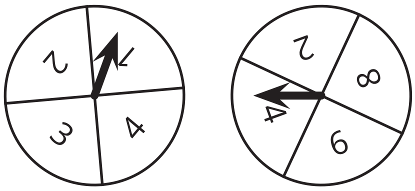
Maths Games Example Solution 1.5 - Yellow

In a board game, a number is made by adding the number spun on Spinner A to the number spun on Spinner B.

Spinner A has the numbers 1, 2, 3 and 4 on it.

Spinner B has the numbers 2, 4, 6 and 8 on it.

In how many ways can you spin a total that is odd?



Spinner A   Spinner B

Strategy 1: Build a Table

Let's begin by listing the four possible results for Spinner A.

Spinner A	
1	
2	
3	
4	

For each of these results, we could have one of four different results on Spinner B.

	Spinner B			
Spinner A	2	4	6	8
1				
2				
3				
4				

Let's calculate the possible sums, to fill in the table.

	Spinner B			
+	2	4	6	8
Spinner A 1	3	5	7	9
Spinner A 2	4	6	8	10
Spinner A 3	5	7	9	11
Spinner A 4	6	8	10	12

We can now mark all of the totals that are odd.

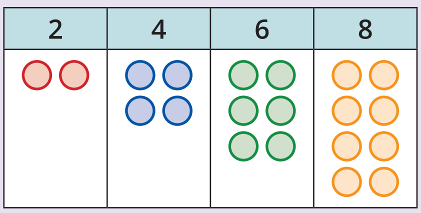
	Spinner B			
+	2	4	6	8
Spinner A 1	(3)	(5)	(7)	(9)
Spinner A 2	4	6	8	10
Spinner A 3	(5)	(7)	(9)	(11)
Spinner A 4	6	8	10	12

There are **8** ways to spin a total that is odd.

Strategy 2: Draw a Diagram, and Find a Pattern

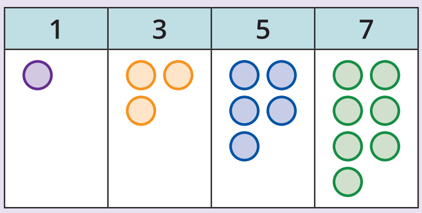
An even number is divisible by **2**.

This means that it can be represented by a rectangular array that is **2** units wide.

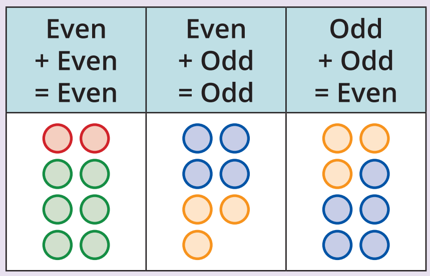


An odd number cannot be represented by an array that is **2** units wide.

The extra **1** is unpaired.



From the diagrams, we can see that:



All **4** of the numbers on Spinner B are even.

So, since the addition of an even number does not change the parity of a number, the parity of the sum of the two spinners is just going to be the parity of the number from Spinner A.

There are **2** odd numbers on Spinner A.

So there are **2** × **4** = **8** ways to spin an odd total.

Answers

1.5 - Green: 2

1.5 - Orange: 12

1.5 - Yellow: 8

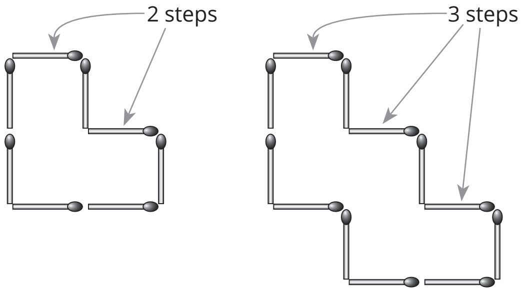
Maths Games – Example Problem 1.6

Example Problem 1.6 - Green

I am using matchsticks to make a staircase pattern.

The diagram shows what the pattern looks like when I have 2 or 3 steps.

How many matchsticks do I need to make 4 steps?

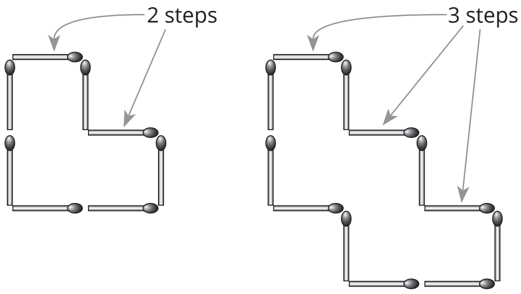


Example Problem 1.6 - Yellow

I am using matchsticks to make a staircase pattern.

The diagram shows what the pattern looks like when I have 2 or 3 steps.

How many matchsticks do I need to make 5 steps?

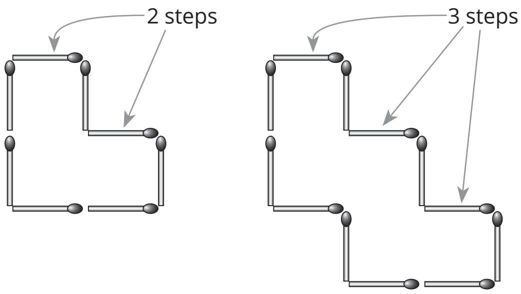


Example Problem 1.6 - Orange

I am using matchsticks to make a staircase pattern.

The diagram shows what the pattern looks like when I have 2 or 3 steps.

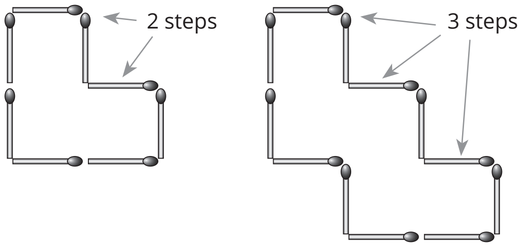
How many matchsticks do I need to make 123 steps?



Maths Games Example Solution 1.6 - Yellow

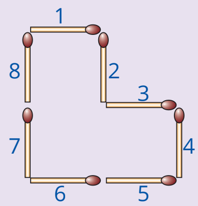
I am using matchsticks to make a staircase pattern. The diagram shows what the pattern looks like when I have 2 or 3 steps.

How many matchsticks do I need to make 5 steps?



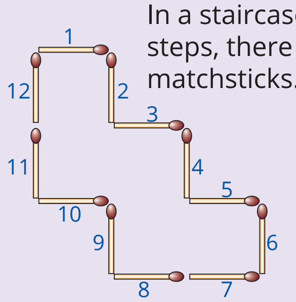
Strategy 1: Build a Table, and Find a Pattern

In a staircase with 2 steps, there are 8 matchsticks.



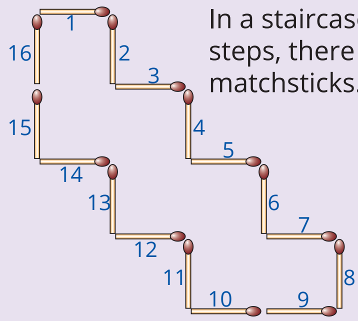
Steps:	2
Matchsticks:	8

In a staircase with 3 steps, there are 12 matchsticks.



Steps:	2	3
Matchsticks:	8	12

In a staircase with 4 steps, there are 16 matchsticks.



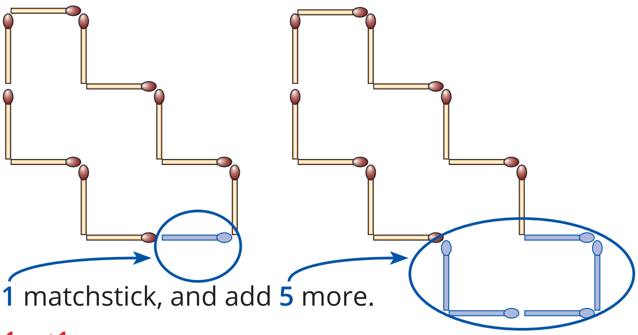
Steps:	2	3	4
Matchsticks:	8	12	16

Can you see a pattern?

Every time we add 1 step, the number of matches increases by 4.

Steps:	2	3	4
Matchsticks:	8	12	16

This is because, to increase the staircase by 1 step,



we take off 1 matchstick, and add 5 more.

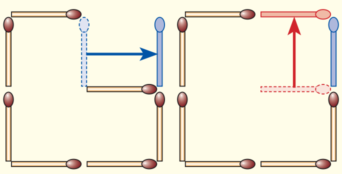
Let's continue the pattern until we reach 5 steps.

Steps:	2	3	4	5
Matchsticks:	8	12	16	20

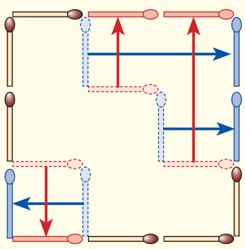
So 20 matchsticks are needed to make a staircase with 5 steps.

Strategy 2: Rearrange the Matchsticks, and Find a Pattern

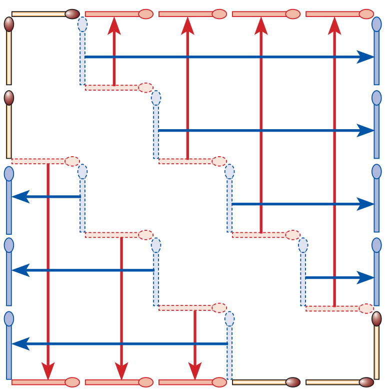
From the staircase pattern, we can push the matchsticks outwards to form a square.



So the 2-step staircase uses the same number of matchsticks as a square with 2 matchsticks to a side.



The 3-step staircase becomes a square with 3 matchsticks to a side.



The 5-step staircase becomes a square with 5 matchsticks to a side.

So for a staircase with 5 steps, I will need 4 x 5 = 20 matchsticks.

Answers

1.6 - Green: 16

1.6 - Orange: 492

1.6 - Yellow: 20

Maths Games – Example Problem 1.7

Example Problem 1.7 - Green

Harry's Number Recycling Machine obeys exactly two rules:

If the inserted number has just 1 digit, double the number and print out the result.
Otherwise, if the inserted number has 2 digits, remove the tens digit and print out the result.

The first number Harry inserts is 2.

Then every answer he gets is inserted back into the machine until twenty numbers are inserted.

What is the twentieth number to be inserted?

Example Problem 1.7 - Yellow

Michelle's Number Recycling Machine obeys exactly two rules:

1. *If an inserted number has exactly 1 digit, double the number.*
2. *If an inserted number has exactly 2 digits, compute the sum of the digits.*

The first number Michelle inserts is 1.

Then every answer she gets is inserted back into the machine until fifty numbers are inserted.

What is the fiftieth number to be inserted?

Example Problem 1.7 - Orange

Harry is playing with Michelle's Number Recycling Machine, which obeys exactly two rules:

1. *If an inserted number has exactly 1 digit, double the number.*
2. *If an inserted number has exactly 2 digits, compute the sum of the digits.*

The first number Harry inserts is 3.

Then every answer he gets is inserted back into the machine until a hundred numbers are inserted.

What is the one-hundredth number to be inserted?

Maths Games Example Solution 1.7 - Yellow

Michelle's Number Recycling Machine obeys exactly two rules:

1. *If an inserted number has exactly 1 digit, double the number.*
2. *If an inserted number has exactly 2 digits, compute the sum of the digits.*

The first number Michelle inserts is 1. Then every answer she gets is inserted back into the machine until fifty numbers are inserted.

What is the fiftieth number to be inserted?

Strategy: Find a Pattern, and Build a Table

Let's see what happens as Michelle inserts numbers.

The first number she inserts is 1.

1 has exactly **1** digit, so the machine doubles the number.

1 ⇨ 2 × 1 = 2 ⇨ 2

The answer she gets is 2. She inserts 2 back into the machine.

2 has exactly **1** digit, so the machine doubles the number.

2 ⇨ 2 × 2 = 4 ⇨ 4

The answer she gets is 4. She inserts 4 back into the machine.

4 has exactly **1** digit, so the machine doubles the number.

4 ⇨ 2 × 4 = 8 ⇨ 8

The answer she gets is 8. She inserts 8 back into the machine.

8 has exactly **1** digit, so the machine doubles the number.

8 ⇨ 2 × 8 = 16 ⇨ 16

The answer she gets is 16. She inserts 16 back into the machine.

16 has **2** digits, so the machine computes the sum of the digits.

16 ⇨ 1 + 6 = 7 ⇨ 7

The answer she gets is 7. She inserts 7 back into the machine.

7 has exactly **1** digit, so the machine doubles the number.

7 ⇨ 2 × 7 = 14 ⇨ 14

The answer she gets is 14. She inserts 14 back into the machine.

14 has **2** digits, so the machine computes the sum of the digits.

14 ⇨ 1 + 4 = 5 ⇨ 5

The answer she gets is 5. She inserts 5 back into the machine.

5 has exactly **1** digit, so the machine doubles the number.

5 ⇨ 2 × 5 = 10 ⇨ 10

The answer she gets is 10. She inserts 10 back into the machine.

10 has **2** digits, so the machine computes the sum of the digits.

10 ⇨ 1 + 0 = 1 ⇨ 1

We've seen 1 before. And the machine always does the same thing when Michelle inserts 1.

So the numbers she inserts will go like this:

1, 2, 4, 8, 16, 7, 14, 5, 10,
1, 2, 4, 8, 16, 7, 14, 5, 10,
1, ...

By arranging the pattern in a table, we can see that every **9th** number is a 10.

This means that the **9th**, **18th**, **27th**, **36th**, and **45th** numbers are all 10.

Continuing the pattern, the following numbers are 1, 2, 4, 8, 16.

So the fiftieth number to be inserted is **16.**

1st	2nd	3rd	4th	5th	6th	7th	8th	9th
1,	2,	4,	8,	16,	7,	14,	5,	10,
10th	11th	12th	13th	14th	15th	16th	17th	18th
1,	2,	4,	8,	16,	7,	14,	5,	10,
19th	...							27th
1,	...							10,
								36th
								10,
								45th
								10,
46th	47th	48th	49th	50th				
1,	2,	4,	8,	16				

Answers

1.7 - Green: 4

1.7 - Orange: 3

1.7 - Yellow: 16

Maths Games – Example Problem 1.8

Example Problem 1.8 - Green

Kat and Matt want to buy a particular board game.

Kat needs another $3 before she can afford the board game on her own.

Matt needs another $7 before he can afford the board game on his own.

If they combine their money, they have just enough to afford the board game.

How much does the board game cost?

Example Problem 1.8 - Yellow

Nat and Scott want to buy a particular jigsaw puzzle.

Nat needs another $18 before she can afford the puzzle on her own.

Scott needs another $10 before he can afford the puzzle on his own.

If they combine their money, they still cannot afford to buy the puzzle.

The puzzle costs a whole number of dollars.

What is the most that the puzzle could cost?

Example Problem 1.8 - Orange

Mara and Tara each have some pencils.

If Mara gives four pencils to Tara, they will have the same number of pencils.

If Tara gives two pencils to Mara, Mara will have 3 times as many pencils as Tara.

How many pencils does Mara have?

Maths Games Example Solution 1.8 - Yellow

Nat and Scott want to buy a particular jigsaw puzzle. Nat needs another \$18 before she can afford the puzzle on her own. Scott needs another \$10 before he can afford the puzzle on his own.

If they combine their money, they still cannot afford to buy the puzzle. The puzzle costs a whole number of dollars. What is the most that the puzzle could cost?

Strategy 1: Build a Table, and Find a Pattern

Let's guess the puzzle costs \$30.
If so, Nat must have \$30 – \$18 = \$12.
Scott must have \$30 – \$10 = \$20.
Together, Nat and Scott would have \$12 + \$20 = \$32.

Cost of Puzzle (\$)	30					
Nat's Money (\$)	12					
Scott's Money (\$)	20					
Total (\$)	32					

If the puzzle costs \$30, Nat and Scott can combine their money and have enough to buy the puzzle.

Let's guess the puzzle costs \$31.
If so, Nat must have \$31 – \$18 = \$13.
Scott must have \$31 – \$10 = \$21.
Together, Nat and Scott would have \$13 + \$21 = \$34.

Cost of Puzzle (\$)	30	31				
Nat's Money (\$)	12	13				
Scott's Money (\$)	20	21				
Total (\$)	32	34				

If the puzzle costs \$31, Nat and Scott's total will actually exceed the amount required, by even more than if the puzzle had cost less. Why might this occur?

We can reduce the cost of the puzzle until Nat and Scott's combined total is insufficient for buying the puzzle.

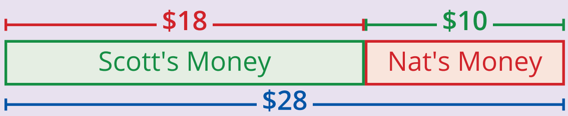
The most the puzzle could cost would be **\$27**.

Cost of Puzzle (\$)	30	31	29	28	27	
Nat's Money (\$)	12	13	11	10	9	
Scott's Money (\$)	20	21	19	18	17	
Total (\$)	32	34	30	28	26	

Strategy 2: Draw a Diagram

Nat is \$18 short, and Scott is \$10 short.

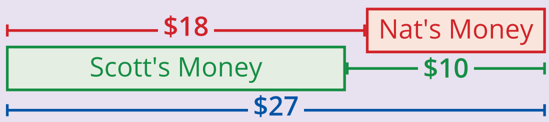
If the puzzle cost \$18 + \$10 = \$28, then Nat and Scott together would have exactly the right amount of money to buy the puzzle.



Together, Nat and Scott cannot afford the puzzle.

This would be the case if the puzzle cost **\$1** less.

The most the puzzle could cost would be **\$27**.



Strategy 3: Reason Algebraically

Let's say that the puzzle costs p dollars.

Nat is \$18 short, so Nat has $p - 18$ dollars.

Scott is \$10 short, so Scott has $p - 10$ dollars.

Together, Nat and Scott would have:

$$(p - 18) + (p - 10) = 2p - 28 \text{ dollars.}$$

Nat and Scott cannot afford the puzzle, even if they combine their money.

Expressing this as an inequality: $2p - 28 < p$

Subtracting p from both sides: $p - 28 < 0$

Adding **28** to both sides: $p < 28$

The puzzle costs a whole number of dollars less than **\$28**.

Therefore the most the puzzle could cost would be **\$27**.

Answers

1.8 - Green: \$10

1.8 - Orange: 16

1.8 - Yellow: \$27

Answers

Set Green

1.1	2
1.2	4
1.3	2
1.4	13
1.5	2
1.6	16
1.7	4
1.8	$10

Set Yellow

1.1	9
1.2	4
1.3	8
1.4	25
1.5	8
1.6	20
1.7	16
1.8	$27

Set Orange

1.1	6
1.2	2
1.3	16
1.4	23
1.5	12
1.6	492
1.7	3
1.8	16