MATHS GAMES

APSMO

2024 MATHS GAMES

IMPORTANT

The information contained in this file is ONLY for the use of registered participants of the 2024 APSMO Maths Games.

Organisation and Procedures

For full details, see the Members' Area

- Maths Games papers are to be conducted under test conditions.

DO	DO NOT
• Supervise students at all times. • Maintain silence. • Provide blank working paper. • Collect, mark and retain the papers.	• Print the papers prior to the scheduled date. • Read the questions aloud to the students. • Interpret the questions for students. • Permit any discussion or movement around the room. • Permit the use of calculators or other electronic devices.

- Papers should be scored by the PICO using the *Solutions and Answers* sheet provided.
- Original student answer sheets should be retained by the PICO until the end of the year.

Absent Students

- A student who is legitimately absent on the date of the Maths Games paper, may sit the paper on their return to school.
- If an absent student does not sit the paper on their return to school they should be marked as 'absent'.
- *Note: This policy differs from the Maths Olympiads Absent Student Policy which has additional requirements*.

APSMO

MATHS GAMES

WEDNESDAY 4 SEPTEMBER 2024

MATHS GAMES
SENIOR

Suggested Time: ***30 Minutes***

Write your answers in the boxes on the back.

←

Keep your answers hidden by folding backwards on this line.

4A. In a multiple choice quiz, there are 5 questions, each worth 1 mark.

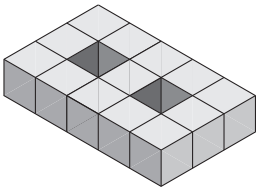
Sam and her two friends all scored different results in the quiz.

Their three individual results are added together for their group mark.

How many different values are possible for the group mark?

Hint: List all of the possible results for the quiz, and then all of the possible group marks if everyone in the group scored different results.

4B. A figure-8 is made by gluing together thirteen 1 cm × 1 cm × 1 cm cubes, as shown in the diagram.



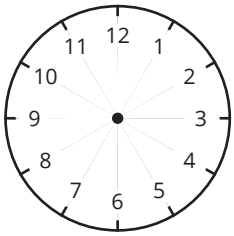
The figure-8 is then completely immersed in paint.

What is the total area of the painted faces, in square centimetres?

Hint: How can you count the painted faces in an organised way? Make sure you count the faces on the underside, and the faces inside the "holes".

4C. Emma's analog clock stopped at 3:20.

At 4:30, she replaced the battery and set the clock to the right time.



Through how many degrees did Emma have to turn the hour hand when she set the clock?

Hint: At 1 o'clock, the hour hand points directly to the 1. At 2 o'clock, it points directly to the 2. Where would the hour hand point at 1:30?

4D. There are 10 pens in a box: 1 green, 2 red, 3 blue and 4 black.

Kane reaches into the box without looking.

What is the smallest number of pens Kane needs to take out, to be certain that he has three pens of the same colour?

Hint: What pens might Kane take out, that cannot be included in a group of 3 of the same colour?

4E. It takes six minutes to fill a barrel at the slow tap.

It takes two minutes to fill a barrel at the fast tap.

If both taps are running into the barrel at the same time, how many seconds will it take to fill the barrel?

Hint: Think about what happens in one minute. Alternatively, think about how many barrels the fast tap would fill, if it is run for six minutes.

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MATHS GAMES

WEDNESDAY 4 SEPTEMBER 2024

MATHS GAMES
SENIOR

4A.

4B.

4C.

4D.

4E.

Student Name:

Fold here. Keep your answers hidden.

APSMO

MATHS GAMES

WEDNESDAY 4 SEPTEMBER 2024

MATHS GAMES
SENIOR

Solutions and Answers

(Items in parentheses are not required)

4A: 10	**4B:** 50 (cm²)	**4C:** 35 (°)	**4D:** 8	**4E:** 90 (seconds)

4A. The question is, How many different values are possible for the group mark?

***Strategy:* Convert to a More Convenient Form**

There are **5** questions in the quiz.

Each question is worth one mark.

We can begin by listing all of the possible results for the quiz:

0	1	2	3	4	5

Sam and her two friends all scored different results in the quiz.

Therefore, when their individual results are added together, the smallest possible value is:

0	**1**	**2**	3	4	5

0 + 1 + 2 = 3

The greatest possible value is:

0	1	2	**3**	**4**	**5**

3 + 4 + 5 = 12

Method: Make an Organised List

The possible values for the group mark are:

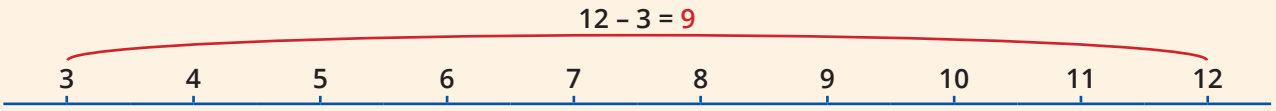
3	4	5	6	7	8	9	10	11	12

10 different values are possible for the group mark.

Method: Use a Number Line

The possible values for the group mark are from **3** to **12** inclusive.

The difference between **3** and **12** is **12 – 3 = 9**.



Using the number line representation, we can see that the difference between two integer values is the same as the number of spaces between those two integers.

As shown on the number line above, there are 9 spaces and **10** integers.

This may be clearer if we consider smaller numbers. For example:

The number of integer values is therefore one more than the difference.

The number of different values that are possible for the group mark is 9 + 1 = **10**.

Follow-Up: *Suppose there had been 10 questions in the quiz. How many different values would have been possible for the group mark?*
[(10 + 9 + 8) – (0 + 1 + 2) + 1 = 27 – 3 + 1 = 25]

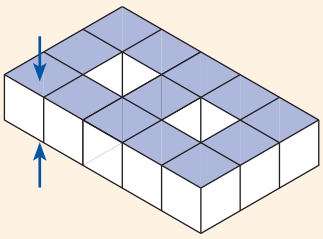
4B. The question is, What is the area of the painted faces, in square centimetres?

Alternatively - What is the surface area of the figure-8 shape?

***Strategy:* Divide a Complex Shape**

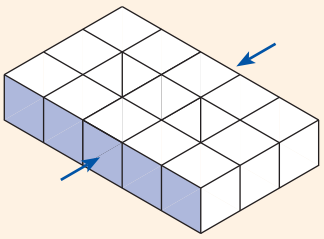
The top face and bottom face of the object both have a surface area of 13 cm².

2 × 13 cm² = 26 cm².



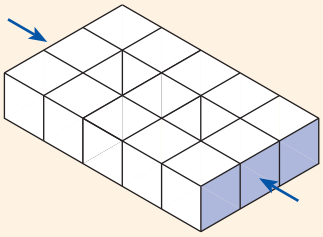
The two long vertical sides both have a surface area of 5 cm².

2 × 5 cm² = 10 cm².



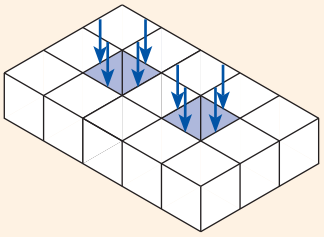
The two short vertical sides both have a surface area of 3 cm².

2 × 3 cm² = 6 cm².



The two "holes" through the object both have a surface area of 4 cm².

2 × 4 cm² = 8 cm².

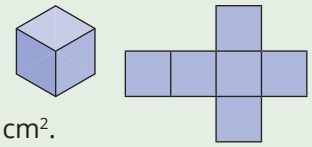


In total, the area of the painted faces is 26 + 10 + 6 + 8 = **50** cm².

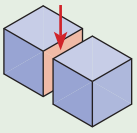
***Strategy:* Divide a Complex Shape (Alternative Method)**

A single cube has six faces.

Since each cube measures 1 cm × 1 cm × 1 cm, the surface area of one cube is 6 cm².

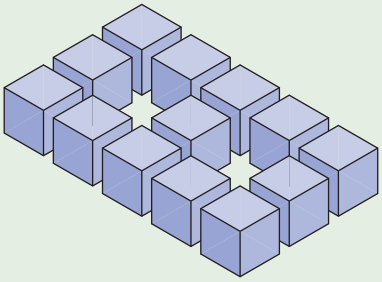


When two cubes are joined together, the total number of exposed faces is reduced by 2.



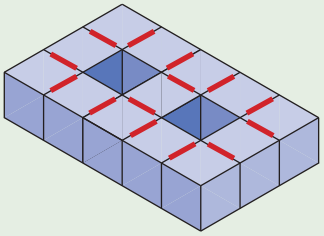
The figure-8 object is made up of **13** individual cubes that each have 6 cm² of surface area.

The total surface area, before joining the cubes together, is 13 × 6 = 78 cm².



When the cubes are joined together, there are **14** joins.

These joins reduce the surface area by 14 × 2 = 28 cm².

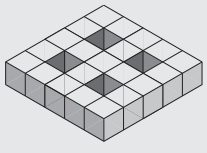


In total, the area of the painted faces is 78 – 28 = **50** cm².

Follow-Up: *What is the surface area of the object in the diagram at the right, which is made up of 21 cubes, each measuring 1 cm × 1 cm × 1 cm?*

[(2 × 21) + (4 × 5) + (4 × 4) = 78 cm².

Alternatively, (21 × 6) – (24 × 2) = 78 cm²]

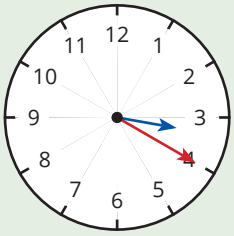


4C. The question is, Through how many degrees did Emma have to turn the hour hand when she set the clock?

Strategy 1: **Draw a Diagram, and Convert to a More Convenient Form**

Under normal working conditions, the hour hand rotates at a constant speed around the clock face.

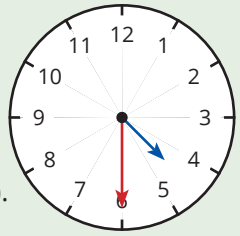
However, Emma's clock stopped at **3:20**.



3:20

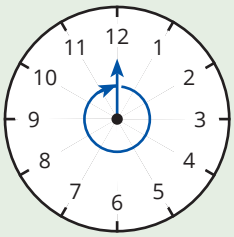
At **4:30**, Emma set the clock to the right time.

To do this, she had to move the hour hand from **3:20** (between the **3** and the **4**), to **4:30** (between the **4** and the **5**).



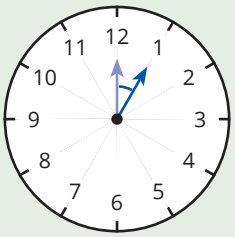
4:30

It takes **12** hours for the hour hand to rotate 360 degrees (i.e. rotate all the way around the clock face).



12 hours

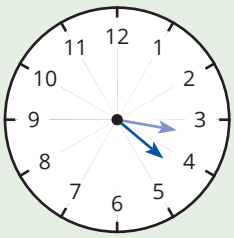
Therefore, in **1** hour, the hour hand will rotate $360 \div 12 = 30$ degrees.



1 hour

Emma is changing the time on her clock from **3:20** to **4:30**.

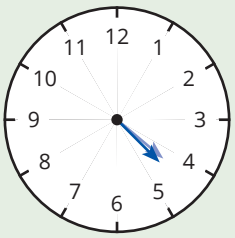
From **3:20** to **4:20** is one hour, so she would have to move the hour hand forward by 30 degrees.



3:20 – 4:20

From **4:20** to **4:30** is **10** minutes, or one-sixth of an hour.

Emma would have to move the hour hand forward by another $30 \div 6 = 5$ degrees.



4:20 – 4:30

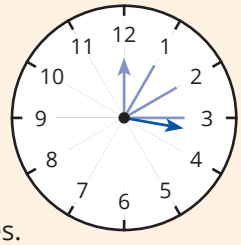
Emma turned the hour hand $30 + 5 = \mathbf{35}$ degrees.

Strategy 2: **Draw a Diagram, and Convert to a More Convenient Form (Alternative Method)**

At **3:20**, it is $3\frac{1}{3}$ hours since **12:00**.

If the hour hand moves 30 degrees in **1** hour,

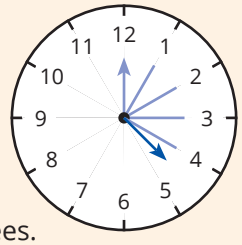
in $3\frac{1}{3}$ hours it will move $3\frac{1}{3} \times 30 = 90 + 10 = 100$ degrees.



At **4:30**, it is $4\frac{1}{2}$ hours since **12:00**.

If the hour hand moves 30 degrees in **1** hour,

in $4\frac{1}{2}$ hours it will move $4\frac{1}{2} \times 30 = 120 + 15 = 135$ degrees.



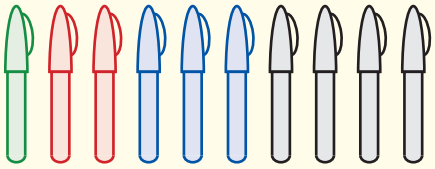
To move the hour hand from **3:20** to **4:30**, Emma would have turned it $135 - 100 = \mathbf{35}$ degrees.

Follow-Up: *After changing the battery and setting the clock correctly to 4:30, Emma discovered that the replacement battery was flat. She went to the shops and bought a new battery. At 5:10, she replaced the battery a second time, and once again, set the clock to the right time. How many degrees did she turn the hour hand this time ?* [20 degrees]

4D. The question is, What is the smallest number of pens Kane needs to take out, to be certain he has three pens of the same colour?

***Strategy:* Solve a Simpler Related Problem**

There are **10** pens in the box:
1 green, **2** red, **3** blue and **4** black.



Kane wants to be certain that he will take out three pens of the same colour.

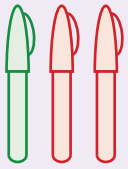
Since it is possible for Kane to take any combination of pens, we can consider a related question:

What is the greatest number of pens that Kane can take out, and still not have three pens of the same colour?

Method 1: Deliberately select pens so that there will not be 3 of the same colour

The green pen cannot be included in a group of **3** of the same colour.

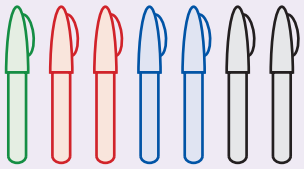
Likewise, the two red pens cannot be included in a group of **3**.



Kane can take two blue pens, and not have **3** of the same colour.

Likewise, he can take two black pens, and still not have a group of **3**.

This means that it is possible for him to take out 1 + 2 + 2 + 2 = **7** pens, and still not have **3** of the same colour.

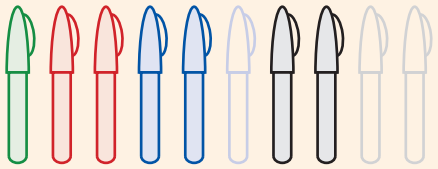


Method 2: Remove the third and subsequent pens

Alternatively, we can remove pens from the original set, one by one, until it is impossible to have three pens of the same colour.

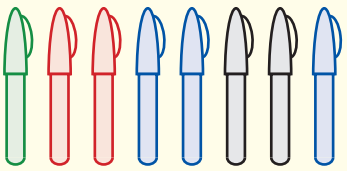
This will result in a set that comprises:

- 1 green pen,
- 2 red,
- 2 blue, and
- 2 black.



After determining the greatest number of pens that Kane can take out and still not have three pens of the same colour, there are just three more pens remaining from the original set: one blue, and two black.

Regardless of which one is taken from the remaining pens, Kane will end up with three pens that are all the same colour.



Kane must take 1 + 2 + 2 + 2 + 1 = **8** pens.

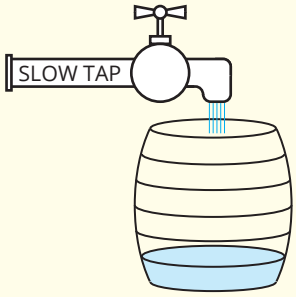
Follow-Up: *What is the smallest number of pens Kane must take out, to be certain he has at least one of each colour?* [10]

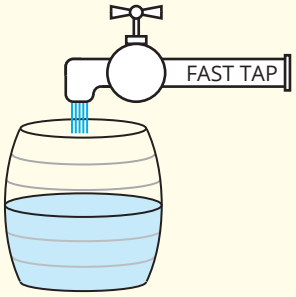
4E. The question is, If both taps are running into the barrel at the same time, how many seconds will it take to fill the barrel?

Strategy 1: **Convert to a More Convenient Form**

It takes **6** minutes to fill a barrel at the slow tap.

In **1** minute, the slow tap will fill $\frac{1}{6}$ of a barrel.





It takes **2** minutes to fill a barrel at the fast tap.

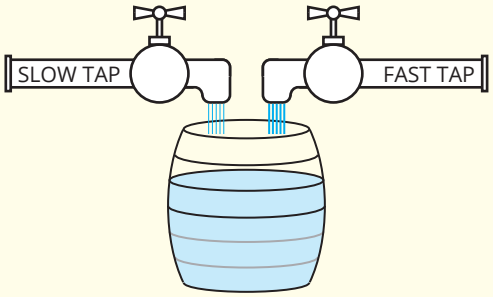
In **1** minute, the fast tap will fill $\frac{1}{2} = \frac{3}{6}$ of a barrel.

In **1** minute, both taps running at the same time will fill $\frac{1}{6} + \frac{3}{6} = \frac{4}{6} = \frac{2}{3}$ of a barrel.

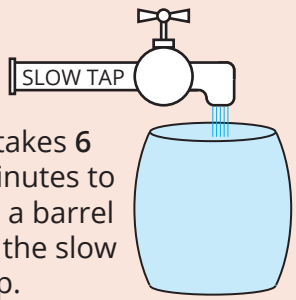
There is then $\mathbf{1} - \frac{2}{3} = \frac{1}{3}$ of a barrel left to fill.

If $\frac{2}{3}$ of a barrel takes **1** minute, then the remaining $\frac{1}{3}$ of a barrel will take another half a minute.

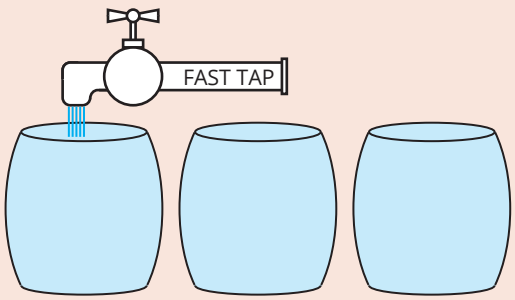
It will take $1\frac{1}{2}$ minutes, or **90** seconds, to fill the barrel.



Strategy 2: **Convert to a More Convenient Form (Alternative Method)**



It takes **6** minutes to fill a barrel at the slow tap.

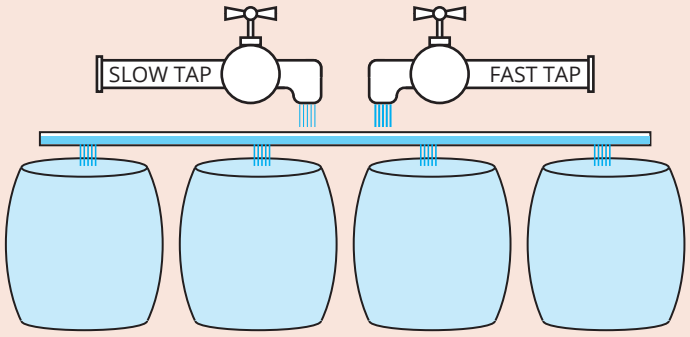


If it takes **2** minutes to fill a barrel at the fast tap, then in $\mathbf{3 \times 2 = 6}$ minutes, the fast tap will fill **3** barrels.

6 minutes equates to $\mathbf{6 \times 60 = 360}$ seconds.

In **360** seconds, the two taps will fill **4** barrels.

Therefore, with both taps running at the same time, it will take $\mathbf{360 \div 4 = 90}$ seconds to fill one barrel.



Follow-Up: *It takes 1 minute to fill the barrel using a fire hose. With the two taps and the fire hose running at the same time, how long will it take to fill the barrel?* [36 seconds]