MATHS GAMES

APSMO

2024 MATHS GAMES

IMPORTANT

The information contained in this file is ONLY for the use of registered participants of the 2024 APSMO Maths Games.

Organisation and Procedures

For full details, see the Members' Area

- Maths Games papers are to be conducted under test conditions.

DO	DO NOT
• Supervise students at all times. • Maintain silence. • Provide blank working paper. • Collect, mark and retain the papers.	• Print the papers prior to the scheduled date. • Read the questions aloud to the students. • Interpret the questions for students. • Permit any discussion or movement around the room. • Permit the use of calculators or other electronic devices.

- Papers should be scored by the PICO using the *Solutions and Answers* sheet provided.
- Original student answer sheets should be retained by the PICO until the end of the year.

Absent Students

- A student who is legitimately absent on the date of the Maths Games paper, may sit the paper on their return to school.
- If an absent student does not sit the paper on their return to school they should be marked as 'absent'.
- *Note: This policy differs from the Maths Olympiads Absent Student Policy which has additional requirements*.

APSMO

WEDNESDAY 31 JULY 2024

MATHS GAMES

SENIOR

Suggested Time: ***30 Minutes***

3A. Jill has used identical square tiles to cover a square patio.
The number of tiles is a multiple of 3, and also a multiple of 4.
None of the tiles are cut.
Find the smallest possible number of tiles on Jill's patio.

Hint: How many square tiles might be used to cover a square patio?

3B. On some school days, Nick gets paid \$15 to deliver leaflets to every house on his way to and from school.
On every other school day, he pays \$5 to catch a bus to and from school.
After 30 school days, Nick had saved a total of \$90.
On how many days did Nick deliver leaflets?

Hint: How much would Nick save if he delivered leaflets on 29 days, and took the bus on one day?

3C. In the following cryptarithm, different letters represent different digits, and a leading digit cannot be 0.
What is the smallest value that could be represented by *FAST*?

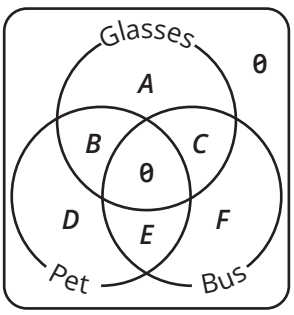
$$\begin{array}{cccc} & R & U & N \\ + & R & U & N \\ \hline F & A & S & T \end{array}$$

Hint: There is only one value that can be represented by the letter *F*.

3D. Four printers will print sixty photographs in three minutes.
How many photographs will five printers print in two minutes?

Hint: How many photographs would four printers print in one minute?

3E. The Venn diagram represents the number of students in Mr Gregg's class who wear glasses, have a pet, or catch a bus to school.
A, ***B***, ***C***, ***D***, ***E***, and ***F*** each represents a different value from 1 to 6.
10 students do not have a pet.
15 students do not wear glasses.
10 students do not catch a bus.
If there are 21 students in the class, how many have a pet and catch a bus?



Hint: A student who owns a pet would be counted in the value ***B*** (if they also wear glasses), ***E*** (if they also catch the bus), or ***D*** (if they do not wear glasses or catch the bus). As shown in the diagram, there are 0 (zero) students who have a pet, wear glasses, and also catch a bus.

Write your answers in the boxes on the back.

←

Keep your answers hidden by folding backwards on this line.

APSMO

WEDNESDAY 31 JULY 2024

MATHS GAMES

SENIOR

3A.

3B.

3C.

3D.

3E.

Student Name:

Fold here. Keep your answers hidden.

APSMO
WEDNESDAY 31 JULY 2024

MATHS GAMES

MATHS GAMES
SENIOR

Solutions and Answers
(Items in parentheses are not required)

3A: 36	**3B:** 12	**3C:** 1046	**3D:** 50	**3E:** 4

3A. The question is, Find the smallest possible number of tiles on Jill's patio.

Strategy 1: **Eliminate All But One Possibility**

Since the number of tiles is a multiple of 3, we can begin by listing multiples of 3,

3	6	9	12	15	18	21	24	27	30	33	36	39	42	45

multiples of 4,

4	8	12	16	20	24	28	32	36	40	44

and square numbers.

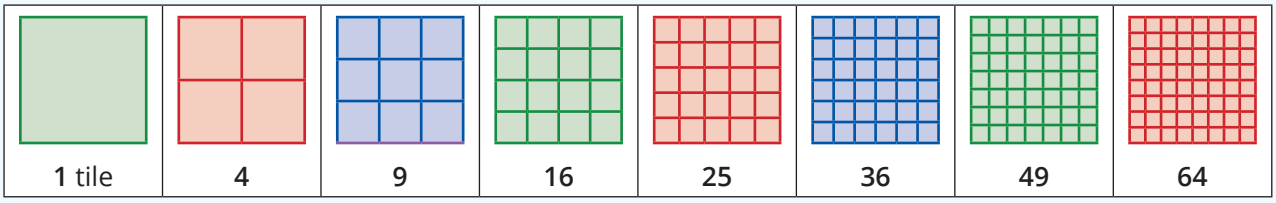
1	4	9	16	25	36	49

By inspection, we can see that the first number to appear in all three lists is **36**.

27	30	33	36	39	42	45
20	24	28	32	36	40	44
1	4	9	16	25	36	49

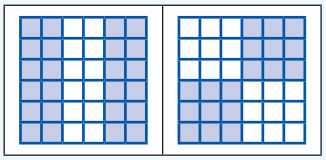
Strategy 2: **Draw a Diagram**

Jill is using square tiles to tile a square patio. This can result in the following tiling patterns:



The first pattern that uses a number of tiles that is both
- a multiple of **3**, and
- a multiple of **4**,

is the one that uses **36** tiles.



Strategy 3: **Consider Prime Factors**

The number of tiles is a multiple of 3, and also a multiple of 4.
- 3 is a prime number.
- The prime factors of 4 are 2 × 2.

We are looking for a square number that has, at a minimum, the prime factors 3 × 2 × 2.

The smallest such square number is therefore (3 × 2)(3 × 2) = **36**.

Follow-Up: *For a square patio using square tiles, what is the smallest number of tiles if the number is divisible by both 4 and 5?* [100]

3B. The question is, On how many days did Nick deliver leaflets?

Strategy 1: **Build a Table, and Find a Pattern**

Suppose Nick delivers leaflets every day.

Over 30 days, he will earn 30 × $15 = $450.

Leaflet days	30
Earnings ($)	450
Cost of bus ($)	0
Savings ($)	450

If Nick delivers leaflets on 29 days, and pays $5 for a bus on the 30th day, he will save 29 × $15 – $5 = $430.

Leaflet days	30	29
Earnings ($)	450	435
Cost of bus ($)	0	5
Savings ($)	450	430

On every day that Nick does not deliver leaflets, he will:

- Not get paid $15, and
- Pay $5 for the bus.

Leaflet days	30	29	28	27	...	22	...	17	...	12	
Earnings ($)	450	435	420	405							
Cost of bus ($)	0	5	10	15							
Savings ($)	450	430	410	390	...	290	...	190	...	90	

–20 –20 –20 –100 –100 –100

For each day, his savings would be reduced by $15 + $5 = $20.

Every 5 days, his savings would be reduced by 5 × $20 = $100.

From the table, we can see that Nick will save $90 in total, if he delivers leaflets on **12** days.

Strategy 2: **Solve a Simpler Related Problem**

In this scenario, on a school day, Nick either:

- Pays $5 for the bus, or
- Earns $15 delivering leaflets on his way to and from school.

Nick would save exactly the same amount if he:

- Pays $5 for the bus ***on every school day***, and
- Earns $20 on the days when he delivers leaflets.

This is because, on a day when he delivers leaflets, he would earn $20 and spend $5, resulting in him saving $20 – $5 = $15 in total.

Using this alternative arrangement:

- Nick pays a total of 30 × $5 = $150 for 30 days of bus trips.
- To save $90, he must have been paid $150 + $90 = $240 in total.

Nick must have worked on $240 ÷ $20 = **12** days.

Strategy 3: **Reason Algebraically**

Let x represent the number of days on which Nick delivered leaflets.

In that 30 day period, he caught the bus on 30 – x days.

We can now construct the following equation:

$$15x - 5(30 - x) = 90$$
$$15x - 150 + 5x = 90$$
$$20x = 240$$
$$x = 12$$

Therefore, Nick delivered leaflets on **12** days.

Follow-Up: *Over the next 30 school days, Nick saved $230. On how many days did he deliver leaflets?* [19]

3C. The question is, What is the smallest possible value represented by ***FAST***?

	R	*U*	*N*
+	*R*	*U*	*N*
F	*A*	*S*	*T*

***Strategy:* Eliminate All But One Possibility**

Since different letters represent different digits, we might use a table to keep track of the digits that have been used.

A leading digit cannot be **0**, so the smallest possible value for ***F*** is 1.

The smallest possible value for ***A*** is then 0.

0	1	2	3	4	5	6	7	8	9
A	*F*								

	R	*U*	*N*
+	*R*	*U*	*N*
1	0	*S*	*T*

If ***A*** represents 0, since there must be regrouping from the hundreds place to the thousands place, it must be that case that:

- ***R*** + ***R*** = 10, or
- ***R*** + ***R*** + 1 = 10 (if there is also regrouping from the tens place into the hundreds place).

Since ***R*** is a whole number, we now know that ***R*** = 5.

0	1	2	3	4	5	6	7	8	9
A	*F*				*R*				

	5	*U*	*N*
+	5	*U*	*N*
1	0	*S*	*T*

The smallest possible value for ***S*** is then 2.

If ***S*** represents 2, then ***U*** must be 1. This is not possible, because ***F*** is 1.

Similarly, if ***S*** represents 3, then ***U*** must be 1, which is not possible.

The next smallest value for ***S*** is 4.

If ***S*** is 4 then ***U*** must represent 2.

0	1	2	3	4	5	6	7	8	9
A	*F*	*U*		*S*	*R*				

	5	2	*N*
+	5	2	*N*
1	0	4	*T*

The smallest possible value for ***T*** is then 3.

If ***T*** represents 3, then ***N*** must be 1, which is not possible.

The next smallest value for ***T*** is 6.

If ***T*** is 6 then ***N*** must represent 3.

0	1	2	3	4	5	6	7	8	9
A	*F*	*U*	*N*	*S*	*R*	*T*			

	5	2	3
+	5	2	3
1	0	4	6

The smallest possible value for ***FAST*** is **1046**.

***Strategy:* Eliminate All But One Possibility (Alternate Method)**

As for Strategy 1, we can begin by trying the following values:

- ***F*** = 1,
- ***A*** = 0, and
- ***R*** = 5.

0	1	2	3	4	5	6	7	8	9
A	*F*				*R*				

Using the remaining digits, the smallest possible value for ***RUN*** would be 523.

523 + 523 = 1046, so if ***RUN*** represents 523 then ***FAST*** would represent 1046.

	5	2	3
+	5	2	3
1	0	4	6

Since every letter represents a different digit, these values satisfy the conditions of the problem.

The smallest possible value for ***FAST*** is **1046**.

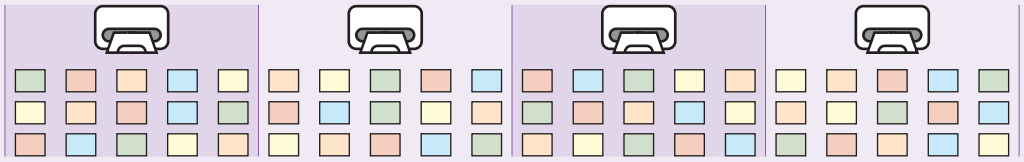
0	1	2	3	4	5	6	7	8	9
A	*F*	*U*	*N*	*S*	*R*	*T*			

Follow-Up: *What is the second-smallest possible value for FAST?* [532 + 532 = 1064]

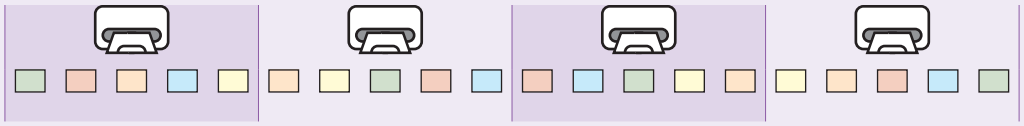
3D. The question is, How many photographs will five printers print in two minutes?

Strategy: **Solve a Simpler Related Problem**

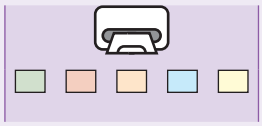
4 printers will print 60 photographs in 3 minutes.



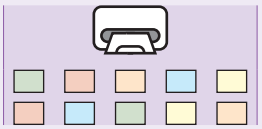
In 1 minute, those 4 printers will print 60 ÷ 3 = 20 photographs.



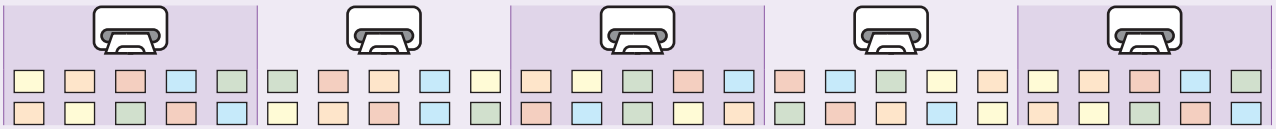
In 1 minute, 1 printer will print 20 ÷ 4 = 5 photographs.



In 2 minutes, 1 printer will print 2 × 5 = 10 photographs.

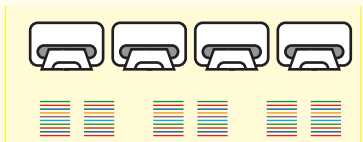


In 2 minutes, 5 printers will print 5 × 10 = **50** photographs.



Strategy: **Solve a Simpler Related Problem (Alternate Method)**

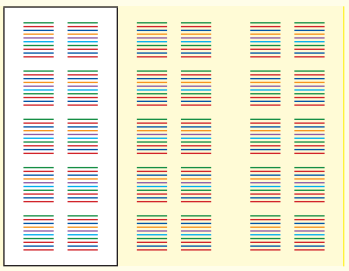
If 4 printers will print 60 photographs in 3 minutes,



in 3 minutes, 5 × 4 = 20 printers will print 5 × 60 = 300 photographs.

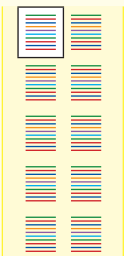
If 20 printers will print 300 photographs in 3 minutes,

in 1 minute, 20 printers will print 300 ÷ 3 = 100 photographs.



If 20 printers will print 100 photographs in 1 minute,

in 1 minute, 2 printers will print 100 ÷ 10 = 10 photographs.



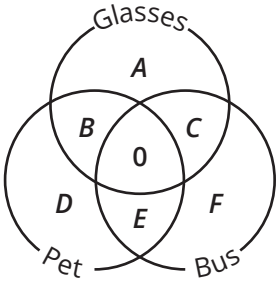
If 2 printers will print 10 photographs in 1 minute,

in 5 minutes, 2 printers will print 5 × 10 = **50** photographs.

Follow-Up: *4 printers will take 15 minutes to print 12 posters. How long will it take for 5 printers to print 5 posters?* [5 minutes]

3E. The question is, How many students both have a pet and catch a bus?

Alternatively: What is the value represented by E?



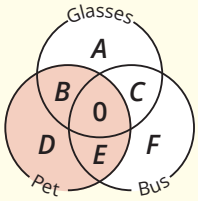
***Strategy:* Eliminate All But One Possibility**

There are **21** students in Mr Gregg's class.

Since $A + B + C + D + E + F = 1 + 2 + 3 + 4 + 5 + 6 = 21$, all of the students in the class can be placed somewhere on this Venn diagram.

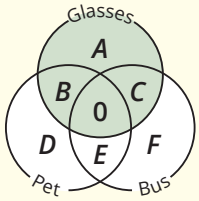
There are **10** students who do not have a pet.

$A + C + F = 10$.



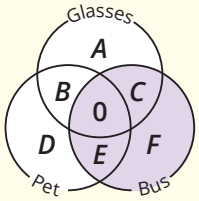
There are **15** students who do not wear glasses.

$D + E + F = 15$.



There are **10** students who do not catch a bus.

$A + B + D = 10$.



Method 1: Guess, Check and Refine, and Reason Logically

The greatest possible sum of three of the numbers is

$4 + 5 + 6 = 15$.

We know that

$D + E + F = 15$

so ***D***, ***E*** and ***F*** must represent **4**, **5** and **6** in some order.

A, ***B*** and ***C*** must represent **1**, **2** and **3** in some order.

Suppose $D = 4$.

If so: $A + B + D = 10$

$A + B = 6$.

Since ***A***, ***B*** and ***C*** must represent the values **1**, **2** and **3**, it is impossible for $A + B$ to equal **6**.

Similarly, if ***F*** represents **4**, then $A + C = 6$, which is likewise impossible.

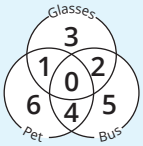
Since:

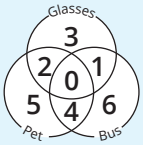
- ***D***, ***E*** and ***F*** represent the values **4**, **5** and **6**, and
- Neither ***D*** nor ***F*** can represent **4**,

E must represent the value **4**.

There are **4** students who both have a pet and catch a bus.

Possible solutions for the Venn diagram:





Method 2: Reason Arithmetically

We have: $A + C + F = 10$

$D + E + F = 15$

$A + B + D = 10$.

Adding them all together:

$A + A + B + C + D + D + E + F + F = 35$.

Since $A + B + C + D + E + F = 21$,

$A + D + F = 35 - 21$

$= 14$.

The greatest possible sum of three of the numbers is

$4 + 5 + 6 = 15$.

We know that $D + E + F = 15$, so ***D***, ***E*** and ***F*** must represent the values **4**, **5** and **6**.

14 is just one less than **15**, so the only possible way to get a sum of **14** for ***A***, ***D*** and ***F*** is if they represent the values **3**, **5** and **6**.

Since:

- ***A***, ***D*** and ***F*** represent the values **3**, **5** and **6**, and
- ***D***, ***E*** and ***F*** represent the values **4**, **5** and **6**,

D and ***F*** must represent **5** and **6**, and so ***E*** must represent the value **4**.

There are **4** students who both have a pet and catch a bus.

Follow-Up: *How many of the students wear glasses, do not have a pet, and do not catch the bus?* [3]