APSMO

2024 MATHS GAMES

Important

The information contained in this file is ONLY for the use of registered participants of the 2024 APSMO Maths Games.

APSMO

2024 MATHS GAMES

Organisation and Procedures

For full details, see the Members' Area

- Maths Games papers are to be conducted under test conditions.

DO	DO NOT
• Supervise students at all times. • Maintain silence. • Provide blank working paper. • Collect, mark and retain the papers.	• Print the papers prior to the scheduled date. • Read the questions aloud to the students. • Interpret the questions for students. • Permit any discussion or movement around the room. • Permit the use of calculators or other electronic devices.

- Papers should be scored by the PICO using the *Solutions and Answers* sheet provided.
- Original student answer sheets should be retained by the PICO until the end of the year.

Absent Students

- A student who is legitimately absent on the date of the Maths Games paper, may sit the paper on their return to school.
- If an absent student does not sit the paper on their return to school they should be marked as 'absent'.
- *Note: This policy differs from the Maths Olympiads Absent Student Policy which has additional requirements*.

APSMO
MATHS GAMES
WEDNESDAY 12 JUNE 2024

MATHS GAMES SENIOR

Suggested Time: ***30 Minutes***

__Write your answers in the boxes on the back.__

__Keep your answers hidden by folding backwards on this line.__

2A. One quarter of the people on a bus paid the full adult bus fare.

Ten people paid a pensioner fare.

Eight people paid a student fare.

Five people paid a child fare.

Three babies, and the bus driver, did not pay any fare.

How many people were on the bus?

Hint: What fraction of the people on the bus did not pay the full adult bus fare?

2B. A jar contains a lot of 10c coins, 20c coins, and 50c coins.

Liz took three coins from the jar.

How many different amounts of money might she have taken?

Hint: What amounts might Liz have, if she only took two coins?

2C. A caterer made enough pies for ten lunch platters.

When orders came in for twelve platters, there wasn't time to make more pies, so they shared the pies equally amongst all twelve platters.

They added extra sandwiches to make up for having two less pies on each platter.

How many pies were there on each of the twelve platters?

Hint: How many pies did they place on the extra platters?

2D. Warren picked 80 avocados from his avocado tree, and divided them equally amongst his neighbours.

Each neighbour received a whole number of avocados.

The number of avocados each neighbour received is not divisible by 10.

What is the smallest number of neighbours that Warren might have?

Hint: You could start by guessing the number of neighbours.

2E. A random number generator works by adding together all of the digits in the current time.

At 9:05, it will calculate 9 + 0 + 5, and print the number 14.

At 23:59, it will calculate 2 + 3 + 5 + 9, and print the number 19.

If the generator is asked to print one number every minute over a 24 hour period, how many times will it print the number 23?

Hint: You could try listing all of the possible hour values.

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MATHS GAMES

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WEDNESDAY 12 JUNE 2024

MATHS GAMES
SENIOR

2A.

Student Name:

Fold here. Keep your answers hidden.

2B.

2C.

2D.

2E.

MATHS GAMES

APSMO

WEDNESDAY 12 JUNE 2024

MATHS GAMES
SENIOR

Solutions and Answers

(Items in parentheses are not required)

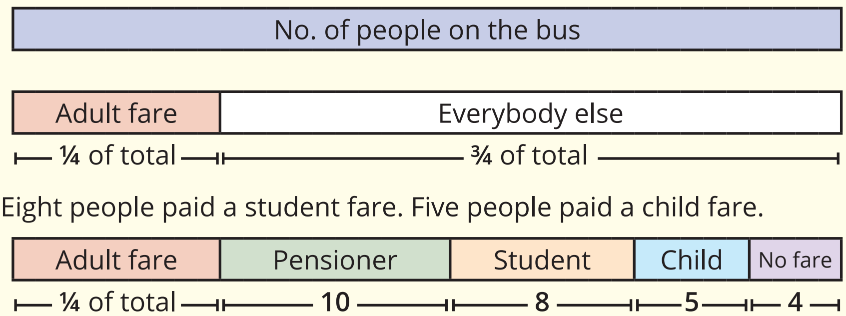
2A: 36	2B: 10	2C: 10	2D: 5	2E: 4

2A. The question is, How many people were on the bus?

Strategy 1: **Work Backwards**

We can use a bar to represent the number of people on the bus.

If one-quarter paid the adult fare, the other **1 – ¼ = ¾** of the people did not pay the adult fare.

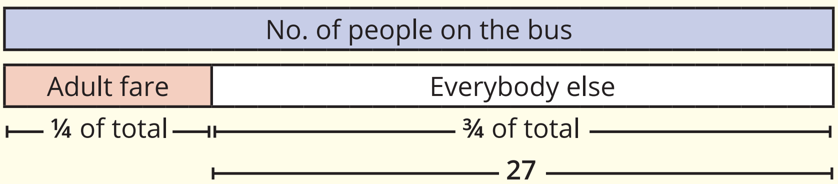


Ten people paid a pensioner fare. Eight people paid a student fare. Five people paid a child fare.

Four people (3 babies, and the driver), paid no fare.

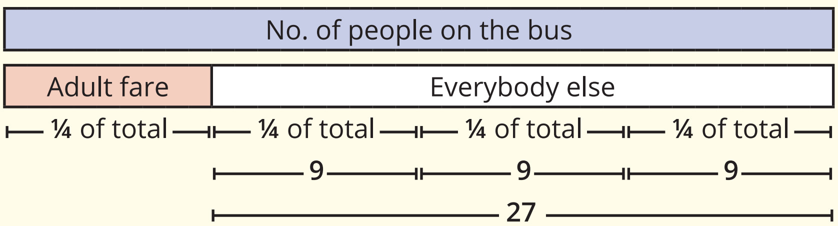
There are **10 + 8 + 5 + 4 = 27** people on the bus who did not pay the adult fare.

This is three-quarters of the total number of people on the bus.



If **27** is three-quarters of the total number of people on the bus, then **27 ÷ 3 = 9** is one-quarter of the total number.

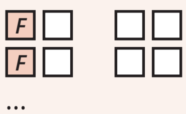
Therefore there are **4 × 9 = 36** people on the bus.



Strategy 2: **Draw a Diagram**

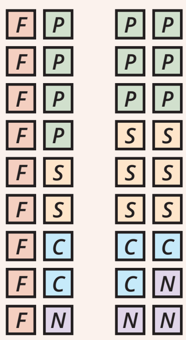
Suppose the bus has four seats in each row.

We will reserve ¼ of the seats - say, one seat in each row, for an adult fare paying passenger.



We can fill in the remaining seats with the other people, as noted in the question:

- **10** pensioners,
- **8** students,
- **5** children, and
- **4** non-paying people.



The **10 + 8 + 5 + 4 = 27** people on the bus who did not pay the adult fare, will occupy the remaining spaces on **9** rows of seats.

This means that there are **9** seats being occupied by adult fare paying passengers.

In total, there are **27 + 9 = 36** people on the bus.

Follow-Up: *At the next stop, some adult fare paying passengers got off the bus. After that, only one-tenth of the people on the bus had paid the adult fare. How many adult fare paying passengers got off the bus?* [6]

2B. The question is, How many different amounts of money might Liz have taken?

***Strategy 1:* Make an Organised List (1)**

Suppose Liz took a **10c** coin first.

Her second coin might be a **10c**, **20c**, or **50c**.

The third coin might likewise be a **10c**, **20c**, or **50c**.

Possible totals are **30c**, **40c**, **50c**, **70c**, **80c**, and **$1.10**.

1st	2nd	3rd	Total
10	10	10	30c
10	10	20	40c
10	10	50	70c
10	20	10	40c
10	20	20	50c
10	20	50	80c
10	50	10	70c
10	50	20	80c
10	50	50	$1.10

If the first coin had been **20c**, she would have **10c** more.

Possible totals would be **40c**, **50c**, **60c**, **80c**, **90c**, and **$1.20**.

If the first coin had been **50c**, she would have **40c** more.

Possible totals would be **70c**, **80c**, **90c**, **$1.10**, **$1.20**, and **$1.50**.

Listing the possible totals from smallest to largest, Liz could have taken:

30c, **40c**, **50c**, **60c**, **70c**, **80c**, **90c**, **$1.10**, **$1.20**, or **$1.50**.

Three coins from the jar could result in **10** different amounts of money.

***Strategy 2:* Make an Organised List (2)**

Suppose that, after Liz takes the three coins, she puts them in order from the least to the greatest value.

By listing the values in ascending order, we will avoid double-counting combinations of coins.

1st coin	10	10	10	10	10	10	20	20	20	50
2nd coin	10	10	10	20	20	50	20	20	50	50
3rd coin	10	20	50	20	50	50	20	50	50	50
Total (c)	30	40	70	50	80	110	60	90	120	150

We can see that Liz could have taken **10** different amounts of money.

***Strategy 3:* Make an Organised List (3)**

The smallest amount that Liz could have is **10c + 10c + 10c = 30c**.

The largest amount would be **50c + 50c + 50c = $1.50**.

We can try making every total between **30c** and **$1.50**, in **10c** increments.

Liz could have taken **10** different amounts of money.

Total	Combination
30c	10 + 10 + 10
40c	10 + 10 + 20
50c	10 + 20 + 20
60c	20 + 20 + 20
70c	10 + 10 + 50
80c	10 + 20 + 50
90c	20 + 20 + 50

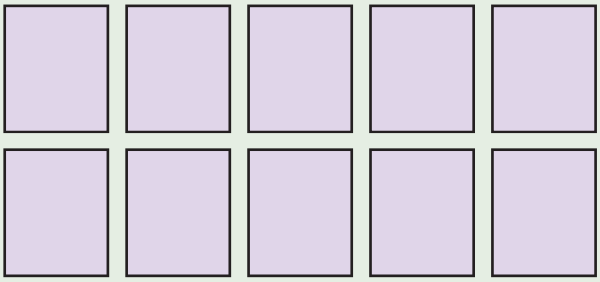
Total	Combination
$1.00	Not possible
$1.10	10 + 50 + 50
$1.20	20 + 50 + 50
$1.30	Not possible
$1.40	Not possible
$1.50	50 + 50 + 50

Follow-Up: *How many different amounts might Liz have taken, if she took 3 coins or less?* [13]

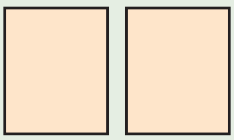
2C. The question is, How many pies were there on each of the twelve platters?

***Strategy 1:* Work Backwards**

A caterer made enough pies for **10** lunch platters.

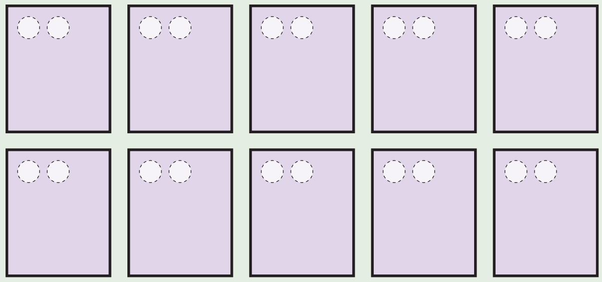


However, orders came in for **12** platters in total.

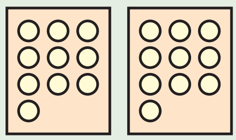


The caterer needed to make another **12 – 10 = 2** new platters.

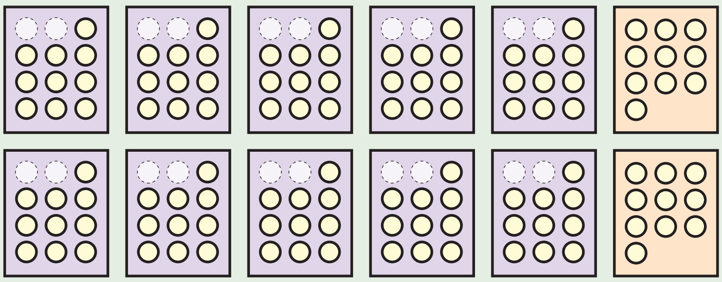
They ended up with **2** less pies on each platter.



Those pies were placed on the **2** extra platters.



After taking **10 × 2 = 20** pies from the original platters, each of the **2** new platters received **20 ÷ 2 = 10** pies.



Since the pies ended up being shared equally between all twelve platters, each of the twelve platters must have **10** pies.

***Strategy 2:* Reason Algebraically**

The caterer made enough pies for **10** lunch platters.

Let x represent the number of pies that the caterer normally makes for **1** lunch platter.

- The caterer made **10**x pies.

The caterer ended up putting $(x - 2)$ pies on each of **12** lunch platters.

- The total number of pies is $12(x - 2)$.

We now have the following:

	$12(x - 2) = 10x$
Expanding brackets:	$12x - 24 = 10x$
Adding **24** to both sides:	$12x = 10x + 24$
Subtracting **10**x from both sides:	$2x = 24$
Dividing both sides by **2**:	$x = 12$

The caterer ended up putting $(x - 2) = 12 - 2 = $ **10** pies on each of **12** lunch platters.

Follow-Up: *After creating 12 platters, the caterer realised that they had misread the order. The customer actually wanted 15, not 12, identical platters. How many pies would the caterer be putting on each of the 15 platters?* [8]

2D. The question is, What is the smallest number of neighbours that Warren might have?

***Strategy 1:* Make an Organised List**

We can consider every possible number of neighbours, starting from **1**.

We are looking for the first whole number of avocados per neighbour, that is not divisible by **10**.

The smallest number of neighbours that Warren might have is **5**.

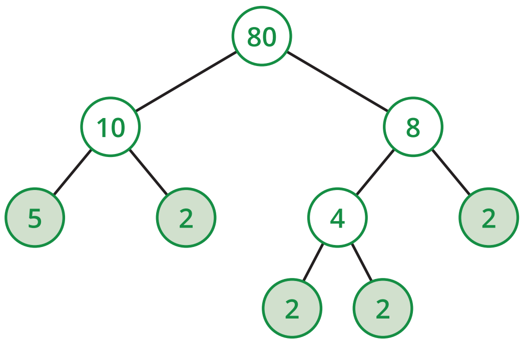
No. of neighbours	Avocados per neighbour	No. of avocados divisible by 10?
1	80 ÷ 1 = 80	Yes
2	80 ÷ 2 = 40	Yes
3	80 ÷ 3 = 26 r.2	
4	80 ÷ 4 = 20	Yes
5	80 ÷ 5 = 16	No

***Strategy 2:* Consider Prime Factors**

Warren is sharing 80 avocados equally amongst his neighbours.

Using a factor tree, we can see that the prime factors of 80 are 5 × 2 × 2 × 2 × 2.

80 is therefore divisible by every value that can be created by multiplying some combination of 5, 2, 2, 2, and 2.



Method 1: List all of the factors of 80

We begin by noting that every whole number is divisible by **1** and itself.

Since 80 has just two distinct prime factors, 2 and 5, we can use a table to list all of the factors of 80.

We can see that the greatest factor of 80 that is not divisible by **10** is 16.

×	1	2	2 × 2	2 × 2 × 2	2 × 2 × 2 × 2
1	1	2	4	8	16
5	5	10	20	40	80

The smallest number of neighbours that Warren might have is **80 ÷ 16 = 5**.

Method 2: Consider the prime factors of 10

Each neighbour receives a number of avocados that is not divisible by **10**.

The prime factors of **10** are 5 × 2.

The greatest number of avocados that is not divisible by (5 × 2), would be 2 × 2 × 2 × 2 = 16.

The smallest number of neighbours that Warren might have is **80 ÷ 16 = 5**.

Follow-Up: *80, which has a prime factorisation of 2 × 2 × 2 × 2 × 5, has 10 factors.*
How many different factors does 2 × 3 × 3 × 3 × 3 = 162 have? [10]

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MATHS GAMES

WEDNESDAY 12 JUNE 2024

MATHS GAMES SENIOR

2E. The question is, over a 24 hour period, how many times will the generator print the number 23?

***Strategy:* Make an Organised List**

The minutes value for a valid time can be any whole number from **0** to **59**.

The sum of the minutes digits is therefore a value between **0** and **5 + 9 = 14** inclusive.

Sum of minutes digits	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

We can then consider what the sum of the hours digits would be, if the generator were to print the number **23**.

Sum of minutes digits	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Sum of hours digits	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9

The hours value can be any whole number from **0** to **23**.

Hours value	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Sum of hours digits	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	10	2	3	4	5

The sum of the hours digits is therefore a value between **0** and **10** inclusive.

Of these sums, the only ones that can contribute to the generator printing **23** are the sums **9** and **10.**

Sum of minutes digits	~~0~~	~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~	~~7~~	~~8~~	~~9~~	~~10~~	~~11~~	~~12~~	13	14
Sum of hours digits	~~23~~	~~22~~	~~21~~	~~20~~	~~19~~	~~18~~	~~17~~	~~16~~	~~15~~	~~14~~	~~13~~	~~12~~	~~11~~	10	9

Hours value	~~0~~	~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~	~~7~~	~~8~~	9	~~10~~	~~11~~	~~12~~	~~13~~	~~14~~	~~15~~	~~16~~	~~17~~	18	19	~~20~~	~~21~~	~~22~~	~~23~~
Sum of hours digits	~~0~~	~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~	~~7~~	~~8~~	9	~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~	~~7~~	~~8~~	9	10	~~2~~	~~3~~	~~4~~	~~5~~

We are looking for times where:

- The sum of the minutes digits is **13**, and the hour value is **19**; or
- The sum of the minutes digits is **14**, and the hour value is either **9** or **18**.

If the sum of the minutes digits is **13**, the possible minutes values are:

- **49: 4 + 9 = 13**
- **58: 5 + 8 = 13**

Times with a digit sum of **23** are therefore

- **19:49: 1 + 9 + 4 + 9 = 23**
- **19:58: 1 + 9 + 5 + 8 = 23**.

If the sum of the minutes digits is **14**, the only possible minutes value is:

- **59: 5 + 9 = 14**

Times with a digit sum of **23** are therefore

- **9:59: 9 + 5 + 9 = 23**
- **18:59: 1 + 8 + 5 + 9 = 23**.

The random number generator will print the number **23** at **4** different times over a **24** hour period: **19:49**, **19:58**, **9:59**, and **18:59**.

Follow-Up: *Over a 24 hour period, how many times will the generator print the number 22?* [9: 19:39, 19:48, 19:57, 9:49, 9:58, 18:49, 18:58, 8:59, 17:59]