2024 Maths Games Junior - Years 5 & 6
Resource Kit 3
Teaching Problem Solving

MATHS GAMES

Problem Solving Strategies

This resource kit focuses on the following problem solving strategies:

1. Solve a Simpler Related Problem

Many hard problems are actually simpler problems that have been extended to larger numbers.

Patterns can sometimes be identified by trying the problem with smaller numbers.

2. Eliminate All But One Possibility

Deciding what a quantity is not, can narrow the field to a very small number of possibilities.

These can then be tested against the conditions of the original problem.

It follows on from strategies introduced in the Preparation Resource Kit and Resource Kits 1 and 2:

Guess, Check and Refine

Draw a Diagram

Find a Pattern

Build a Table

Work Backwards

Make an Organised List

Resource Kit 3 focuses on:

Solve a Simpler Related Problem

Eliminate All But One Possibility

Set Yellow

Example problems for which full worked solutions are included.

Set Green

Problems that are designed to be similar to Set Yellow, but with fewer difficult elements.

Set Orange

Problems that are similar in mathematical structure to the corresponding Yellow problems.

Further questions and solution methods can be found in the APSMO resource book "Building Confidence in Maths Problem Solving", available from www.apsmo.edu.au.

How to use these problems

At the start of the lesson, present the problem and ask the students to think about it. Encourage students to try to solve it in any way they like. When the students have had enough time to consider their solutions, ask them to describe or present their methods, taking particular note of different ways of arriving at the same solution.

Each question includes at least one solution method that the majority of students should be able to follow. By participating in lessons that demonstrate achievable problem solving techniques, students may gain increased confidence in their own ability to address unfamiliar problems.

Finally, the consideration of different solution methods is fundamental to the students' development as effective and sophisticated problem solvers. Even when students have solved a problem to their own satisfaction, it is important to expose them to other methods and encourage them to judge whether or not the other methods are more efficient.

Preparation Kit

Guess, Check and Refine

This involves making a reasonable guess of the answer, and checking it against the conditions of the problem. An incorrect guess may provide more information that may lead to the answer.

Draw a Diagram

A diagram may reveal information that may not be obvious just by reading the problem.

It is also useful for keeping track of where the student is up to in a multi-step problem.

Resource Kit 1

Find a Pattern

A frequently used problem solving strategy is that of recognising and extending a pattern.

Students can often simplify a difficult problem by identifying a pattern in the problem situation.

Build a Table

A table displays information so that it is easily located and understood.

A table is an excellent way to record data so the student doesn't have to repeat their efforts.

Resource Kit 2

Work Backwards

If a problem describes a procedure and then specifies the final result, this method usually makes the problem much easier to solve.

Make an Organised List

Listing every possibility in an organised way is an important tool.

How students organise the data often reveals additional information.

Resource Kit 3

Solve a Simpler Related Problem

Many hard problems are actually simpler problems that have been extended to larger numbers.

Patterns can sometimes be identified by trying the problem with smaller numbers.

Eliminate All But One Possibility

Deciding what a quantity is not, can narrow the field to a very small number of possibilities.

These can then be tested against the conditions of the original problem.

Resource Kit 4

Convert to a More Convenient Form

There are times when changing some of the conditions of a problem makes a solution clearer or more convenient.

Divide a Complex Shape

Sometimes it is possible to divide an unusual shape into two or more common shapes that are easier to work with.

Set Yellow

3.1) There are 10 points evenly spaced from each other along a metre ruler.

The first point is at 15 cm.

The tenth point is at 51 cm.

Where, in centimetres, is the third point?

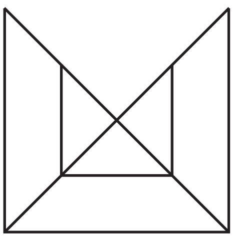
3.2) Find a three-digit number where:

- The hundreds digit is one greater than the tens digit.
- The ones digit is double the hundreds digit.
- The sum of the three digits is 15.

3.3) What is the value of ☐ that makes this number sentence true?

$3 \times 4 \times 50 = 30 \times \square$

3.4) How many triangles, of any size, can be drawn by tracing over lines in this diagram?



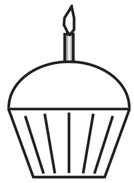
Set Yellow

3.5) Anna, Grace and Lily all have birthdays in summer (December, January and February), but in different months.

Anna's birthday is twenty days before Grace's birthday.

Grace's birthday is a week after January 26 (Australia Day).

In what month is Lily's birthday?



3.6) In the following cryptarithm, different letters represent different digits.

The letter ***O*** represents the digit 0.

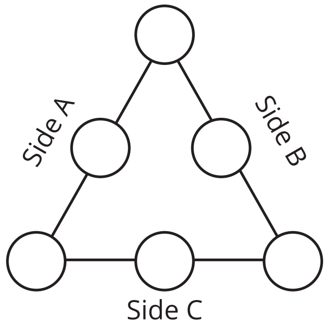
How many different values are possible for the number represented by ***WOOF***?

$$\begin{array}{r} D\ O\ G \\ +\ D\ O\ G \\ \hline W\ O\ O\ F \end{array}$$

3.7) The numbers 1, 2, 3, 4, 5, and 6 are placed in the diagram, one in each circle.

The sum of the three numbers along Side A is 13, along Side B is 13, and along Side C is 6.

What number is in the circle at the top of the diagram?



3.8) In a trivia game, each player is asked 10 questions.

You get 10 points for each correct answer.

If you don't answer a question correctly, you lose 5 points.

At the end of the game, Clint's total was 55 points.

How many questions did Clint answer correctly?

Set Green

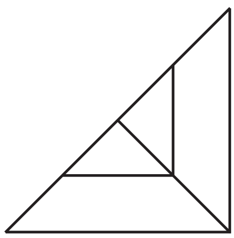
3.1) There are 5 points evenly spaced from each other along a metre ruler.
The first point is at 10 cm.
The fifth point is at 30 cm.
Where, in centimetres, is the fourth point?

3.2) Find a two-digit number where:
- The tens digit is 3 more than the ones digit.
- The ones digit is half the tens digit.

3.3) What is the value of ☐ that makes this number sentence true?

$3 \times 50 = 30 \times \square$

3.4) How many triangles, of any size, can be drawn by tracing over lines in this diagram?



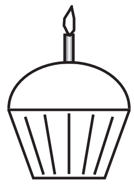
Set Green

3.5) Charlie, Oscar and Maria all have birthdays in autumn (March, April and May), but in different months.

Charlie's birthday is ten days after Oscar's birthday.

Oscar's birthday is the 25th of April (ANZAC Day).

In what month is Maria's birthday?



3.6) In the following cryptarithm, different letters represent different digits.

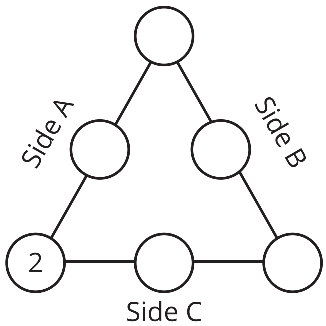
What is the greatest possible value represented by *HA*?

$$\begin{array}{rr} & A \\ & A \\ + & A \\ \hline H & A \end{array}$$

3.7) The numbers 1, 2, 3, 4, 5, and 6 are placed in the diagram, one in each circle.

The sum of the three numbers along Side A is 13, along Side B is 13, and along Side C is 6.

What number is in the circle at the top of the diagram?



3.8) In a trivia game, each player is asked 5 questions.

You get 2 points for each correct answer.

If you don't answer a question correctly, you lose 1 point.

At the end of the game, Jennifer's total was 4 points.

How many questions did Jennifer answer correctly?

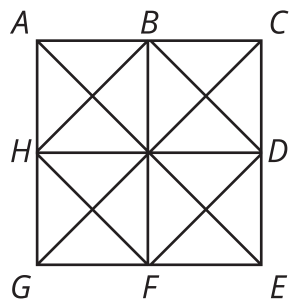
Set Orange

3.1) I am building a 50-metre-long wire fence along one side of a straight road.
The wires will be attached to posts, which are set into the ground at 5 metre intervals.
To begin with there are no posts along this stretch of road.
How many posts do I need to construct this fence?

3.2) Dr. Bolton was born in an interesting year.
The tens digit was twice the thousands digit.
The ones digit was three times the tens digit.
The hundreds digit was equal to the sum of the other three digits.
In what year was Dr. Bolton born?

3.3) What is the value of (5 × 34) + (34 × 3) + (2 × 34)?

3.4) Square *ACEG* is drawn at the right.
Points *B*, *D*, *F*, and *H* are halfway along the sides of the square.
What is the total number of squares of all sizes which can be traced using only the lines drawn?



Set Orange

3.5) Peter, Quinn, Rob and Stephen are all different ages: 9, 10, 11 and 12.

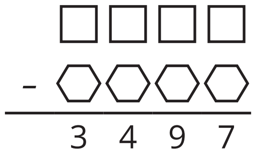
Peter is older than both Rob and Stephen.

Quinn is two years younger than Rob.

How old is Stephen?

3.6) In this subtraction, the squares (□) contain the digits 3, 4, 6, and 9 in some order and the hexagons (⬡) contain the digits 4, 5, 8, and 9 in some order.

What four-digit number is represented by the squares?



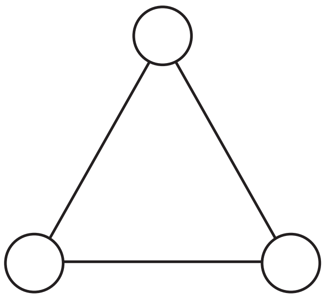
3.7) Anna drew three circles joined by three lines.

She wrote a number in each circle.

Then, she added the numbers from each pair of circles, and wrote the sum on the line joining them.

She found that she had all of the numbers 1, 2, 3, 4, 5 and 6 somewhere on her diagram.

What were the numbers in the circles, from smallest to largest?



3.8) I am building a 50-metre-long wire fence along one side of a straight road.

The wires will be attached to posts, which are set into the ground at 5 metre intervals.

To begin with there are no posts along this stretch of road.

How many posts do I need to construct this fence?

Maths Games – Example Problem 3.1

Example Problem 3.1 - Green

There are 5 points evenly spaced from each other along a metre ruler.

The first point is at 10 cm.

The fifth point is at 30 cm.

Where, in centimetres, is the fourth point?

Example Problem 3.1 - Yellow

There are 10 points evenly spaced from each other along a metre ruler.

The first point is at 15 cm.

The tenth point is at 51 cm.

Where, in centimetres, is the third point?

Example Problem 3.1 - Orange

I am building a 50-metre-long wire fence along one side of a straight road.

The wires will be attached to posts, which are set into the ground at 5 metre intervals.

To begin with there are no posts along this stretch of road.

How many posts do I need to construct this fence?

Maths Games Example Solution 3.1 - Yellow

There are 10 points evenly spaced from each other along a metre ruler. The first point is at 15 cm. The tenth point is at 51 cm. Where, in centimetres, is the third point?

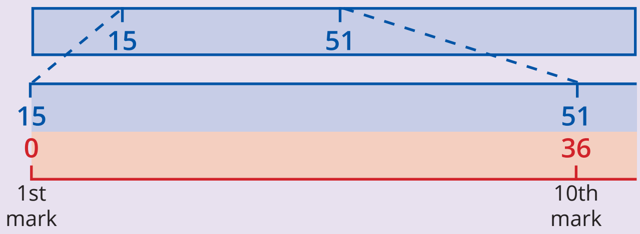
Strategy 1: Solve a Simpler Related Problem (1)

Let's draw a diagram of the metre ruler.

It has the first point at 15 cm, and the tenth point at 51 cm.

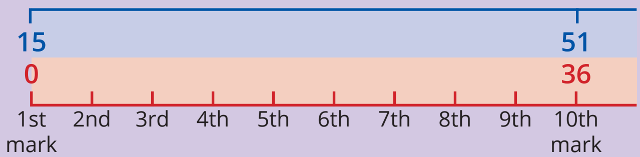
Suppose we start measuring the distance from the first point, instead of the end of the metre ruler.

If the first point occurred at 15 – 15 = 0 cm, then the tenth point would be at 51 – 15 = 36 cm.



There are 10 points.

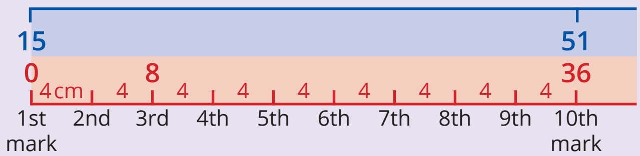
With the 1st point at 0 cm, there would be 9 more to mark the end of each section.



9 sections later, we are 36 cm from the 0 cm point.

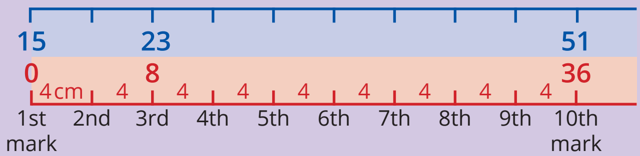
So each section must be 36 ÷ 9 = 4 cm wide.

The 3rd point must therefore be 2 × 4 cm = 8 cm away from the 1st point.



Returning to our original question, the 1st point was at 15 cm.

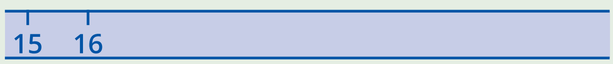
Since the 3rd point is 8 cm away from the 1st point, we can see that the 3rd point is at 15 + 8 = 23 cm.



Strategy 2: Solve a Simpler Related Problem (2)

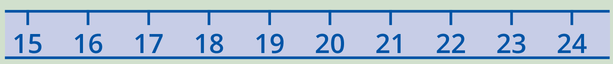
Suppose, instead of knowing where the 10th point was, we knew where the 2nd point was.

With the first point at 15 cm, let's say that the 2nd point is 1 cm further along, at 16 cm.



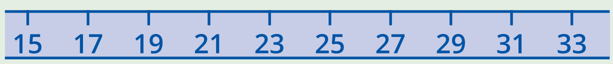
Setting out all of the remaining points, we can see that the 10th point will occur at 24 cm.

That's 24 – 15 = 9 cm away from the first point.



If the 2nd point was 2 cm further along than the first point, then the 10th point will occur at 33 cm.

That's a further 33 – 24 = 9 cm away from the first point.

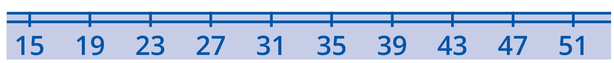


The 10th point gets another 9 cm away every time we increase each gap by 1 cm.

In the original question, the 10th point occurs at 51 cm, which is 51 - 15 = 36 cm away from the first point.

To get the 10th point 36 cm away from the first point, let's try making each gap 36 ÷ 9 = 4 cm wide.

If the 2nd point was 4 cm further along than the first point, then the 10th point will occur at 51 cm.



From the diagram, we can see that the 3rd point occurs at 23 cm.

Answers

3.1 - Green: 25

3.1 - Orange: 11

3.1 - Yellow: 23

Maths Games – Example Problem 3.2

Example Problem 3.2 - Green

Find a two-digit number where:

- The tens digit is 3 more than the ones digit.
- The ones digit is half the tens digit.

Example Problem 3.2 - Yellow

Find a three-digit number where:

- The hundreds digit is one greater than the tens digit.
- The ones digit is double the hundreds digit.
- The sum of the three digits is 15.

Example Problem 3.2 - Orange

Dr. Bolton was born in an interesting year.

The tens digit was twice the thousands digit.

The ones digit was three times the tens digit.

The hundreds digit was equal to the sum of the other three digits.

In what year was Dr. Bolton born?

Maths Games Example Solution 3.2 - Yellow

Find a three-digit number where:

- The hundreds digit is one greater than the tens digit.
- The ones digit is double the hundreds digit.
- The sum of the three digits is 15.

Strategy: Eliminate All But One Possibility

We can list all of the possible options, and then cross out any that do not fit the conditions of the problem.

Condition 1: The hundreds digit is one greater than the tens digit.

Let's list all of the possible tens digits, and then the corresponding hundreds digit.

Hundreds	1	2	3	4	5	6	7	8	9	10
Tens	0	1	2	3	4	5	6	7	8	9
Ones										

We can see that it's not possible for the hundreds digit to be **10**, since **10** is not a single digit number.

Hundreds	1	2	3	4	5	6	7	8	9	~~10~~
Tens	0	1	2	3	4	5	6	7	8	~~9~~
Ones										

Condition 2: The ones digit is double the hundreds digit.

Hundreds	1	2	3	4	5	6	7	8	9	~~10~~
Tens	0	1	2	3	4	5	6	7	8	~~9~~
Ones	2	4	6	8	10	12	14	16	18	

Again, it's not possible for any of the digits to be a value that is not a single digit number.

Hundreds	1	2	3	4	~~5~~	~~6~~	~~7~~	~~8~~	~~9~~	~~10~~
Tens	0	1	2	3	~~4~~	~~5~~	~~6~~	~~7~~	~~8~~	~~9~~
Ones	2	4	6	8	~~10~~	~~12~~	~~14~~	~~16~~	~~18~~	

Condition 3: The sum of the three digits is 15.

Hundreds	~~1~~	~~2~~	~~3~~	**4**	~~5~~	~~6~~	~~7~~	~~8~~	~~9~~	~~10~~
Tens	~~0~~	~~1~~	~~2~~	**3**	~~4~~	~~5~~	~~6~~	~~7~~	~~8~~	~~9~~
Ones	~~2~~	~~4~~	~~6~~	**8**	~~10~~	~~12~~	~~14~~	~~16~~	~~18~~	
Digit Sum	3	7	11	**15**						

The three-digit number must be **438**.

Answers

3.2 - Green: 63

3.2 - Orange: 1926

3.2 - Yellow: 438

Maths Games – Example Problem 3.3

Example Problem 3.3 - Green

What is the value of ☐ that makes this number sentence true?

$3 \times 50 = 30 \times \square$

Example Problem 3.3 - Yellow

What is the value of ☐ that makes this number sentence true?

$3 \times 4 \times 50 = 30 \times \square$

Example Problem 3.3 - Orange

What is the value of $(5 \times 34) + (34 \times 3) + (2 \times 34)$?

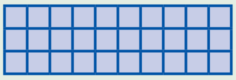
Maths Games Example Solution 3.3 - Yellow

What is the value of □ that makes this number sentence true? 3 × 4 × 50 = 30 × □

Strategy 1: Solve a Simpler Related Problem

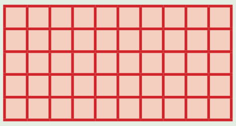
We begin with the equation 3 × 4 × 50 = 30 × □

Noticing that 30 = 3 × 10,



3 × 4 × 50 = 3 × 10 × □

We also know that 50 = 5 × 10:



3 × 4 × 5 × 10 = 3 × 10 × □

Let's rearrange the order we use to multiply the values, so that the common values (**3** and **10**) occur at the start of the expression.

3 × 10 × 4 × 5 = 3 × 10 × □

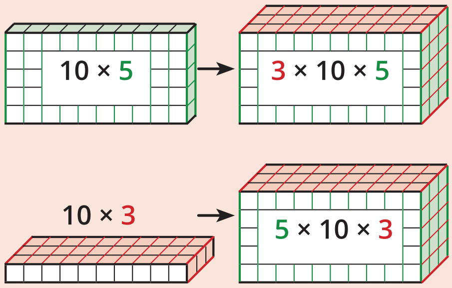
Since both sides are equal, it must be the case that: 4 × 5 = □

Therefore, □ is **20**.

Multiplication is both **associative:** (the way we group the numbers does not matter),

and **commutative**: (the order of the numbers does not matter).

For example, we can see that



3 × 10 × 5 is the same as 5 × 10 × 3.

Strategy 2: Draw a Diagram

Let's start by rearranging the expression

3 × 4 × 50

to

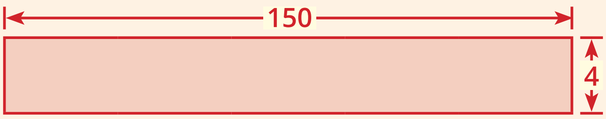
3 × 50 × 4.

Then we would have

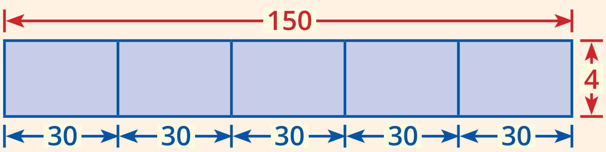
3 × 50 × 4 = 30 × □

150 × 4 = 30 × □

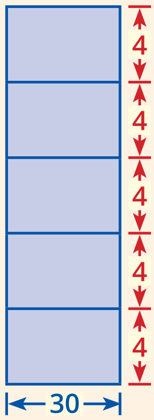
We can represent 150 × 4 with an area model:



We can then break up the 150 × 4 area model into five 30 × 4 area models:



If we stack the five 30 × 4 area models in a different way, we would get a model with the same area, but this time it would represent 30 × (5 × 4).



Since we've been working with the same area all along, we can see that 3 × 4 × 50 is equal to 30 × (5 × 4).

Therefore if 3 × 4 × 50 = 30 × □,

and 3 × 4 × 50 = 30 × (5 × 4),

then □ must be **5 × 4 = 20.**

Answers

3.3 - Green: 5

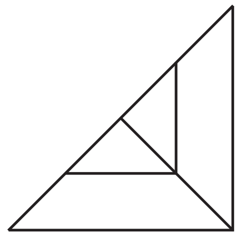
3.3 - Orange: 340

3.3 - Yellow: 20

Maths Games – Example Problem 3.4

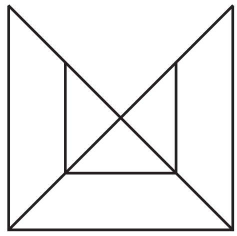
Example Problem 3.4 - Green

How many triangles, of any size, can be drawn by tracing over lines in this diagram?



Example Problem 3.4 - Yellow

How many triangles, of any size, can be drawn by tracing over lines in this diagram?

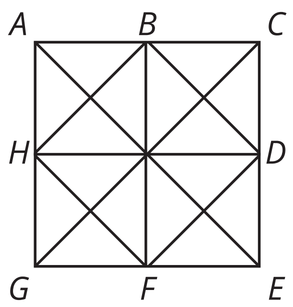


Example Problem 3.4 - Orange

Square *ACEG* is drawn at the right.

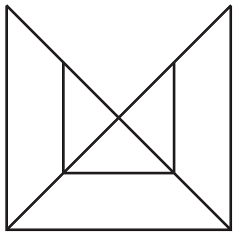
Points *B*, *D*, *F*, and *H* are halfway along the sides of the square.

What is the total number of squares of all sizes which can be traced using only the lines drawn?



Maths Games Example Solution 3.4 - Yellow

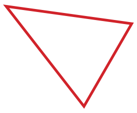
How many triangles, of any size, can be drawn by tracing over lines in this diagram?



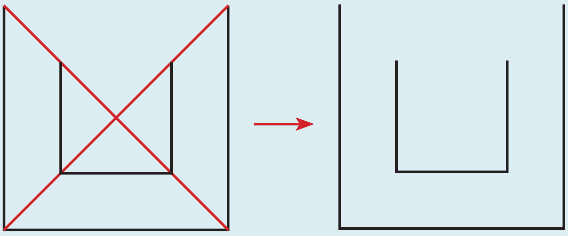
Strategy: Solve a Simpler Related Problem

A triangle has **3** sides.

This means that each side of the triangle must be connected to both of the other sides.



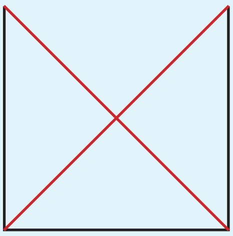
In our diagram, if we remove the "X" in the middle, none of the inner lines are connected to any of the outer lines.



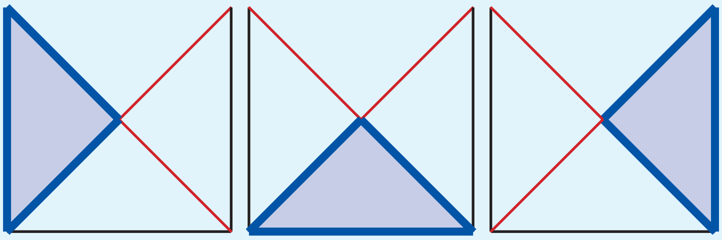
So there can't be a triangle that includes a line from the inner U-shape, and also includes a line from the outer U-shape.

Let's simplify the problem by breaking it into two parts.

We'll start by finding the triangles that are connected to the outer U-shape.

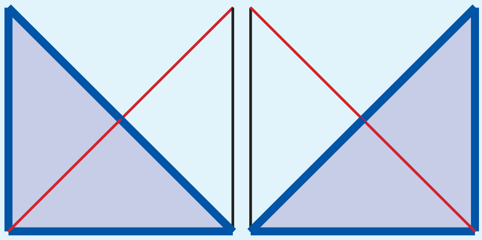


There are 3 triangles which look like this:



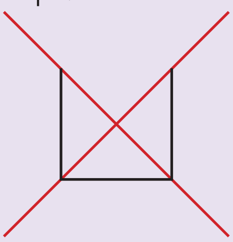
There are also some larger triangles, made up of 2 of these smaller triangles.

We can't make a bigger triangle here - it is not possible to combine all three of the small triangles to make a single triangle.

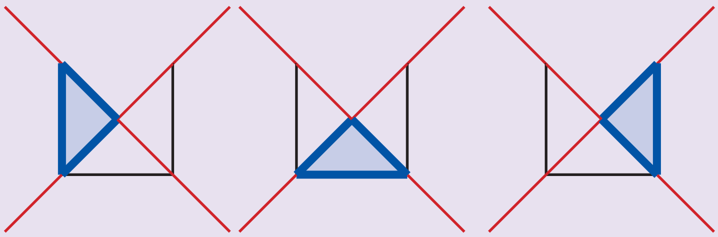


Next, let's find the triangles that are connected to the inner U-shape.

This question is very similar to the one we solved for the outer U-shape.

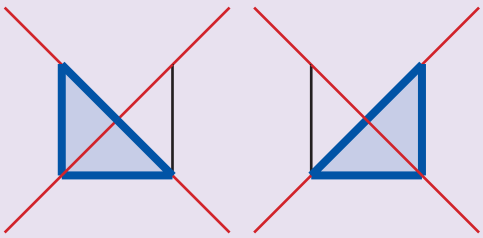


There are 3 triangles which look like this.



Similarly, there are 2 larger triangles.

Again, we can't make a bigger triangle here.



In total, 3 + 2 + 3 + 2 = **10** triangles can be drawn on these lines.

Answers

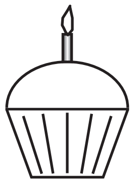
3.4 - Green: 6

3.4 - Orange: 10

3.4 - Yellow: 10

Maths Games – Example Problem 3.5

Example Problem 3.5 - Green



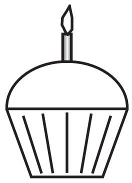
Charlie, Oscar and Maria all have birthdays in autumn (March, April and May), but in different months.

Charlie's birthday is ten days after Oscar's birthday.

Oscar's birthday is on the 25th of April (ANZAC Day).

In what month is Maria's birthday?

Example Problem 3.5 - Yellow



Anna, Grace and Lily all have birthdays in summer (December, January and February), but in different months.

Anna's birthday is twenty days before Grace's birthday.

Grace's birthday is a week after January 26 (Australia Day).

In what month is Lily's birthday?

Example Problem 3.5 - Orange

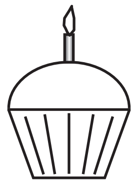
Peter, Quinn, Rob and Stephen are all different ages: 9, 10, 11 and 12.

Peter is older than both Rob and Stephen.

Quinn is two years younger than Rob.

How old is Stephen?

Maths Games Example Solution 3.5 - Yellow



Anna, Grace and Lily all have birthdays in summer (December, January and February), but in different months.

Anna's birthday is twenty days before Grace's birthday.

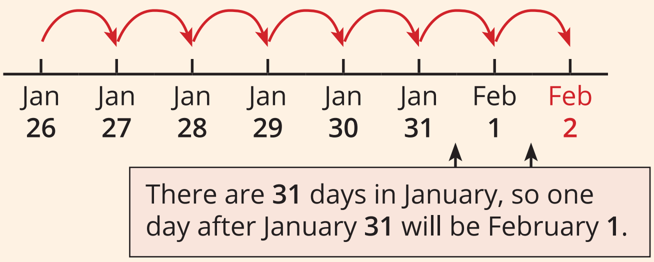
Grace's birthday is a week after January 26 (Australia Day).

In what month is Lily's birthday?

Strategy: Eliminate All But One Possibility

Option 1: Use a split strategy.

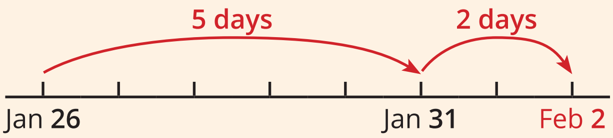
Grace's birthday is 1 week, or 7 days, after January **26**.



Alternatively, we can split the **7** days.

January **31** is **31** – **26** = **5** days after January **26**.

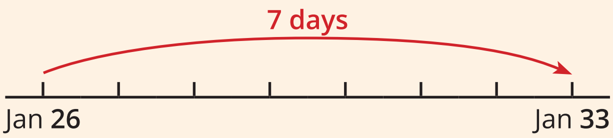
So **7** days after January **26** will be **2** days after January **31**.



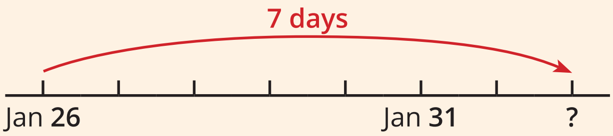
Therefore Grace's birthday is February **2**.

Option 2: Use a compensation strategy.

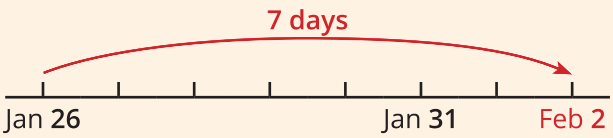
If January had an unlimited number of days, then **7** days after January **26** would be January **33**.



That can't be right. January only has **31** days.



Grace's birthday must be **33** – **31** = **2** days into the next month.



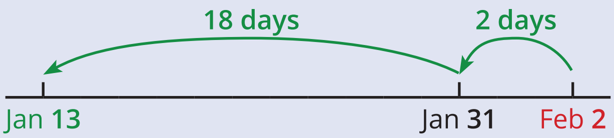
Therefore Grace's birthday is February **2**.

Anna's birthday is **20** days before Grace's birthday.

2 days before Grace's birthday is January **31**.

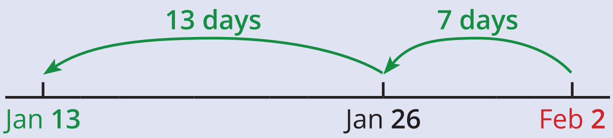
This means that Anna's birthday is **20** – **2** = **18** days before January **31**.

This is January **31** – **18** = January **13**.



Alternatively, we know that **7** days before Grace's birthday is January **26**.

So Anna's birthday is **20** – **7** = **13** days before January **26**, which is January **26** – **13** = January **13**.



Grace's birthday is on February **2**. Anna's birthday is on January **13**.

Since the three girls all have birthdays in different summer months, Lily's birthday must be in **December**.

Answers

3.5 - Green: March

3.5 - Orange: 10

3.5 - Yellow: December

Maths Games – Example Problem 3.6

Example Problem 3.6 - Green

In the following cryptarithm, different letters represent different digits.

What is the value represented by *HA*?

$$\begin{array}{rr} & A \\ & A \\ + & A \\ \hline H & A \end{array}$$

Example Problem 3.6 - Yellow

In the following cryptarithm, different letters represent different digits.

The letter ***O*** represents the digit 0.

How many different values are possible for the number represented by ***WOOF***?

$$\begin{array}{rrrr} & D & O & G \\ + & D & O & G \\ \hline W & O & O & F \end{array}$$

Example Problem 3.6 - Orange

In this subtraction, the squares (□) contain the digits 3, 4, 6, and 9 in some order and the hexagons (⬡) contain the digits 4, 5, 8, and 9 in some order.

What four-digit number is represented by the squares?

$$\begin{array}{rrrrr} & \square & \square & \square & \square \\ - & ⬡ & ⬡ & ⬡ & ⬡ \\ \hline & 3 & 4 & 9 & 7 \end{array}$$

Maths Games Example Solution 3.6 - Yellow

In the following cryptarithm, different letters represent different digits.

The letter *O* represents the digit 0.

How many different values are possible for the number represented by *WOOF*?

$$\begin{array}{r} D\ O\ G \\ +\ D\ O\ G \\ \hline W\ O\ O\ F \end{array}$$

Strategy 1: Eliminate Possibilities

$$\begin{array}{r} D\ 0\ G \\ +\ D\ 0\ G \\ \hline W\ 0\ 0\ F \end{array}$$

We begin by noting that the letter *O* represents the digit 0.

O represents 0, so *G* must represent a digit that is not 0.

With 0 in the tens place, we can see that there are only four valid options for *G*.

$$\begin{array}{r} D\ 0\ 1 \\ +\ D\ 0\ 1 \\ \hline W\ 0\ 0\ 2 \end{array} \quad \begin{array}{r} D\ 0\ 2 \\ +\ D\ 0\ 2 \\ \hline W\ 0\ 0\ 4 \end{array} \quad \begin{array}{r} D\ 0\ 3 \\ +\ D\ 0\ 3 \\ \hline W\ 0\ 0\ 6 \end{array} \quad \begin{array}{r} D\ 0\ 4 \\ +\ D\ 0\ 4 \\ \hline W\ 0\ 0\ 8 \end{array}$$

$$\begin{array}{r} D\ 0\ 5 \\ +\ D\ 0\ 5 \\ \hline W\ 0\ \cancel{1}\ 0 \end{array} \quad \begin{array}{r} D\ 0\ 6 \\ +\ D\ 0\ 6 \\ \hline W\ 0\ \cancel{1}\ 2 \end{array} \quad \begin{array}{r} D\ 0\ 7 \\ +\ D\ 0\ 7 \\ \hline W\ 0\ \cancel{1}\ 4 \end{array} \quad \begin{array}{r} D\ 0\ 8 \\ +\ D\ 0\ 8 \\ \hline W\ 0\ \cancel{1}\ 6 \end{array} \quad \begin{array}{r} D\ 0\ 9 \\ +\ D\ 0\ 9 \\ \hline W\ 0\ \cancel{1}\ 8 \end{array}$$

Since the tens place value column consists entirely of 0s, we can be sure that there is no trading between the tens place and the hundreds place.

In the hundreds place, we have *D* + *D* = ☐, where ☐ is a value that ends in a 0.

This means that the only possible value for *D* must be 5, since 5 + 5 = 10.

We now have the following options.

$$\begin{array}{r} 5\ 0\ 1 \\ +\ 5\ 0\ 1 \\ \hline W\ 0\ 0\ 2 \end{array} \quad \begin{array}{r} 5\ 0\ 2 \\ +\ 5\ 0\ 2 \\ \hline W\ 0\ 0\ 4 \end{array} \quad \begin{array}{r} 5\ 0\ 3 \\ +\ 5\ 0\ 3 \\ \hline W\ 0\ 0\ 6 \end{array} \quad \begin{array}{r} 5\ 0\ 4 \\ +\ 5\ 0\ 4 \\ \hline W\ 0\ 0\ 8 \end{array}$$

Finally, since 5 + 5 = 10, we can see that the only possible value for *W* is 1.

We have already used a 1 for the options for *G*.

Since *W* cannot be any value other than 1, it must be the case that *G* cannot be equal to 1.

$$\begin{array}{r} 5\ 0\ \cancel{1} \\ +\ 5\ 0\ \cancel{1} \\ \hline 1\ 0\ 0\ 2 \end{array} \quad \begin{array}{r} 5\ 0\ 2 \\ +\ 5\ 0\ 2 \\ \hline 1\ 0\ 0\ 4 \end{array} \quad \begin{array}{r} 5\ 0\ 3 \\ +\ 5\ 0\ 3 \\ \hline 1\ 0\ 0\ 6 \end{array} \quad \begin{array}{r} 5\ 0\ 4 \\ +\ 5\ 0\ 4 \\ \hline 1\ 0\ 0\ 8 \end{array}$$

There are **3** possible values for the number represented by *WOOF*.

Strategy 2: Solve a Simpler Related Problem, and Eliminate Possibilities

By recognising that there is no trading between the tens place and the hundreds place, we can consider the digits in the hundreds and the thousands place values as a separate addition problem.

$$\begin{array}{r} D\ 0\ G \\ +\ D\ 0\ G \\ \hline W\ 0\ 0\ F \end{array} \rightarrow \begin{array}{r} D \\ +\ D \\ \hline W\ 0 \end{array} \quad \begin{array}{r} G \\ +\ G \\ \hline 0\ F \end{array}$$

Beginning with *D* + *D* = *W*0, we can see that the only possible values for *D* and *W* are 5 and 1 respectively.

$$\begin{array}{r} D \\ +\ D \\ \hline W\ 0 \end{array} \rightarrow \begin{array}{r} 5 \\ +\ 5 \\ \hline 1\ 0 \end{array}$$

The remaining options for *G* and *F* are now 2, 3, 4, 6, 7, 8, and 9.

$$\begin{array}{r} G \\ +\ G \\ \hline 0\ F \end{array} \rightarrow \begin{array}{r} 2 \\ +\ 2 \\ \hline 0\ 4 \end{array} \quad \begin{array}{r} 3 \\ +\ 3 \\ \hline 0\ 6 \end{array} \quad \begin{array}{r} 4 \\ +\ 4 \\ \hline 0\ 8 \end{array}$$

WOOF could therefore be 1004, 1006 or 1008.

There are **3** possible values for the number represented by *WOOF*.

Answers

3.6 - Green: 15

3.6 - Orange: 9346

3.6 - Yellow: 3

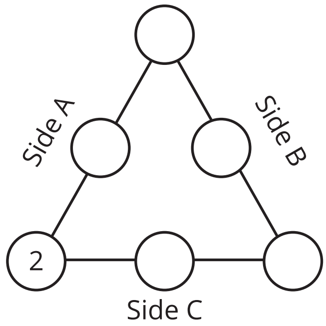
Maths Games – Example Problem 3.7

Example Problem 3.7 - Green

The numbers 1, 2, 3, 4, 5, and 6 are placed in the diagram, one in each circle.

The sum of the three numbers along Side A is 13, along Side B is 13, and along Side C is 6.

What number is in the circle at the top of the diagram?

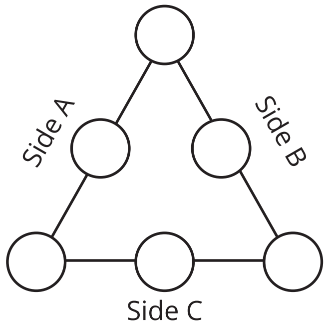


Example Problem 3.7 - Yellow

The numbers 1, 2, 3, 4, 5, and 6 are placed in the diagram, one in each circle.

The sum of the three numbers along Side A is 13, along Side B is 13, and along Side C is 6.

What number is in the circle at the top of the diagram?



Example Problem 3.7 - Orange

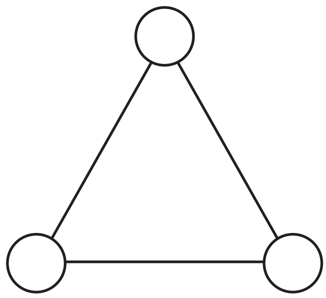
Anna drew three circles joined by three lines.

She wrote a number in each circle.

Then, she added the numbers from each pair of circles, and wrote the sum on the line joining them.

She found that she had all of the numbers 1, 2, 3, 4, 5 and 6 somewhere on her diagram.

What were the numbers in the circles, from smallest to largest?

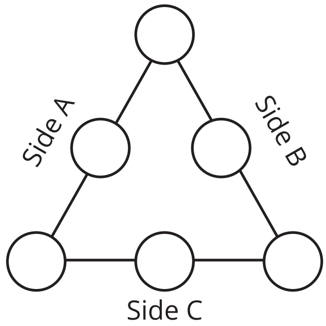


Maths Games Example Solution 3.7 - Yellow

The numbers 1, 2, 3, 4, 5, and 6 are placed in the diagram, one in each circle.

The sum of the three numbers along Side A is 13, along Side B is 13, and along Side C is 6.

What number is in the circle at the top of the diagram?



Strategy: Eliminate All But One Possibility

Using the numbers 1, 2, 3, 4, 5 and 6, we need to find:

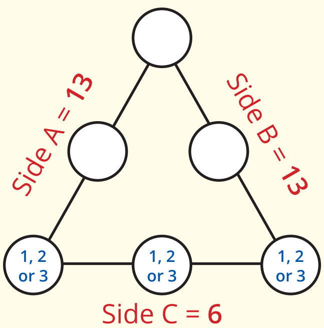
- Three numbers that add to 13,
- Another three numbers that add to 13, and
- Three numbers that add to 6.

Let's start by considering how to make 6.

It's a smaller total, so we'll try smaller numbers.

The three smallest numbers are 1, 2, and 3.

Since 1 + 2 + 3 = 6, we know that this is the only possible combination that will work for Side C.

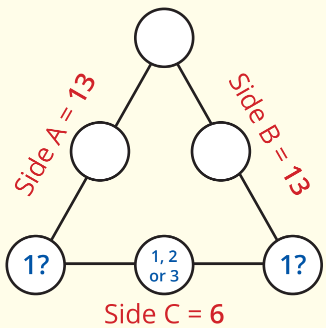


Suppose the value in the bottom left circle is 1.

Then Side A's total of 13 would include a 1.

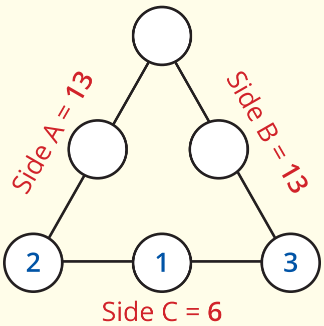
However, since 1 + 5 + 6 = 12 we can see that it's not possible to include the 1 and still make 13.

The same thing would happen if we had 1 in the bottom right circle.



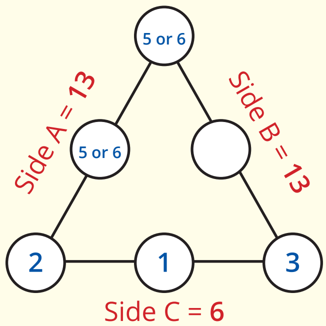
The only position left for 1 is the middle of Side C.

Since both Side A and Side B add up to 13, it doesn't matter which way around we position the 2 and the 3.



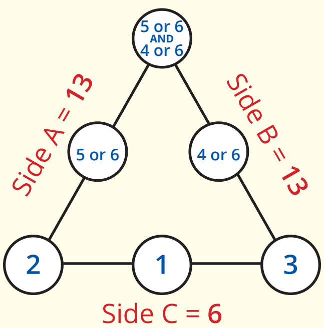
Since Side A's total of 13 includes a 2, we need to find a combination of two of the remaining numbers that add up to 13 - 2 = 11.

Since the remaining numbers are 4, 5 and 6, the only way to make 11 will be 5 + 6.



Since Side B's total of 13 includes a 3, we need to find a combination of two of the remaining numbers that add up to 13 - 3 = 10.

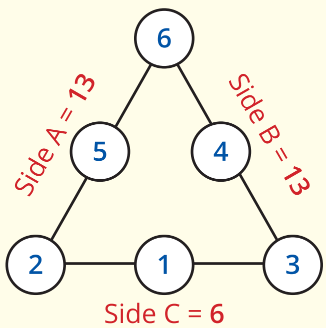
Since the remaining numbers are 4, 5 and 6, the only way to make 10 will be 4 + 6.



We can see that the topmost circle must satisfy the following:

- Either 5 or 6, AND
- Either 4 or 6.

The only way this could possibly work is for the value to actually be 6.



Therefore the number in the circle at the top of the diagram must be **6**.

Answers

3.7 - Green: 6

3.7 - Orange: 1, 2, 4

3.7 - Yellow: 6

Maths Games – Example Problem 3.8

Example Problem 3.8 - Green

In a trivia game, each player is asked 5 questions.
You get 2 points for each correct answer.
If you don't answer a question correctly, you lose 1 point.
At the end of the game, Jennifer's total was 4 points.
How many questions did Jennifer answer correctly?

Example Problem 3.8 - Yellow

In a trivia game, each player is asked 10 questions.
You get 10 points for each correct answer.
If you don't answer a question correctly, you lose 5 points.
At the end of the game, Clint's total was 55 points.
How many questions did Clint answer correctly?

Example Problem 3.8 - Orange

I am building a 50-metre-long wire fence along one side of a straight road.
The wires will be attached to posts, which are set into the ground at 5 metre intervals.
To begin with there are no posts along this stretch of road.
How many posts do I need to construct this fence?

Maths Games Example Problem 3.8 - Solution

In a trivia game, each player is asked 10 questions. You get 10 points for each correct answer.

If you don't answer a question correctly, you lose 5 points. At the end of the game, Clint's total was 55 points.

How many questions did Clint answer correctly?

Strategy 1: Build a Table

In this trivia game, each player is asked **10** questions.

You get **10** points for each correct answer, and you lose **5** points if you don't answer correctly.

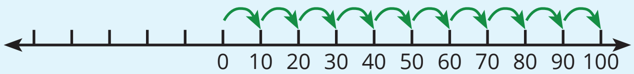
No. questions answered correctly	0	1	2	3	4	5	6	7	8	9	10
No. questions not answered correctly	10	9	8	7	6	5	4	3	2	1	0
Score	-50	-35	-20	-5	10	25	40	55	70	85	100

Clint's total was **55** points, so Clint must have answered **7** questions correctly.

Strategy 2: Draw a Diagram, and Find a Pattern

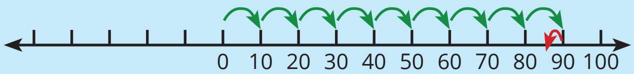
Suppose Clint answered all **10** questions correctly.

He would score **10** × **10** = **100** points.



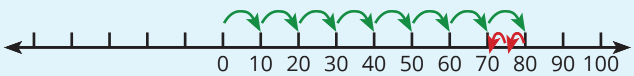
If Clint answered **9** questions correctly, he would score **9** × **10** – **1** × **5** = **85** points.

That's **100** – **85** = **15** less than the highest score.



If Clint answered **8** questions correctly, he would score **8** × **10** – **2** × **5** = **70** points.

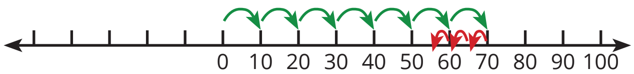
That's **100** – **70** = **30** less than the highest score.



For every incorrect answer, Clint loses **15** points from the high score of **100**. Why does this occur?

A score of **55** means that Clint lost **100** – **55** = **45** points.

Therefore, Clint answered **45** ÷ **15** = **3** questions incorrectly, and **10** – **3** = **7** questions correctly.



Strategy 3: Solve a Simpler Related Problem

Suppose we change the scoring for the trivia game, as follows.

- Every time a player is asked a question, they automatically score **5** points.
- If they answer correctly, they pick up another **10** points.

  This means that they will score a total of **5** + **10** = **15** points for a correct response.
- If they do not answer correctly, they lose the automatic **5** points.

  This means that they will score a total of **5** – **5** = **0** points for an incorrect response.

Using this new scoring method, every participant would receive an extra **10** × **5** = **50** points.

Under the original scoring method, Clint scored **55** points, so using the new method, Clint would have scored **55** + **50** = **105** points.

Using the new method, each correct answer scores **15** points.

To reach a total of **105** points, Clint would have answered **105** ÷ **15** = **7** questions.

Answers

3.8 - Green: 3

3.8 - Yellow: 7

3.8 - Orange: 11

Answers

Set Green

3.1	25
3.2	63
3.3	5
3.4	6
3.5	March
3.6	15
3.7	6
3.8	3

Set Yellow

3.1	23
3.2	438
3.3	20
3.4	10
3.5	December
3.6	3
3.7	6
3.8	7

Set Orange

3.1	11
3.2	1926
3.3	340
3.4	10
3.5	10
3.6	9346
3.7	1, 2, 4
3.8	11