2024 Maths Games Junior - Years 5 & 6
Resource Kit 2
Teaching Problem Solving

MATHS GAMES

Problem Solving Strategies

This resource kit focuses on the following problem solving strategies:

1. Work Backwards

If a problem describes a procedure and then specifies the final result, this method usually makes the problem much easier to solve.

2. Make an Organised List

Listing every possibility in an organised way is an important tool.

How students organise the data often reveals additional information.

It follows on from strategies introduced in the preparation resource kit and resource kit 1:

Guess, Check and Refine

Draw a Diagram

Find a Pattern

Build a Table

Resource Kit 2 focuses on:

Work Backwards

Make an Organised List

Set Yellow

Example problems for which full worked solutions are included.

Set Green

Problems that are designed to be similar to Set Yellow, but with fewer difficult elements.

Set Orange

Problems that are similar in mathematical structure to the corresponding Yellow problems.

Further questions and solution methods can be found in the APSMO resource book "Building Confidence in Maths Problem Solving", available from www.apsmo.edu.au.

How to use these problems

At the start of the lesson, present the problem and ask the students to think about it. Encourage students to try to solve it in any way they like. When the students have had enough time to consider their solutions, ask them to describe or present their methods, taking particular note of different ways of arriving at the same solution.

Each question includes at least one solution method that the majority of students should be able to follow. By participating in lessons that demonstrate achievable problem solving techniques, students may gain increased confidence in their own ability to address unfamiliar problems.

Finally, the consideration of different solution methods is fundamental to the students' development as effective and sophisticated problem solvers. Even when students have solved a problem to their own satisfaction, it is important to expose them to other methods and encourage them to judge whether or not the other methods are more efficient.

Preparation Kit

Guess, Check and Refine

This involves making a reasonable guess of the answer, and checking it against the conditions of the problem. An incorrect guess may provide more information that may lead to the answer.

Draw a Diagram

A diagram may reveal information that may not be obvious just by reading the problem.

It is also useful for keeping track of where the student is up to in a multi-step problem.

Resource Kit 1

Find a Pattern

A frequently used problem solving strategy is that of recognising and extending a pattern.

Students can often simplify a difficult problem by identifying a pattern in the problem situation.

Build a Table

A table displays information so that it is easily located and understood.

A table is an excellent way to record data so the student doesn't have to repeat their efforts.

Resource Kit 2

Work Backwards

If a problem describes a procedure and then specifies the final result, this method usually makes the problem much easier to solve.

Make an Organised List

Listing every possibility in an organised way is an important tool.

How students organise the data often reveals additional information.

Resource Kit 3

Solve a Simpler Related Problem

Many hard problems are actually simpler problems that have been extended to larger numbers.

Patterns can sometimes be identified by trying the problem with smaller numbers.

Eliminate All But One Possibility

Deciding what a quantity is not, can narrow the field to a very small number of possibilities.

These can then be tested against the conditions of the original problem.

Resource Kit 4

Convert to a More Convenient Form

There are times when changing some of the conditions of a problem makes a solution clearer or more convenient.

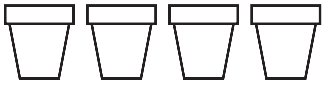
Divide a Complex Shape

Sometimes it is possible to divide an unusual shape into two or more common shapes that are easier to work with.

Set Yellow

2.1) Find the largest factor of 100 that is not divisible by 5.

2.2) Don has four pot plants which he arranges in a straight line.



One pot has herbs, one pot has tomatoes, one pot has daisies, and one pot has bamboo.

In how many different ways can he arrange his four plants?

2.3) In this subtraction, *PR*5*T* and 47*Y*6 represent 4-digit numbers.

What number does *PR*5*T* represent?

$$\begin{array}{cccc} & P & R & 5 & T \\ - & 4 & 7 & Y & 6 \\ \hline & 1 & 9 & 9 & 8 \end{array}$$

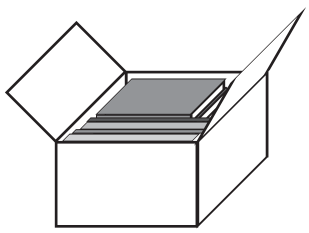
2.4) Some children donated boxes of books for a school fundraiser.

On average, there were 10 books in each box.

Tom then brought in a box of 20 books.

This raised the average to 12 books per box.

How many boxes were there before Tom arrived?



Set Yellow

2.5) Noah has some pairs of table tennis bats (racquets, or paddles).

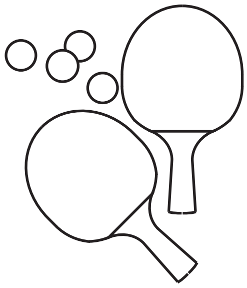
Each pair came with 4 table tennis balls.

Noah lost 8 of the balls.

Then, he gave half of the remaining balls to his friend.

Noah has 6 balls left.

How many bats does he have?



2.6) The average of five numbers is 8.

Two of the numbers are 2 and 5.

The other three numbers are equal.

What is the value of one of the three equal numbers?

2.7) Sandy writes every whole number from 1 to 100 without skipping any numbers.

How many times will Sandy write the digit “2”?

2.8) Rory has four clear plastic pockets on his pencil case.

The pockets contain letter tiles that spell out “*RORY*”.

The two “*R*” tiles look exactly the same as each other.

One day, Rory rearranged the tiles so that they spelled “*YRRO*”.

The next day, he rearranged them to spell a different four-letter word.

How many different four-letter words can be made using his tiles, including the words “*RORY*” and “*YRRO*”?

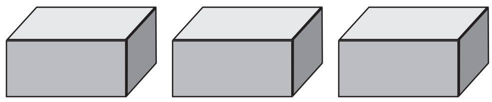
Set Green

2.1) Find the largest factor of 30 that is not divisible by 3.

2.2) Lara has three coloured bricks which she arranges in a row.

One brick is green, one is blue, and one is red.

In how many different ways can she arrange her three bricks?



2.3) In this subtraction, *ABC* and 476 represent 3-digit numbers.

What number does *ABC* represent?

$$\begin{array}{rccc} & A & B & C \\ - & 4 & 7 & 6 \\ \hline & 1 & 9 & 8 \end{array}$$

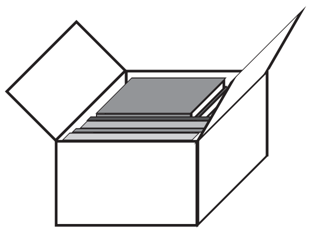
2.4) Some children donated boxes of books for a school fundraiser.

There were 10 books in each box.

Tim then brought in a box of 5 books.

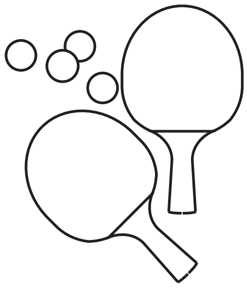
After they shared the books equally between the boxes, there were 11 books in each box.

How many boxes were there before Tom arrived?



Set Green

2.5) Noah has some pairs of table tennis bats (racquets, or paddles).
Each pair came with 2 table tennis balls.
Noah lost 5 of the balls.
Then, he gave two of the remaining balls to his friend.
Noah has 3 balls left.
How many bats does he have?



2.6) The average of three numbers is 8.
Two of the numbers are 2 and 5.
What is the value of the third number?

2.7) Sandy writes every whole number from 1 to 30 without skipping any numbers.
How many times will Sandy write the digit “2”?

2.8) Ava has three clear plastic pockets on her pencil case.
The pockets contain letter tiles that spell out “*AVA*”.
The two “*A*” tiles look exactly the same as each other.
One day, Ava rearranged the tiles so that they spelled “*VAA*”.
How many different three-letter words can be made using her tiles, including the words “*AVA*” and “*VAA*”?

Set Orange

2.1) Find the largest factor of 1000 that is not divisible by 2.

2.2) Angela, Ben, Caroline and David need to sit in a row on the stage to receive awards.
Angela and Caroline have to sit together.
Ben and David have to sit together.
In how many different ways can they sit?

2.3) In the addition shown, different letters represent different digits.
What four-digit number would be represented by *ABCD*?

$$\begin{array}{cccccc} & & & & 8 & A \\ & & & 2 & B & 9 \\ & & 1 & C & 6 & 7 \\ + & 1 & D & 7 & 5 & 4 \\ \hline & 1 & 2 & 3 & 4 & 5 \end{array}$$

2.4) The average of 5 weights is 13 grams.
This set of 5 weights is then increased by another weight of 7 grams.
What is the average of the six weights?

Set Orange

2.5) Ms Williams spent $\frac{3}{5}$ of her money on clothing.

She then spent $\frac{2}{5}$ of her remaining money at the gas station.

Lastly, she spent $\frac{1}{5}$ of the remaining money on hamburgers.

She ended the day with $48.

How much money did Ms Williams have at the start of the day?

2.6) The average height of four adults is 180 centimetres.

Two of the adults are each 170 centimetres tall, and the third is 185 centimetres tall.

How tall, in centimetres, is the fourth adult?

2.7) The number 10 is even.

10 has digits 1 and 0. The sum of the digits is 1 + 0 = 1, which is odd.

The number 24 is also even.

24 has digits 2 and 4. The sum of the digits is 2 + 4 = 6, which is even.

How many even 2-digit numbers have an odd number as the sum of their digits?

2.8) Mark wants to create a password for his computer.

He wants it to consist of one letter chosen from the word *MARK*, one even-numbered digit, and one odd-numbered digit.

These three characters can appear in any order.

How many different passwords could Mark create?

Maths Games – Example Problem 2.1

Example Problem 2.1 - Green

Find the largest factor of 30 that is not divisible by 3.

Example Problem 2.1 - Yellow

Find the largest factor of 100 that is not divisible by 5.

Example Problem 2.1 - Orange

Find the largest factor of 1000 that is not divisible by 2.

Maths Games Example Solution 2.1 - Yellow

Find the largest factor of 100 that is not divisible by 5.

Strategy 1: Make an Organised List

To list the factors of **100** in an organised way, we can consider every whole number, beginning with 1.

If the number divides evenly into **100** with no remainder, then both the number, and its factor pair, will be factors of **100**.

We can stop as soon as we find a factor pair where the second number is less than or equal to the number we are testing.

(Why is this?)

Number	Factor pair for 100	Factors found so far
1	100 ÷ 1 = 100	1, 100
2	100 ÷ 2 = 50	1, 2, 50, 100
3	100 ÷ 3 = 33 r. 1	*3 is not a factor*
4	100 ÷ 4 = 25	1, 2, 4, 25, 50, 100
5	100 ÷ 5 = 20	1, 2, 4, 5, 20, 25, 50, 100
6	100 ÷ 6 = 16 r. 4	*6 is not a factor*
7	100 ÷ 7 = 14 r. 2	*7 is not a factor*
8	100 ÷ 8 = 12 r. 4	*8 is not a factor*
9	100 ÷ 9 = 11 r. 1	*9 is not a factor*
10	100 ÷ 10 = 10	1, 2, 4, 5, 10, 20, 25, 50, 100

We can see that the factors of **100** are 1, 2, 4, 5, 10, 20, 25, 50, and 100.

We know that numbers that end in a **5** or a **0**, will be divisible by **5**. (Why is this?)

Therefore, the largest factor of **100** that is not divisible by **5**, is **4**.

Strategy 2: Use Number Sense

To find factors of **100** that are not divisible by **5**, we can just keep dividing by **5**, as long as the result has no remainder.

100 ÷ 5 = 20

20 ÷ 5 = 4

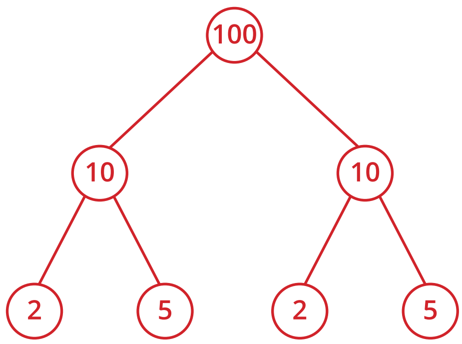
Since **4** is not divisible by **5**, we can see that **4** must be the largest factor of **100** that is not divisible by **5**.

Strategy 3: Consider Prime Factors

Apart from **1**, all of the factors of a number can be expressed as products of its prime factors.

We can use a factor tree to find the prime factors of **100**.

It involves breaking **100** into factor pairs, and then breaking each of those factors into factor pairs, until every resulting factor is prime.



We can see that **100** can be expressed as a product of the following prime factors:

100 = 2 × 5 × 2 × 5

= 2 × 2 × 5 × 5

Therefore, the largest factor of **100** that is not divisible by **5** is 2 × 2 = **4**.

Answers

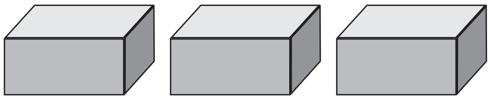
2.1 - Green: 10

2.1 - Orange: 125

2.1 - Yellow: 4

Maths Games – Example Problem 2.2

Example Problem 2.2 - Green

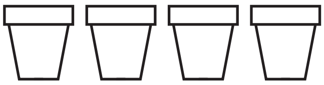


Lara has three coloured bricks which she arranges in a row.

One brick is green, one is blue, and one is red.

In how many different ways can she arrange her three bricks?

Example Problem 2.2 - Yellow



Don has four pot plants which he arranges in a straight line.

One pot has herbs, one pot has tomatoes, one pot has daisies, and one pot has bamboo.

In how many different ways can he arrange his four plants?

Example Problem 2.2 - Orange

Angela, Ben, Caroline and David need to sit in a row on the stage to receive awards.

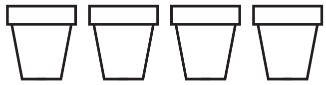
Angela and Caroline have to sit together.

Ben and David have to sit together.

In how many different ways can they sit?

Maths Games Example Solution 2.2 - Yellow

Don has four pot plants which he arranges in a straight line. One pot has herbs, one pot has tomatoes, one pot has daisies, and one pot has bamboo.



In how many different ways can he arrange his four plants?

Strategy 1: Make an Organised List

Don has four pot plants. There is one each of herbs, tomatoes, daisies and bamboo.

Let's begin by listing them alphabetically. →

We'll start with all of the options where the left-most pot is bamboo.

How many possibilities would there be if the daisies came next?

How many possibilities would there be if the herbs came next?

How many possibilities would there be if the tomatoes came next?

Every time we change the first pot plant, we'll get the same number of different possibilities.

There are **4** different ways to select the plant for the first (left-most) position.

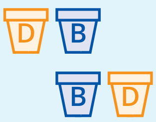
For each possible plant in the first position, there are **3** possible plants that can be placed in the second position, followed by **2** possible plants for the 3rd position, and **1** remaining plant for the fourth position.

All together, there are **4 × 3 × 2 × 1 = 24** ways to arrange Don's pot plants.

Strategy 2: Count in an Organised Way

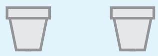
Let's put the bamboo down first.

We can now put the daisies either to the left or to the right of the bamboo.

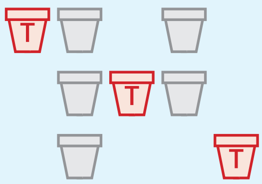


There are **2** possible positions for the daisies, relative to the bamboo.

So far, we have placed bamboo and daisies in some order.

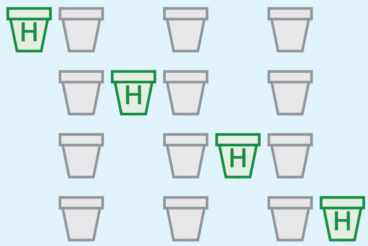


We can now put the tomatoes in one of **3** different positions relative to the plants that are already there.



With **2** ways to arrange the bamboo and daisies, and **3** ways to arrange the tomatoes around them, there are **2 × 3 = 6** different arrangements for these three plants.

The herbs can now be placed in one of **4** different positions relative to the other plants.



Therefore there are **2 × 3 × 4 = 24** ways to arrange Don's pot plants.

Answers

2.2 - Green: 6

2.2 - Orange: 8

2.2 - Yellow: 24

Maths Games – Example Problem 2.3

Example Problem 2.3 - Green

In this subtraction, *ABC* and 476 represent 3-digit numbers.
What number does *ABC* represent?

$$\begin{array}{cccc} & A & B & C \\ - & 4 & 7 & 6 \\ \hline & 1 & 9 & 8 \end{array}$$

Example Problem 2.3 - Yellow

In this subtraction, *PR*5*T* and 47*Y*6 represent 4-digit numbers.
What number does *PR*5*T* represent?

$$\begin{array}{ccccc} & P & R & 5 & T \\ - & 4 & 7 & Y & 6 \\ \hline & 1 & 9 & 9 & 8 \end{array}$$

Example Problem 2.3 - Orange

In the addition shown, different letters represent different digits.
What four-digit number would be represented by *ABCD*?

$$\begin{array}{cccccc} & & & & 8 & A \\ & & & 2 & B & 9 \\ & & 1 & C & 6 & 7 \\ + & 1 & D & 7 & 5 & 4 \\ \hline & 1 & 2 & 3 & 4 & 5 \end{array}$$

Maths Games Example Solution 2.3 - Yellow

In this subtraction, *PR5T* and 47*Y*6 represent 4-digit numbers.

What number does *PR5T* represent?

$$\begin{array}{cccc} & P & R & 5 & T \\ - & 4 & 7 & Y & 6 \\ \hline & 1 & 9 & 9 & 8 \end{array}$$

Strategy: Work Backwards (1)

In the ones place, we have *T* – 6 = 8.

Working backwards, we can see that

T = 6 + 8 = 14

$$\begin{array}{cccc} & P & R & 5 & T \\ - & 4 & 7 & Y & 6 \\ \hline & 1 & 9 & 9 & 8 \end{array}$$

Since *T* can only represent a single digit here, the value 14 must be the result after trading a ten from the tens place.

Therefore, *T* = 4, and we'll note the trading.

$$\begin{array}{cccc} & P & R & \overset{4}{\cancel{5}} & {}^{1}4 \\ - & 4 & 7 & Y & 6 \\ \hline & 1 & 9 & 9 & 8 \end{array}$$

In the tens place, we now have 4 – *Y* = 9.

Since 4 < 9, that's not going to work for the subtraction algorithm. So we must have traded a ten from the hundreds place.

14 – *Y* = 9

Therefore, *Y* = 5, and once again we'll note the trading.

$$\begin{array}{cccc} & P & \overset{(R\text{-}1)}{\cancel{R}} & \overset{14}{\cancel{5}} & {}^{1}4 \\ - & 4 & 7 & 5 & 6 \\ \hline & 1 & 9 & 9 & 8 \end{array}$$

In the hundreds place, we now have (*R*-1) – 7 = 9.

Working backwards, we can see that

R – 1 = 7 + 9 = 16
R = 17

$$\begin{array}{cccc} & \overset{(P\text{-}1)}{\cancel{P}} & \overset{16}{\cancel{7}} & \overset{14}{\cancel{5}} & {}^{1}4 \\ - & 4 & 7 & 5 & 6 \\ \hline & 1 & 9 & 9 & 8 \end{array}$$

Since *R* can only represent a single digit here, the value 17 must be the result after trading a ten from the thousands place.

Therefore, *R* = 7, and we'll note the trading.

In the thousands place, we now have (*P*-1) – 4 = 1.

Working backwards, we can see that

P – 1 = 4 + 1 = 5
P = 6

$$\begin{array}{cccc} & \overset{5}{\cancel{6}} & \overset{16}{\cancel{7}} & \overset{14}{\cancel{5}} & {}^{1}4 \\ - & 4 & 7 & 5 & 6 \\ \hline & 1 & 9 & 9 & 8 \end{array}$$

Therefore, *PR5T* represents **6754**.

Strategy: Work Backwards (2)

We can work backwards from the answer to the subtraction.

If *PR5T* – 47*Y*6 = 1998, then we know that 47*Y*6 + 1998 = *PR5T*.

$$\begin{array}{cccc} & 4 & 7 & Y & 6 \\ + & 1 & 9 & 9 & 8 \\ \hline & P & R & 5 & T \end{array}$$

Starting with the ones place:

6 + 8 = 14

4 goes in the ones place, where it says *T* in the answer.

$$\begin{array}{cccc} & 4 & 7 & \overset{1}{Y} & 6 \\ + & 1 & 9 & 9 & 8 \\ \hline & P & R & 5 & 4 \end{array}$$

So *T* = 4, and we'll carry the 10 from the 14 to the next place value.

In the tens place, we have

1 + *Y* + 9 = ...5

Since 1 + 9 = 10, the sum will be greater than 5, so it must be 15:

1 + *Y* + 9 = 15

$$\begin{array}{cccc} & 4 & \overset{1}{7} & \overset{1}{5} & 6 \\ + & 1 & 9 & 9 & 8 \\ \hline & P & R & 5 & 4 \end{array}$$

So *Y* = 5, and we'll carry the 10 from the 15 to the next place value.

In the hundreds place,

1 + 7 + 9 = 17

So *R* = 7, and we'll carry the 10 from the 17 to the next place value.

$$\begin{array}{cccc} & \overset{1}{4} & \overset{1}{7} & \overset{1}{5} & 6 \\ + & 1 & 9 & 9 & 8 \\ \hline & P & 7 & 5 & 4 \end{array}$$

In the thousands place,

1 + 4 + 1 = 6

So *P* = 6.

$$\begin{array}{cccc} & \overset{1}{4} & \overset{1}{7} & \overset{1}{5} & 6 \\ + & 1 & 9 & 9 & 8 \\ \hline & 6 & 7 & 5 & 4 \end{array}$$

So *PR5T* represents **6754**.

Let's check: **6754** – **4756** = **1998**.

Answers

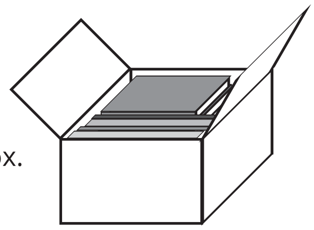
2.3 - Green: 674

2.3 - Orange: 5320

2.3 - Yellow: 6754

Maths Games – Example Problem 2.4

Example Problem 2.4 - Green



Some children donated boxes of books for a school fundraiser.

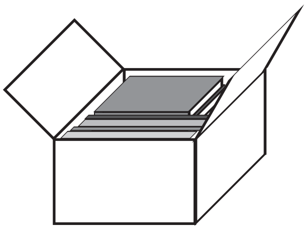
There were 10 books in each box.

Tim then brought in a box of 5 books.

After they shared the books equally between the boxes, there were 11 books in each box.

How many boxes were there before Tom arrived?

Example Problem 2.4 - Yellow



Some children donated boxes of books for a school fundraiser.

On average, there were 10 books in each box.

Tom then brought in a box of 20 books.

This raised the average to 12 books per box.

How many boxes were there before Tom arrived?

Example Problem 2.4 - Orange

The average of 5 weights is 13 grams.

This set of 5 weights is then increased by another weight of 7 grams.

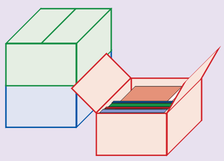
What is the average of the six weights?

Maths Games Example Solution 2.4 - Yellow

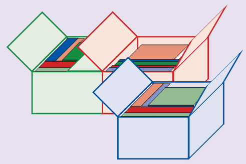
Some children donated boxes of books for a school fundraiser. On average, there were 10 books in each box. Tom then brought in a box of 20 books. This raised the average to 12 books per box.
How many boxes were there before Tom arrived?

Strategy 1: Work Backwards

Let's begin by thinking about how we would find the average number of books in each box.



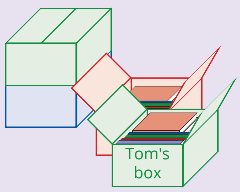
First, we would find the total number of books in all of the boxes.



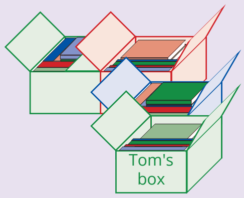
Then, we would divide the books equally between the boxes.

We know that, on average, there were **10** books in each box.
So after sharing the books equally between the boxes, there were **10** books in each box.

Let's see what happens when Tom adds his box of **20** books.



The total number of books goes up by **20**, and the total number of boxes goes up by **1**.



When we divide the books equally between the boxes, there are now **12** books in each box, including Tom's box.

- To get from **20** books down to **12** books in Tom's box, we need to take out **20 – 12 = 8** books.
- To get from **10** books up to **12** books in the other boxes, we would need to add **12 – 10 = 2** books to each box.

The **8** books being taken out of Tom's box must be added to **8 ÷ 2 = 4** other boxes.
So there must have been **4** boxes before Tom arrived.

Strategy 2: Guess, Check and Refine, and Build a Table

Let's guess that there were **2** boxes to begin with.
With an average of **10** books per box, that's **2 × 10 = 20** books.
After Tom arrived, there would have been **20** + **20** = **40** books and **2** + **1** = **3** boxes.
That's **40 ÷ 3 = $13\frac{1}{3}$** books on average per box, which is impossible.

No. of boxes before Tom arrived	2		
No. of books before Tom arrived	20		
No. of books with Tom's 20 books	40		
No. of boxes including Tom's box	3		
Average after Tom arrived	$13\frac{1}{3}$		

If there were **3** boxes to begin with, that's **3 × 10 = 30** books.
After Tom arrived, there would have been **30** + **20** = **50** books, **3** + **1** = **4** boxes, and **50 ÷ 4 = $12\frac{1}{2}$** books on average per box.

If there were **4** boxes to begin with, that's **4 × 10 = 40** books.
After Tom arrived, there would have been **40** + **20** = **60** books, **4** + **1** = **5** boxes, and **60 ÷ 5 = 12** books on average per box.

No. of boxes before Tom arrived	2	3	4
No. of books before Tom arrived	20	30	40
No. of books with Tom's **20** books	40	50	60
No. of boxes including Tom's box	3	4	5
Average after Tom arrived	$13\frac{1}{3}$	$12\frac{1}{2}$	12

That matches the question.
So there must have been **4** boxes to begin with.

Answers

2.4 - Green: 5

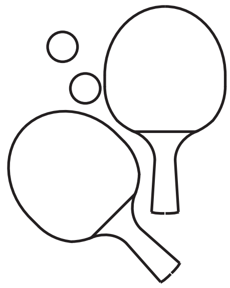
2.4 - Orange: 12 grams

2.4 - Yellow: 4

Maths Games – Example Problem 2.5

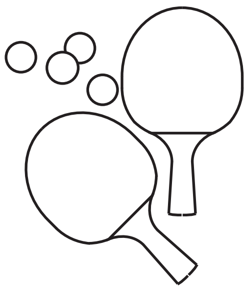
Example Problem 2.5 - Green

Noah has some pairs of table tennis bats (racquets, or paddles).
Each pair came with 2 table tennis balls.
Noah lost 5 of the balls.
Then, he gave two of the remaining balls to his friend.
Noah has 3 balls left.
How many bats does he have?



Example Problem 2.5 - Yellow

Noah has some pairs of table tennis bats (racquets, or paddles).
Each pair came with 4 table tennis balls.
Noah lost 8 of the balls.
Then, he gave half of the remaining balls to his friend.
Noah has 6 balls left.
How many bats does he have?

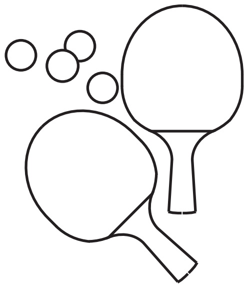


Example Problem 2.5 - Orange

Ms Williams spent $\frac{3}{5}$ of her money on clothing.
She then spent $\frac{2}{5}$ of her remaining money at the gas station.
Lastly, she spent $\frac{1}{5}$ of the remaining money on hamburgers.
She ended the day with $48.
How much money did Ms Williams have at the start of the day?

Maths Games Example Solution 2.5 - Yellow

Noah has some pairs of table tennis bats (racquets, or paddles). Each pair came with 4 table tennis balls. Noah lost 8 of the balls. Then, he gave half of the remaining balls to his friend. Noah has 6 balls left.



How many bats does he have?

Strategy 1: Work Backwards

Let's use a bar to represent the number of bats.

No. of bats

If each pair of bats comes with 4 balls, then it's the same as each bat coming with 2 balls.

No. of bats	*No. of bats*
No. of balls	

After Noah lost 8 balls,
and gave away half of the remaining balls,
he had 6 balls left.

Lost 8 balls	No. of balls remaining	
Lost 8 balls	*Gave away half*	6

Before giving away half of the remaining balls, Noah must have had 2 × 6 = 12 balls.

Lost 8 balls	12

Before losing 8 balls, Noah must have had 12 + 8 = 20 balls.

20

Therefore, since each bat comes with 2 balls, Noah must have 20 ÷ 2 = 10 bats.

10	10

Strategy 2: Build a Table, and Draw a Diagram

	No. of Bats	No. of Balls	After losing 8 balls	After giving away half
If Noah had just 1 pair of bats, he would have 4 table tennis balls. That's not enough for him to have lost 8 balls.		○○○○		
With 3 pairs of bats, he'd have 3 × 4 = 12 balls. After losing 8 balls he would have 12 – 8 = 4 balls. After giving away half of the remaining balls, he would have 4 ÷ 2 = 2 balls left.		○○○○ ○○○○ ○○○○	○○○○ ○○○○ ○○○○	○○○○
With 4 pairs of bats, he'd have 4 × 4 = 16 balls. After losing 8 balls he would have 16 – 8 = 8 balls. After giving away half of the remaining balls, he would have 8 ÷ 2 = 4 balls left. This is closer to the target of having 6 balls left.		○○○○ ○○○○ ○○○○ ○○○○	○○○○ ○○○○ ○○○○ ○○○○	○○○○ ○○○○
With 5 pairs of bats, he'd have 5 × 4 = 20 balls. After losing 8 balls he would have 20 – 8 = 12 balls. After giving away half of the remaining balls, he would have 12 ÷ 2 = 6 balls left. That matches the question.		○○○○ ○○○○ ○○○○ ○○○○ ○○○○	○○○○ ○○○○ ○○○○ ○○○○ ○○○○	○○○○ ○○○○ ○○○○

So from the diagram in the table, we can see that Noah has **10** table tennis bats.

Answers **2.5 - Green:** 10 **2.5 - Orange:** $250

2.5 - Yellow: 10

Maths Games – Example Problem 2.6

Example Problem 2.6 - Green

The average of three numbers is 8.

Two of the numbers are 2 and 5.

What is the value of the third number?

Example Problem 2.6 - Yellow

The average of five numbers is 8.

Two of the numbers are 2 and 5.

The other three numbers are equal.

What is the value of one of the three equal numbers?

Example Problem 2.6 - Orange

The average height of four adults is 180 centimetres.

Two of the adults are each 170 centimetres tall, and the third is 185 centimetres tall.

How tall, in centimetres, is the fourth adult?

Maths Games Example Solution 2.6 - Yellow

The average of five numbers is 8. Two of the numbers are 2 and 5. The other three numbers are equal.

What is the value of one of the three equal numbers?

Strategy 1: Solve a Simpler Related Problem, and Work Backwards

Finding the average is like finding the amount that would be an "equal share".

To find the average of five numbers, we would:

- Add the five numbers together, and
- Divide the result by **5**.

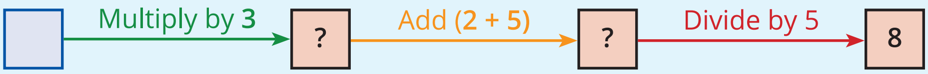
Let's try putting in a value for the three equal numbers - say, 1.

To find the average, we would

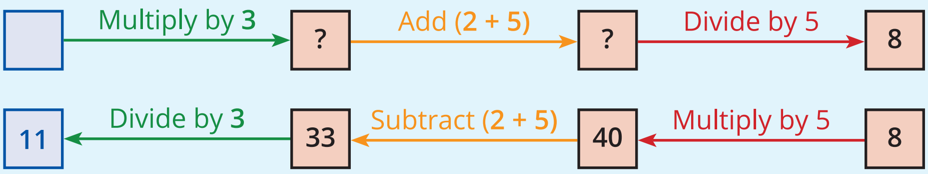
- Add the five numbers together: 1 + 1 + 1 + 2 + 5 = 10

  Since there are 3 of the equal numbers, we can think of 1 + 1 + 1 as (3 × 1). The sum can be written as (3 × 1) + 2 + 5 = 10.
- Divide the result by 5: 10 ÷ 5 = 2.

Let's use a box ( ) to represent one of these three equal numbers, and go through the actions to create the average, which we know is **8**.



By reversing each operation, we can work backwards from the average back to the value of the three equal numbers.

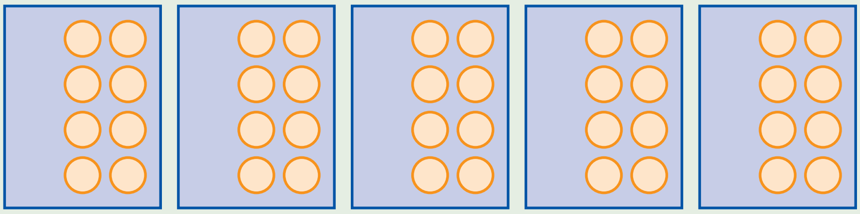


Working backwards from the average, we can find that the value of each of the three equal numbers is **11**.

Strategy 2: Draw a Diagram

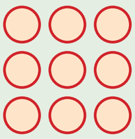
Let's pretend that the five numbers are actually the numbers of oranges in these five boxes.

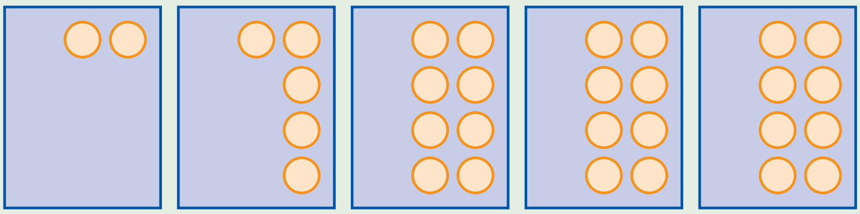
Since the average is **8**, if we shared all of the oranges equally between the boxes, we'd have **8** in each box.



Two of the numbers are **2** and **5**.

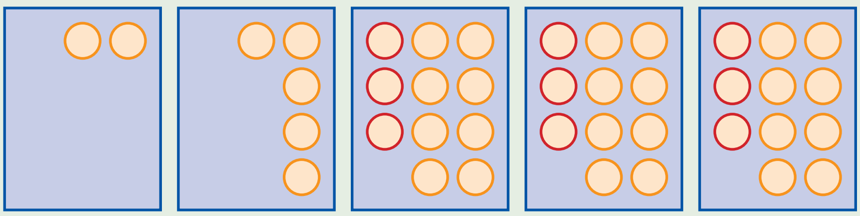
To represent this, we'll move some oranges out of the first couple of boxes so that one box has **2**, and the other has **5**.





Since the other three numbers are equal, we need to distribute the oranges we took out, equally amongst the three other boxes.

We can see that each of the three equal numbers must be **11**.



Answers

2.6 - Green: 17

2.6 - Orange: 195

2.6 - Yellow: 11

Maths Games – Example Problem 2.7

Example Problem 2.7 - Green

Sandy writes every whole number from 1 to 30 without skipping any numbers.

How many times will Sandy write the digit “2”?

Example Problem 2.7 - Yellow

Sandy writes every whole number from 1 to 100 without skipping any numbers.

How many times will Sandy write the digit “2”?

Example Problem 2.7 - Orange

The number 10 is even.

10 has digits 1 and 0. The sum of the digits is 1 + 0 = 1, which is odd.

The number 24 is also even.

24 has digits 2 and 4. The sum of the digits is 2 + 4 = 6, which is even.

How many even 2-digit numbers have an odd number as the sum of their digits?

Maths Games Example Solution 2.7 - Yellow

Sandy writes every whole number from 1 to 100 without skipping any numbers.

How many times will Sandy write the digit “2”?

Strategy 1: Make an Organised List

Sandy starts from **1** and then writes every whole number up to **100**.

So the first time Sandy writes the digit "2" will be for the number 2.	2
The next time Sandy writes the digit "2" will be for the number 12.	2, 12
Let's write all of the numbers that will have 2 in the ones place.	2, 12, 22, 32, 42, 52, 62, 72, 82, 92

We can see that there are **10** such numbers.

This is because there would be **10** possible digits in the **10**s place if we include **0** (to represent the number **0**2, which we would simply write as 2).

Now that we have done this, we can use the same idea to write all of the numbers that have the digit 2 in the tens place.	20, 21, 22, 23, 24, 25, 26, 27, 28, 29

Again, there are **10** such numbers.

Since the number **22** appears twice - once in each list - we'll put it aside for now. There are **9** + **9** = **18** numbers that include a single digit **2**.	2, 12, 32, 42, 52, 62, 72, 82, 92 20, 21, 23, 24, 25, 26, 27, 28, 29
The number **22** has two digits "**2**". Therefore Sandy would write the digit "2" a total of **18** + **2** = **20** times.	2, 12, 32, 42, 52, 62, 72, 82, 92 20, 21, 23, 24, 25, 26, 27, 28, 29 22,

Strategy 2: Build a Table, and Use Number Sense

Tens \ Ones	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	10	11	12	13	14	15	16	17	18	19
2	20	21	22	23	24	25	26	27	28	29
3	30	31	32	33	34	35	36	37	38	39
4	40	41	42	43	44	45	46	47	48	49
5	50	51	52	53	54	55	56	57	58	59
6	60	61	62	63	64	65	66	67	68	69
7	70	71	72	73	74	75	76	77	78	79
8	80	81	82	83	84	85	86	87	88	89
9	90	91	92	93	94	95	96	97	98	99

Since **100** does not include the digit "**2**", we can think of Sandy's list as comprising every number from **1** to **99**.

Each number from **1** to **99** has a digit in the ones place.

Each number from **10** to **99** has a digit in the tens place.

We can use a table to show how these numbers are constructed.

We can see that:

- There are **10** "2"s in the ones place, and
- there are **10** "2"s in the tens place.

Therefore Sandy must have written the digit "2" **10** + **10** = **20** times.

Answers

2.7 - Green: 13

2.7 - Orange: 25

2.7 - Yellow: 20

Maths Games – Example Problem 2.8

Example Problem 2.8 - Green

Ava has three clear plastic pockets on her pencil case.

The pockets contain letter tiles that spell out "*AVA*".

The two "*A*" tiles look exactly the same as each other.

One day, Ava rearranged the tiles so that they spelled "*VAA*".

How many different three-letter words can be made using her tiles, including the words "*AVA*" and "*VAA*"?

Example Problem 2.8 - Yellow

Rory has four clear plastic pockets on his pencil case.

The pockets contain letter tiles that spell out "*RORY*".

The two "*R*" tiles look exactly the same as each other.

One day, Rory rearranged the tiles so that they spelled "*YRRO*".

The next day, he rearranged them to spell a different four-letter word.

How many different four-letter words can be made using his tiles, including the words "*RORY*" and "*YRRO*"?

Example Problem 2.8 - Orange

Mark wants to create a password for his computer.

He wants it to consist of one letter chosen from the word *MARK*, one even-numbered digit, and one odd-numbered digit.

These three characters can appear in any order.

How many different passwords could Mark create?

Maths Games Example Problem 2.8 - Solution

Rory has four clear plastic pockets on his pencil case. The pockets contain letter tiles that spell out "*RORY*".

One day, Rory rearranged the tiles so that they spelled "*YRRO*".

The next day, he rearranged them to spell a different four-letter word.

How many different four-letter words can be made using his tiles, including the words "*RORY*" and "*YRRO*"?

Strategy 1: Make an Organised List

Let's list the options alphabetically.

First we will think about the words starting with ***O***.

- We have ***R***, ***R***, ***Y*** left.
- It's like they are all ***R***s and the only one that is different is ***Y***. So we just need to think about the different places the ***Y*** can be.

O R R Y

O R Y R

O Y R R

Next, let's think about the words starting with ***R***.

- We have ***O***, ***R***, ***Y*** left.
- What options are possible if the second letter is ***O***?

R O R Y

R O Y R

- What options are possible if the second letter is the other ***R***?

R R O Y

R R Y O

- What options are possible if the second letter is ***Y***?

R Y O R

R Y R O

Finally, let's think about the words starting with ***Y***.

- We have ***O***, ***R***, ***R*** left.
- We've done one like this before.
- It's like they are all ***R***s and the only one that is different is ***O***. So we need to think about the different places the ***O*** can be.

Y O R R

Y R O R

Y R R O

Since we counted them in an organised way, we can be sure that we have found all of the possible arrangements.

So Rory can use his tiles to make **12** different four-letter words.

Strategy 2: Count in an Organised Way

Let's begin by pretending that all of the letter tiles are different. So, instead of ***RORY***, we'll imagine that the letter tiles have the letters "***ROrY***".

There are four clear plastic pockets on Rory's pencil case.

Since we have 4 different tiles, there would be 4 different ways to choose a tile for the first pocket.

1st pocket could be: R

or O

or r

or Y

Let's suppose we placed the ***r*** in the first pocket.

Then there would be 3 remaining options for the second pocket.

Whichever letter we chose for the first pocket, there would be 3 options remaining for the second pocket.

2nd pocket could be: r R

or r O

or r Y

There would then be 2 different options remaining for the 3rd pocket,

3rd pocket could be: r Y R

or r Y O

and then just 1 option left for the last pocket.

r Y R O

With 4 ways to choose the first tile, 3 ways for the second, 2 ways for the third and 1 way for the fourth, there are 4 × 3 × 2 × 1 = **24** ways to arrange these tiles.

However, the **24** combinations would include both ***ROrY*** and ***rORY***, and ***YRrO*** and ***YrRO***, and so on. With two different Rs, we have ended up double-counting each word.

So there are really just **24** ÷ **2** = **12** different four-letter arrangements for Rory's tiles.

Answers

2.8 - Green: 3

2.8 - Orange: 600

2.8 - Yellow: 12

Answers

Set Green

2.1	10
2.2	6
2.3	674
2.4	5
2.5	10
2.6	17
2.7	13
2.8	3

Set Yellow

2.1	4
2.2	24
2.3	6754
2.4	4
2.5	10
2.6	11
2.7	20
2.8	12

Set Orange

2.1	125
2.2	8
2.3	5320
2.4	12 grams
2.5	$250
2.6	195
2.7	25
2.8	600