Important

The information contained in this file is ONLY for the use of registered participants of the 2024 APSMO Maths Games.

Organisation and Procedures

For full details, see the Members' Area

- Maths Games papers are to be conducted under test conditions.

DO	DO NOT
• Supervise students at all times. • Maintain silence. • Provide blank working paper. • Collect, mark and retain the papers.	• Print the papers prior to the scheduled date. • Read the questions aloud to the students. • Interpret the questions for students. • Permit any discussion or movement around the room. • Permit the use of calculators or other electronic devices.

- Papers should be scored by the PICO using the *Solutions and Answers* sheet provided.
- Original student answer sheets should be retained by the PICO until the end of the year.

Absent Students

- A student who is legitimately absent on the date of the Maths Games paper, may sit the paper on their return to school.
- If an absent student does not sit the paper on their return to school they should be marked as 'absent'.
- *Note: This policy differs from the Maths Olympiads Absent Student Policy which has additional requirements*.

APSMO

MATHS GAMES

WEDNESDAY 4 SEPTEMBER 2024

MATHS GAMES JUNIOR

Suggested Time: ***30 Minutes***

Write your answers in the boxes on the back.

Keep your answers hidden by folding backwards on this line.

4A. Brandon correctly works out the value of

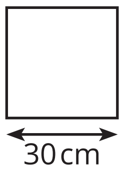
4 × 5 × 4 × 5 × 4 × 5

How many digits are in his answer?

Hint: Can you group the numbers to make this easier to work out?

4B. Harry has two squares of wrapping paper.

Each square has a side length of 30 cm.



He overlaps the two squares, and sticks them together to make a rectangle of wrapping paper that measures 50 cm × 30 cm.

What is the area of the overlapping region, in square centimetres?

Hint: You can draw a diagram to show how the two squares overlap.

4C. Lucy makes a chain using 40 paper clips.

The first paper clip is silver. She then repeats the same pattern for the entire length of the paper clip chain, as follows:

silver, gold, pink, silver, gold, pink, silver, gold, pink ...

How many silver paper clips are there in the chain?

Hint: How long is the repeating pattern?

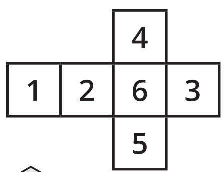
4D. A box contains 6 pens: 1 green pen, 2 red pens, and 3 blue pens.

Annie reaches into the box without looking.

What is the smallest number of pens she must take out to be certain that she is holding at least one of each colour?

Hint: How many pens would Annie need to take out, to be certain she has at least two colours?

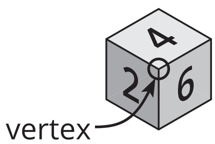
4E. The diagram shows the net of a number cube.



When the net is folded to make a cube, there are three faces that meet at each vertex (corner).

For example, 2, 4 and 6 all meet at one vertex.

What is the smallest sum of three numbers around a single vertex?



Hint: There are 8 vertices (corners) on a cube.

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WEDNESDAY 4 SEPTEMBER 2024

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JUNIOR

4A.

4B.

4C.

4D.

4E.

Student Name:

Fold here. Keep your answers hidden.

MATHS GAMES

APSMO

WEDNESDAY 4 SEPTEMBER 2024

MATHS GAMES JUNIOR

Solutions and Answers

(Items in parentheses are not required)

4A: 4	**4B:** 300 (cm²)	**4C:** 14	**4D:** 6	**4E:** 7

4A. The question is, How many digits are in the value of 4 × 5 × 4 × 5 × 4 × 5 ?

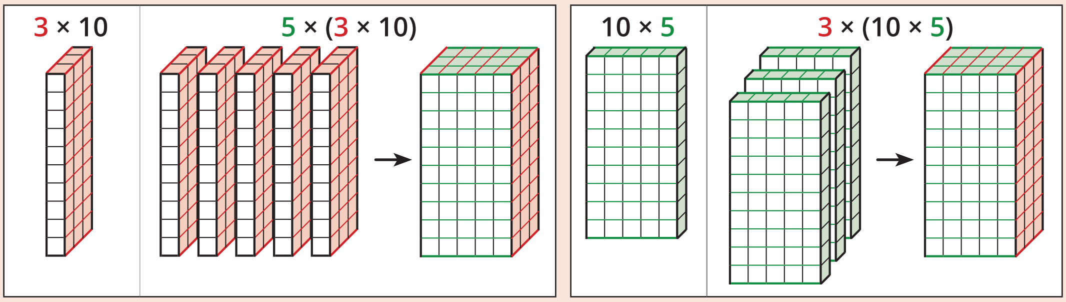
Multiplication is associative and commutative.

The numbers being multiplied can be grouped in different ways, and it will not affect the result.

Additionally, changing the order of the numbers being multiplied will not affect the result.

For example,
5 × (3 × 10)
= 3 × (10 × 5),

as can be seen in the diagrams.



Strategy 1: **Convert to a More Convenient Form**

Method 1

  4 × 5 × 4 × 5 × 4 × 5
= 20 × 20 × 20
= 400 × 20
= 8000

Method 2

  4 × 5 × 4 × 5 × 4 × 5
= 2 × 10 × 2 × 10 × 2 × 10
= 2 × 2 × 2 × 10 × 10 × 10
  8 × 1000
= 8000

4 × 5 × 4 × 5 × 4 × 5 = 8000.

The number of digits in Brandon's answer is **4.**

Strategy 2: **Perform the Multiplication**

  4 × 5 × 4 × 5 × 4 × 5
= 20 × 4 × 5 × 4 × 5
= 80 × 5 × 4 × 5
= 400 × 4 × 5
= 1600 × 5
= 8000

The number of digits in Brandon's answer is **4.**

Follow-Up: *How many digits are in the value of 4 × 4 × 4 × 4 × 5 × 5 × 5 × 5 ?* [6]

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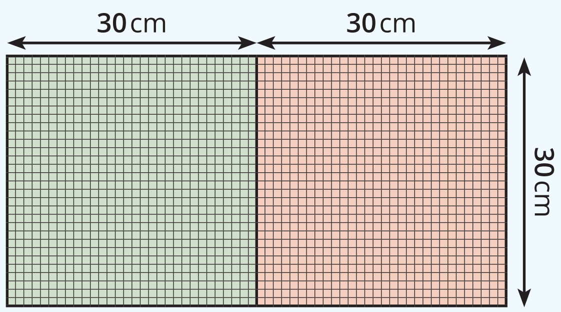
4B. The question is, What is the area of the overlapping region, in square centimetres?

Strategy 1: **Divide a Complex Shape**

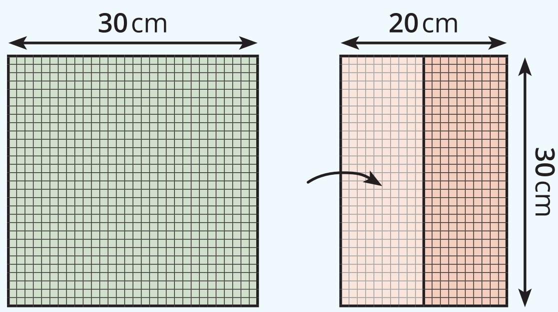
Harry's two squares of wrapping paper each measure **30** cm × **30** cm.

Side by side, they can combine to make a rectangle **60** cm wide.

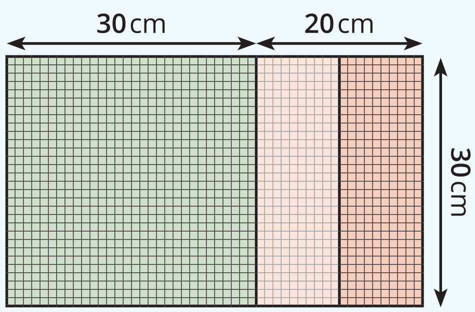


To make a rectangle **50** cm wide, the total width needs to be reduced by **60 – 50 = 10** cm.

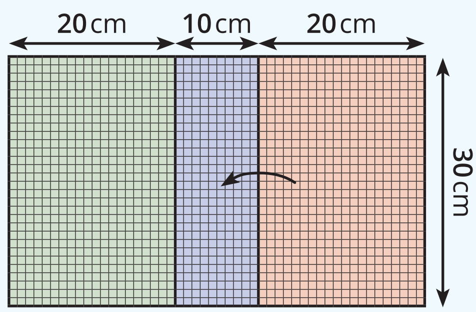
To reduce the total width by **10** cm, we can imagine folding one of the sheets so that its width is reduced to **30 – 10 = 20** cm.



The two sheets can then be placed side by side,



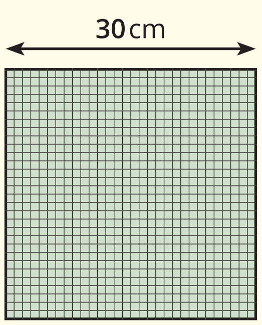
and the folded part laid over the other sheet.



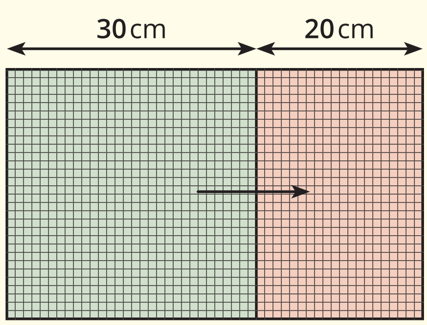
The overlapping region has an area of **10** cm × **30** cm = **300** cm².

Strategy 2: **Divide a Complex Shape (Alternative Method)**

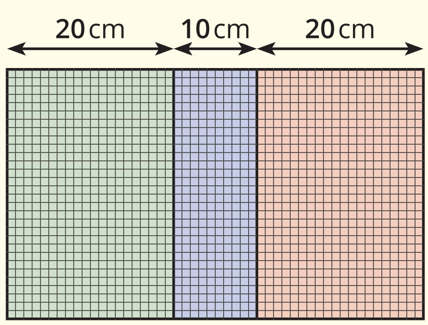
We can start with two squares in a stack.



To widen to **50** cm, we can move one sheet out by **50 – 30 = 20** cm.



The **10** cm wide overlapping region will have an area of **10** cm × **30** cm = **300** cm².



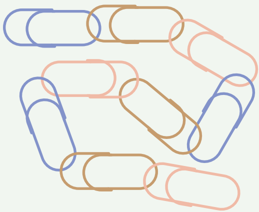
Follow-Up: *What would the width of the resulting rectangle be, if the overlapping region had an area of 150 cm²?* [55 cm]
What would be the perimeter of the resulting rectangle be in this case? [55 + 30 + 55 + 30 = 170 cm]

4C. The question is, How many silver paper clips are there in the chain?

***Strategy 1:* Convert to a More Convenient Form, and Find a Pattern**

The paper clips occur in the following sequence:

silver, gold, pink, silver, gold, pink, silver, gold, pink …



We can arrange the sequence so that it is more convenient to see the pattern - like this:

silver, gold, pink,

silver, gold, pink,

silver, gold, pink,

…

Using our new arrangement, we can list all **40** of the paper clips in the chain.

silver	gold	pink	3
silver	gold	pink	6
silver	gold	pink	9
silver	gold	pink	12
silver	gold	pink	15
silver	gold	pink	18
silver	gold	pink	21
silver	gold	pink	24
silver	gold	pink	27
silver	gold	pink	30
silver	gold	pink	33
silver	gold	pink	36
silver	gold	pink	39
silver			

There are **14** silver paper clips in Lucy's paper clip chain.

Alternatively, we can see that, for every group of **3** paper clips in the chain, there will be **1** silver paper clip.

In the first **30** paper clips, there will be **30** ÷ **3** = **10** silver ones.

silver	gold	pink	3
…	…	…	
silver	gold	pink	30

The **40** – **30** = **10** remaining paper clips in the chain are:

silver	gold	pink	33
silver	gold	pink	36
silver	gold	pink	39
silver			

There are **10** + **4** = **14** silver paper clips in Lucy's paper clip chain.

***Strategy 2:* Convert to a More Convenient Form, and Find a Pattern (Alternative Method)**

The paper clips follow the pattern

silver, gold, pink,

silver, gold, pink,

silver, gold, pink,

…

We note that:

- There are **3** colours in the repeating pattern.
- Every **3**rd paper clip is pink.

The positions of the pink paper clips are all multiples of **3**:

3	6	9	12	15	18	21	24	27	30	33	36	39	42

↑ too far (42)

The **39**th paper clip is pink.

It is the **39** ÷ **3** = **13**th pink paper clip in the chain.

Since the pattern has silver following pink, the **40**th paper clip is silver. It will be the **14**th silver paper clip in the chain.

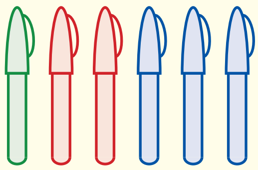
There are **14** silver paper clips in Lucy's paper clip chain.

Follow-Up: *Lucy adds more paper clips until there are 100 paper clips in the chain. What colour is the 100th paper clip?* [Silver]

4D. The question is, What is the smallest number of pens she must take out to be certain that she is holding at least one of each colour?

***Strategy:* Solve a Simpler Related Problem**

There are **6** pens in the box: **1** green, **2** red, and **3** blue.



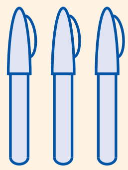
Annie wants to be certain that she will be holding at least one pen of each colour: at least one green, one red, and one blue.

Since it is possible for Annie to take any combination of pens, we can consider a related question:

What is the greatest number of pens that Annie can take out, and still not be holding one pen of each colour?

Method 1: Deliberately select pens so that not all of the colours are represented.

Let's suppose that Annie wants to take out as many pens as possible, but she must stop as soon as she has one of each colour.



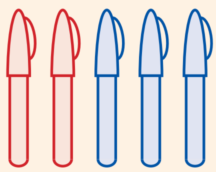
She might then begin by taking out as many pens as possible, that are all the same colour.

Since there are more blue pens than any other colour, it would make sense for Annie to begin by taking out all three blue pens.

The next most common colour is red.

Annie can take out both of the red pens, and still not have every colour.

We can see that it is possible for Annie to take **3** + **2** = **5** pens out of the box, and still not have one pen of every colour.

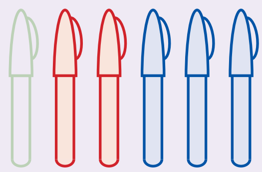


Method 2: Remove the smallest number of pens so that it is impossible to pick up every colour.

Alternatively, we can remove pens from the original set until it is no longer possible to take out one of each of the original colours.

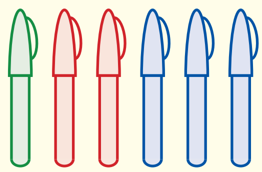
The most uncommon colour is green. There is just one green pen.

We can see that, after selectively removing just one pen, it is no longer possible to take out one green, one red, and one blue pen.



We have determined that the greatest number of pens that Annie can take out and still not have one pen of each colour, is **5**.

To be certain that she is holding at least one of every colour, Annie must take out every pen from the box.



Annie must take **6** pens out of the box.

Follow-Up: *What is the smallest number of pens Annie must take out, to be certain she has two pens of the same colour?* [4]

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4E. The question is, What is the smallest sum of three numbers around a single vertex?

		4	
1	2	6	3
		5	

Strategy 1: Divide a Complex Shape

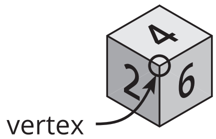
On a cube, each face is connected to four other faces.

For each face, there is only one face that it does not touch.

From the net, we can see that, for example, **6** is connected to **2**, **4**, **3**, and **5**.

6 is ***not*** connected to **1**.

		4	
1	2	6	3
		5	



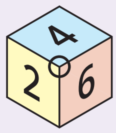
Likewise, we can see that:

- **2** is ***not*** connected to **3**.
- **4** is ***not*** connected to **5**.

		4	
1	2	6	3
		5	

By colouring (or otherwise marking) faces that are ***not*** connected, we can see that three faces that are all different colours, must all be mutually connected.

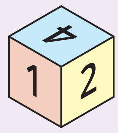
If three faces are mutually connected, then they must meet at a vertex.



To find the smallest sum of three numbers around a single vertex, we can then find:

- The smallest red face (**1** < **6**),
- The smallest yellow face (**2** < **3**), and
- The smallest blue face (**4** < **5**).

The smallest sum is **1 + 2 + 4 = 7**.



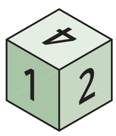
Strategy 2: Divide a Complex Shape, and Make an Organised List

We can construct the cube from the net, and then inspect each vertex.

To assist with looking at the cube in an organised way, we shall say that **5** is on the bottom face, and **4** is on the top face.

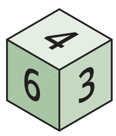
		4	
1	2	6	3
		5	

Looking at the four vertices around the top face, we have:



1 + 4 + 2 = 7

2 + 4 + 6 = 12



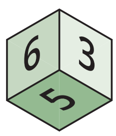
6 + 4 + 3 = 13

3 + 4 + 1 = 8

Looking at the four vertices around the bottom face, we have:

1 + 5 + 2 = 8

2 + 5 + 6 = 13



6 + 5 + 3 = 14

3 + 5 + 1 = 9

Of the **8** possible sums, the smallest sum is **7**.

Follow-Up: *What is the greatest sum of three numbers around a single vertex?* [6 + 5 + 3 = 14]

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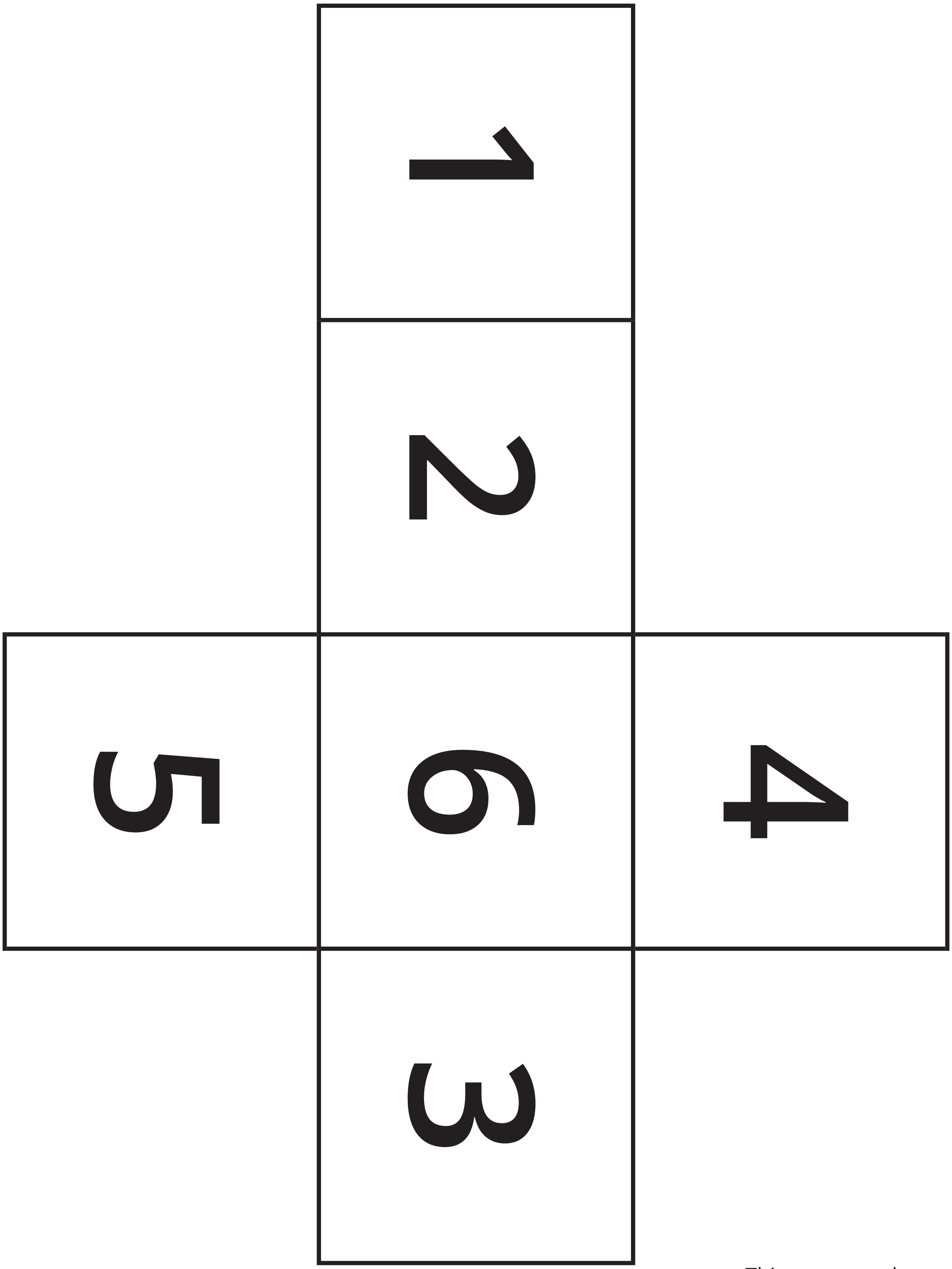

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