MATHS GAMES

APSMO

2024 MATHS GAMES

IMPORTANT

The information contained in this file is ONLY for the use of registered participants of the 2024 APSMO Maths Games.

Organisation and Procedures

For full details, see the Members' Area

- Maths Games papers are to be conducted under test conditions.

DO	DO NOT
• Supervise students at all times. • Maintain silence. • Provide blank working paper. • Collect, mark and retain the papers.	• Print the papers prior to the scheduled date. • Read the questions aloud to the students. • Interpret the questions for students. • Permit any discussion or movement around the room. • Permit the use of calculators or other electronic devices.

- Papers should be scored by the PICO using the *Solutions and Answers* sheet provided.
- Original student answer sheets should be retained by the PICO until the end of the year.

Absent Students

- A student who is legitimately absent on the date of the Maths Games paper, may sit the paper on their return to school.
- If an absent student does not sit the paper on their return to school they should be marked as 'absent'.
- *Note: This policy differs from the Maths Olympiads Absent Student Policy which has additional requirements*.

APSMO

WEDNESDAY 8 MAY 2024

MATHS GAMES

MATHS GAMES JUNIOR

Suggested Time: ***30 Minutes***

Write your answers in the boxes on the back.

←

Keep your answers hidden by folding backwards on this line.

1A. I am making a pattern out of equilateral triangles.

Each triangle has a perimeter of 3 cm.

The diagrams show what the pattern looks like with 1, 2, 3, and 4 triangles.

Find the perimeter, in centimetres, for a pattern made up of 10 triangles.

Hint: You could build a table to show how the perimeter changes, every time a new triangle is added.

1B. Vicky is two years older than Henry, who is two years older than Tyler.

Tyler's age is a multiple of 7. Vicky's age is a multiple of 5.

What is the youngest possible age for Henry?

Hint: What are some possible ages for Vicky?

1C. An office has twice as many blue pens as black pens.

There are four more blue pens than red pens.

If there are 31 pens in total, how many blue pens are there?

Hint: You could guess a number of blue pens, and then check the total number of pens.

1D. Casey plays hockey every third Saturday morning.

He plays basketball every second Saturday afternoon.

The next date on which Casey will play both sports is May 25 (25th of May).

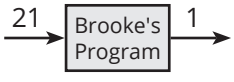
What was the date the last time Casey played both sports on the same day?

Hint: You could draw a calendar.

1E. Brooke has written a computer program that takes an input number and prints an output number.

- If the input number has one digit, it multiplies the number by 3, and prints the result.

- If the input number has two digits, it removes the tens digit, and prints the ones digit only.



Brooke's first input number is 2. She then continuously takes the output number that the program prints, and uses it as the next input number.

What would be Brooke's 40th input number?

Hint: Look for a pattern.

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WEDNESDAY 8 MAY 2024

MATHS GAMES

MATHS GAMES JUNIOR

1A.

1B.

1C.

1D.

1E.

Student Name:

Fold here. Keep your answers hidden.

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WEDNESDAY 8 MAY 2024

MATHS GAMES JUNIOR

Solutions and Answers

(Items in parentheses are not required)

1A: 12 (cm)	**1B:** 23 (years)	**1C:** 14	**1D:** April 13	**1E:** 24

1A. The question is, Find the perimeter, in centimetres, for a pattern made up of 10 triangles.

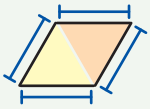
Strategy 1: **Draw a Diagram, and Build a Table**

One equilateral triangle has a perimeter of **3** cm.

Since the triangle has three equal sides, each side length is **3 ÷ 3 = 1** cm.

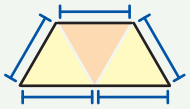
When the pattern is made up of **2** triangles, the perimeter is made up of **4** side lengths.

The perimeter of the 2-triangle pattern is **4** cm.



When the pattern is made up of **3** triangles, the perimeter is made up of **5** side lengths.

The perimeter of the 3-triangle pattern is **5** cm.



We can build a table to show how the perimeter grows.

Triangles	1	2	3
Perimeter (cm)	3	4	5

(+1, +1 above; +1, +1 below)

Adding one triangle increases the perimeter by **1** cm.

We can continue the table to find the perimeter when there are **10** triangles.

Triangles	1	2	3	...	10
Perimeter (cm)	3	4	5	...	12

(+1, +1, +7 above; +1, +1, +7 below)

When there are **10** triangles in the pattern, the perimeter is **12** cm.

Alternatively, the number of centimetres in the perimeter is **2** more than than the number of triangles.

Triangles	1	2	3	...	10
Perimeter (cm)	3	4	5	...	12

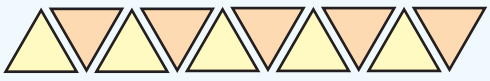
(+1, +1 above; +2, +2; +1, +1 below)

When there are **10** triangles in the pattern, the perimeter is **10 + 2 = 12** cm.

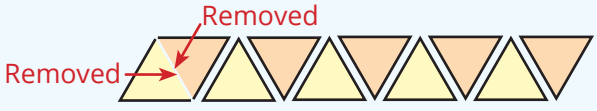
Strategy 2: **Find a Pattern**

Each triangle has a perimeter of **3** cm.

With **10** triangles, the total perimeter if the triangles are separate would be **10 × 3 = 30** cm.



When we join two triangles together, the total perimeter is reduced by **2** side lengths, or **2** cm.



A pattern with **10** triangles has **9** joins.

Each join reduces the total perimeter by **2** cm.

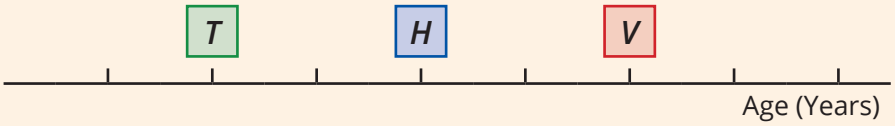
When there are **10** triangles in the pattern, the perimeter is **30 – (9 × 2) = 12** cm.

Follow-Up: *I made a pattern using all of my triangles. If this pattern has a perimeter of 24 cm, how many triangles do I have?* [22]

1B. The question is, What is the youngest possible age for Henry?

***Strategy 1:* Draw a Diagram, and Build a Table**

Vicky is **2** years older than Henry, who is **2** years older than Tyler.



We can try different multiples of 5 for Vicky's age, and then fill in the ages for Henry and Tyler.

The first time Tyler's age is a multiple of 7, occurs when Tyler is 21 years old.

T		H		V
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Age (Years)

Alternatively, we can try different multiples of 7 for Tyler's age.

The first time Vicky's age is a multiple of 5 occurs when Vicky is 25 years old.

T		H		V
7	8	9	10	11
14	15	16	17	18
21	22	23	24	25

Age (Years)

The youngest possible age occurs when Tyler is **21** and Vicky is **25**. Henry would be **23** years old.

***Strategy 2:* Build a Table**

We can build a **100** grid table, and mark the multiples of 5 (Vicky's age) and multiples of 7 (Tyler's age).

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

There are a few instances where we see a difference of **4** between a multiple of 7 and a multiple of 5.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The first such instance occurs for the values $3 \times 7 = 21$, and $4 \times 5 = 25$.

When Tyler is 21 and Vicky is 25, Henry would be **23** years old.

Follow-Up: *When Tyler, Henry and Vicky first met, all of their ages were prime numbers. How old was Henry at that time?* [5]

1C. The question is, How many blue pens are there in the office?

Strategy 1: **Build a Table, and Find a Pattern**

There are twice as many blue pens as black pens.

If there are 2 blue pens, there would be **1** black pen.

If there are 4 blue pens, there would be **2** black pens.

Blue	2	4						
Black	1	2						

Since there are **4** more blue pens than red pens, there cannot be fewer than 4 blue pens.

We can continue to fill in the table, with:

- The number of red pens being **4** less than the number of blue pens.
- The total number of pens.

Blue	~~2~~	4	6	8				
Black	~~1~~	2	3	4				
Red		0	2	4				
Total		6	11	16				

Every time we increase the number of blue pens by **2**, the total number of pens increases by **5**.

Using this pattern, we can keep increasing the total number of pens by **5** until we reach a total of **31**.

We can see that the result of 14 blue, **7** black and 10 red pens does add up to **31** pens in total.

There are **14** blue pens in the office.

		+2	+2	+2	+2	+2		
Blue	~~2~~	4	6	8	10	12	14	
Black	~~1~~	2	3	4			7	
Red		0	2	4			10	
Total		6	11	16	21	26	31	
		+5	+5	+5	+5	+5		

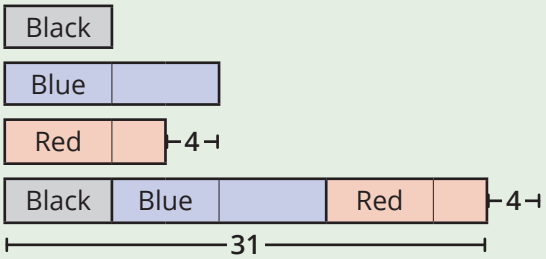
Strategy 2: **Draw a Diagram, and Reason Logically**

We can use a bar to represent the number of black pens.

There are twice as many blue pens as black pens.

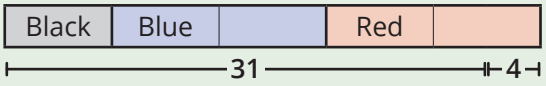
There are **4** more blue pens than red pens.

In total, there are **31** pens.



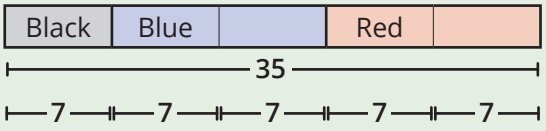
If we add another **4** red pens, there would be the same number of red pens as there are blue pens.

In total, there would be **31** + **4** = **35** pens.



We can now form **5** sets of pens, where each set contains the same number of pens.

Each set would have **35** ÷ **5** = **7** pens.



There are **2** × **7** = **14** blue pens in the office.

Follow-Up: *What fraction of the pens in the office are red?* [10/31]

1D. The question is, What was the date of the last time Casey played both sports on the same day?

Strategy 1: **Build a Table (1)**

Casey plays hockey every **3**rd Saturday, and basketball every **2**nd Saturday.

On **May 25**, Casey will play both hockey and basketball.

We can begin by constructing a calendar, with **May 25** occurring on a Saturday.

May						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

Let's circle the dates on which Casey played hockey, and colour in the dates on which he played basketball.

Since there are no other dates during May when Casey played both sports, we will have to extend the calendar back to April.

May						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
			1	2	3	(4)
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	(25)
26	27	28	29	30	31	

Since April has **30** days, we will construct the calendar to show that **April 30** occurs immediately before **May 1**.

April - May						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
	1	2	3	4	5	6
7	8	9	10	11	12	(13)
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	1	2	3	(4)
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	(25)
26	27	28	29	30	31	

Casey played both sports on **April 13**.

Strategy 2: **Build a Table (2)**

Let's begin with a Saturday on which Casey plays both sports.

Saturday	1	2	3	4	5	6	7	8
Hockey	X							
Basketball	X							

Marking the table with Saturdays on which Casey plays hockey or basketball, we can see that Casey plays both sports on the same day, once every **6** weeks.

Saturday	1	2	3	4	5	6	7	8
Hockey	X			X			X	
Basketball	X		X		X		X	

The next date on which Casey will play both sports will be **May 25**.

3 weeks prior, the date was **May 4**.

Date				May 4	May 11	May 18	May 25
Saturday	1	2	3	4	5	6	7
Hockey	X			X			X
Basketball	X		X		X		X

Counting backwards from **May 4**, we find that **April 27** was the Saturday **4** weeks prior to **May 25.**

April 27	April 28	April 29	April 30	May 1	May 2	May 3	May 4
Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat

6 weeks prior to **May 25**, Casey played both sports on Saturday **April 13**.

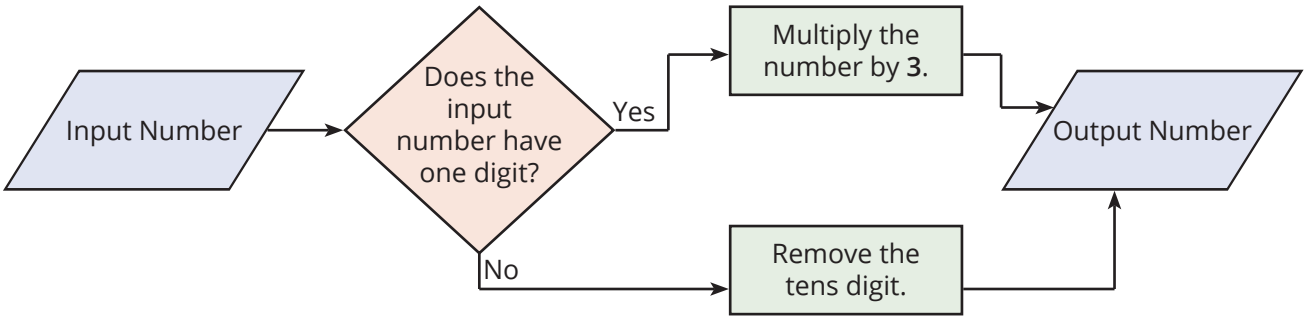
Date	April 13	April 20	April 27	May 4	May 11	May 18	May 25
Saturday	1	2	3	4	5	6	7
Hockey	X			X			X
Basketball	X		X		X		X

Follow-Up: *What would be the first date after May 25 when Casey will play both sports?* [July 6]

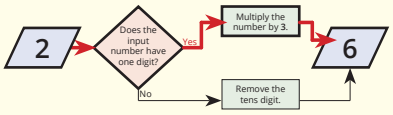
1E. The question is, What would be Brooke's 40th input number?

***Strategy:* Find a Pattern, and Build a Table**

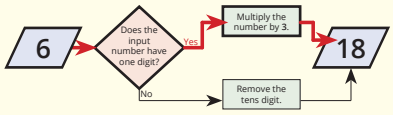
Brooke's computer program takes an input number, and performs an operation on it.



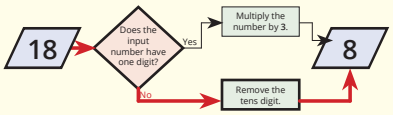
Brooke's first input number is **2**.



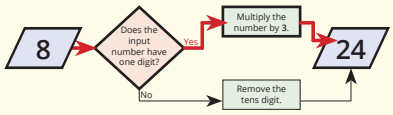
She enters the output number **6** as her 2nd input number.



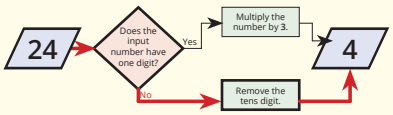
She enters the output number **18** as her 3rd input number.



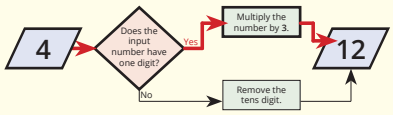
She enters the output number **8** as her 4th input number.



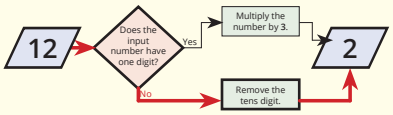
She enters the output number **24** as her 5th input number.



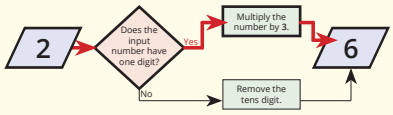
She enters the output number **4** as her 6th input number.



She enters the output number **12** as her 7th input number.



We have had an input number of **2** before.



The sequence repeats for every 7th input number.

We notice that the input number is 12 when the position is a multiple of 7.

Both 35 and 42 are multiples of 7 that are close to 40.

By building a table and continuing the pattern, we can see that the 40th input number is **24**.

Position in sequence	1	2	3	4	5	6	7
Input number	2	6	18	8	24	4	12
Position in sequence	8	9	10	11	12	13	14
Input number	2	6	18	8	24	4	12

...

Position in sequence	29	30	31	32	33	34	35
Input number	2	6	18	8	24	4	12
Position in sequence	36	37	38	39	40	41	42
Input number	2	6	18	8	24	4	12

Follow-Up: *What would the 100th input number be?* [6]