# APSMO

2024: DIVISION J
WEDNESDAY 12 JUNE 2024

MATHS OLYMPIAD

OLYMPIAD 2

*Total Time Allowed:* ***30 Minutes***

***Write your answers in the boxes on the back.***

***Keep your answers hidden by folding backwards on this line.***

**2A.** Aikan, Aiden, and Alden each have a different favourite single-digit number.
The product of their numbers is 70.
What is the sum of their numbers?

**2B.** Mr Abacus has a maths club.
16 students come to the club.
They are either 9 or 10 years old.
There are as many 9 year olds as 10 year olds in the club.
If the students add Mr Abacus' age to their combined ages, the sum is 199.
How old is Mr Abacus?

**2C.** Amelia and Cooper both drove 300 km from Sydney to Canberra.
For the first 150 km, Amelia drove at an average speed of 60 km/hr.
She drove the second 150 km at an average speed of 90 km/hr.
Cooper drove the whole 300 km at an average speed of 75 km/hr.
Cooper's trip took less time.
How many minutes shorter was his trip than Amelia's?

**2D.** Figure 1 is a 5 unit x 8 unit rectangle made of cardboard.
A 3 unit x 3 unit square from one of its corners was cut out and attached to the side of the card. Then, a 1 unit x 1 unit square was cut from another corner and attached to to side of the 3 x 3 square.
What is the perimeter of the final shape seen in Figure 2?

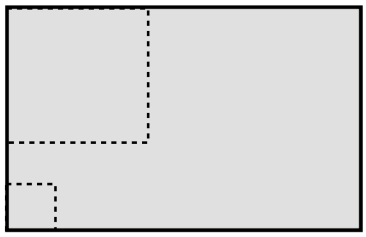

**Figure 1**

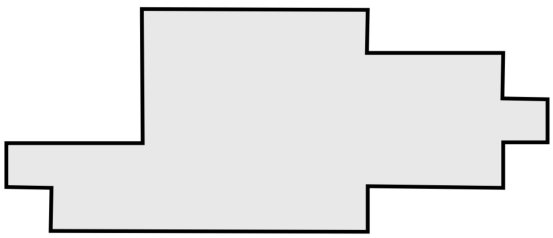

**Figure 2**

**2E.** The positive whole numbers are written down in order.
This forms a string of digits:

12345678910111213141 5...

What is the 200th digit in that string?

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**Student Name:**

**2A.**

**2B.**

**2C.**

**2D.**

**2E.**

*Fold Here. Keep your answers hidden.*

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## Solutions and Answers

**For teacher use only. Not for Distribution.**

| **2A:** 14 | **2B:** 47 | **2C:** 10 | **2D:** 34 | **2E:** 0 |
|---|---|---|---|---|

**2A.** The question is:

Aikan, Aiden, and Alden each have a different favourite single-digit number. The product of their numbers is 70.

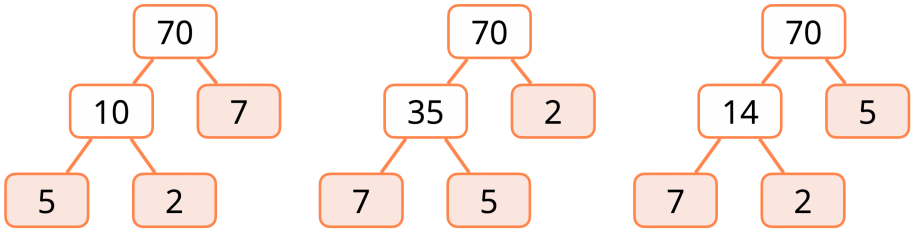
What is the sum of their numbers?

**METHOD 1 *Strategy*:** *Create a Factor Tree.*

We know that the product of Aikan, Aiden and Alden's favourite numbers is 70.

We also know that their favourite numbers are single-digits.

We can create a factor tree of 70 to identify its single digit factors.




No matter what factor tree we create, we find that the 3 single-digit numbers that multiply to give 70 are 2, 5, and 7.

The sum of Aikan, Aiden and Alden's numbers is:

2 + 5 + 7 = **14**

**METHOD 2 *Strategy*:** *Use the Process of Elimination.*

From the problem, we know that one single-digit and the product of the 2 other single-digits must multiply to 70.

We can use the process of elimination to identify the 3 single-digits, starting with the largest single digit number 9.

9 cannot be multiplied with another to reach 70.

8 is not a factor of 70.

7 is a factor of 70. It can be multiplied by 10.

10 has 2 factors that are single-digits, 5 and 2.

The sum of Aikan, Aiden and Alden's numbers is:

2 + 5 + 7 = **14**

**2B.** The question is:

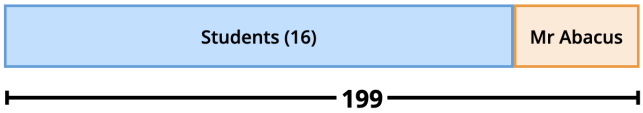
Mr Abacus has a maths club. 16 students come. They are either 9 or 10 years old. There are as many 9 year olds as 10 year olds in the club. If the students add Mr Abacus' age to their combined ages, the sum is 199. How old is Mr Abacus?

**METHOD 1 *Strategy*:** *Use a Number Sentence and Reason Logically.*

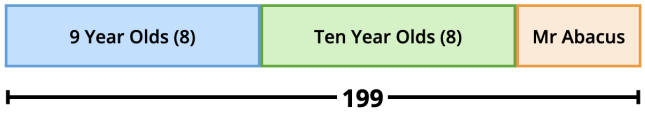
We are told that half of the 16 students are 9 years old, and the other half of the students are 10 years old.
Their ages added together:
$8 \times 9 + 8 \times 10$
$= 72 + 80$
$= 152$
When Mr Abacus' age is added to the total, the sum is 199.
We can subtract the combined students' age from the total sum to find Mr Abacus' age.
$199 - 152 =$ **47**
Mr Abacus is **47 years old**.

**METHOD 2 *Strategy*:** *Use a Bar Diagram.*

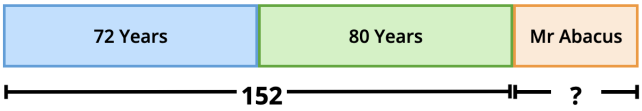
We can use the information in the problem to create a bar diagram to determine the age of Mr Abacus.
We can show that the sum of the ages of the students and Mr Abacus is 199.



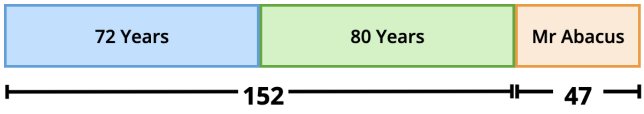

We can divide the 16 students into 2 equal groups.




The sum of the 9 year olds ages is 72 and the sum of the 10 year olds ages is 80.
The sum of all the students combined is 152.




Subtracting 152 from 199 will give Mr Abacus' age.




Mr Abacus is **47 years old**.

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**2C.** The question is:

Amelia and Cooper both drove 300 km from Sydney to Canberra. For the first 150 km, Amelia drove at an average speed of 60 km/hr. She drove the second 150 km at an average speed of 90 km/hr. Cooper drove the whole 300 km at an average speed of 75 km/h. Cooper's trip took less time. How many minutes shorter was his trip than Amelia's?

**METHOD 1 *Strategy*:** *Use a Number Sentence and Reason Logically.*

We can calculate the time taken by dividing the distance by the speed.

**Amelia's trip:**

First, Amelia drove at 60 km/h for 150 km.

We can calculate how far she drove each minute by dividing the speed by time:

60 ÷ 60 = 1km per minute.

150 km ÷ 1 = 150.

It will take Amelia **150 minutes** for her to drive 150 km.

Next, Amelia drove at 90 km/h for 150 km.

We can calculate how far she drove each minute by dividing the speed by the time:

90 ÷ 60 = 1.5 km per minute.

150 km ÷ 1.5 = 100.

Therefore it will take Amelia **100 minutes** to drive the last 150 km.

Altogether, Amelia's trip took **150 + 100 = 250 minutes.**

**Cooper's trip:**

Cooper drove at 75 km/h for 300 km.

We can calculate how far he drove each minutes by dividing the speed by time:

75 ÷ 60 = 1.25 km per minute.

300 km ÷ 1.25 = 240.

Therefore Cooper's trip took **240 minutes**.

Cooper's trip was **10 minutes shorter** than Amelia's trip.

**METHOD 2 *Strategy*:** *Draw a Table.*

We can draw a table to record Alice and Cooper's speed, distance travelled and time taken.

**Alice's Trip**

| Speed | 60 km/h | 60 km/h | 60 km/h | 90 km/h | 90 km/h | 90 km/h |
|---|---|---|---|---|---|---|
| Distance | 60 km | 120 km | 150 km | 195 km | 285 km | 300 km |
| Time | 60 mins | 120 mins | 150 mins | 180 mins | 240 mins | **250 min** |

**Cooper's Trip**

| Speed | 75 km/h | 75 km/h | 75 km/h | 75 km/h |
|---|---|---|---|---|
| Distance | 75 km | 150 km | 225 km | 300 km |
| Time | 60 mins | 120 mins | 180 mins | **240 mins** |

Amelia's trip took 250 minutes. Cooper's trip took 240 minutes.

Cooper's trip was **10 minutes shorter** than Amelia's trip.

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**2D.** The question is:

Figure 1 is a 5 unit x 8 unit rectangle made of cardboard. A 3 unit x 3 unit square from one of its corners was cut out from one of its corners and attached to the side of the card. Then, a 1 unit x 1 unit square was cut from another corner and attached to to side of the 3 x 3 square. What is the perimeter of the final shape seen in Figure 2?

Figure 1

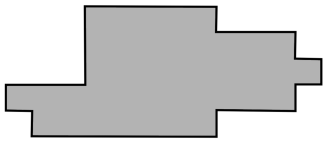
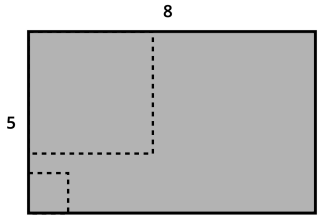
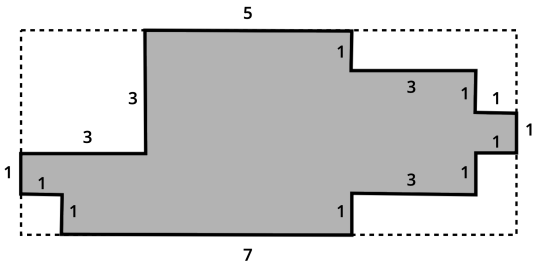


Figure 2

**METHOD 1 *Strategy*:** *Determine the lengths of the 16 sides of the region.*

Using the measurements provided in the problem, we can use the diagram to label each side with its length.
Then we can add the length of all 16 sides to find the total perimeter.




Start with the highest right vertex and work clockwise to find the perimeter:

=1+3+1+1+1+1+1+3+1+7+1+1+1+3+3+5

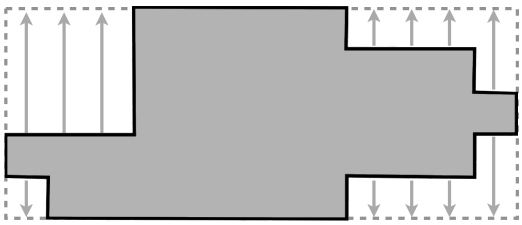
=10x1+4x3+5+7

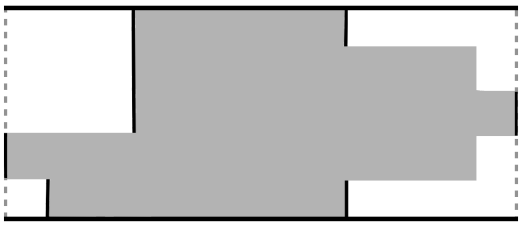
=10+12+12

=**34 units**

**METHOD 2 *Strategy*:** *Form an outer rectangle with an equivalent perimeter.*

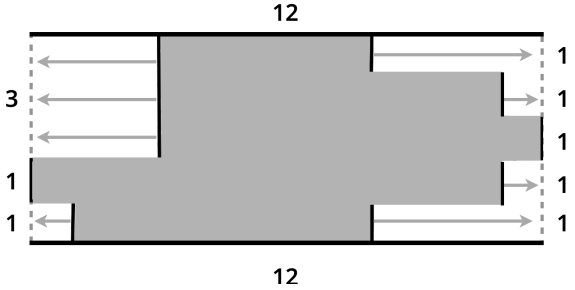
Using the measurements provided in the problem, we can determine the length of the horizontal components of the perimeter. By moving all of the horizontal components of the perimeter out towards the edge of the original rectangle, we can calculate the horizontal length as **12 units.**

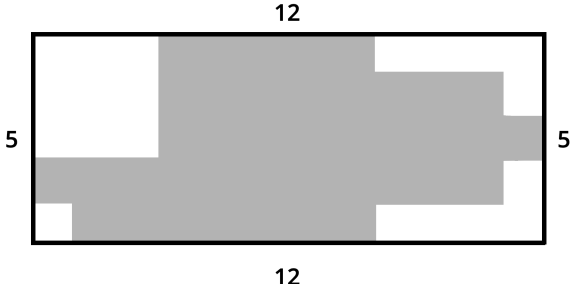




We can likewise determine the length of the vertical components of the perimeter.

By moving all of the vertical components of the perimeter out towards the edge of the original rectangle, we can calculate the vertical length as **5 units.**




The perimeter of the rectangle is 2 × (12 + 5)

= 2 × 17

= **34 units.**

**1E.** The question is:

The positive whole numbers are written down in order.

This forms a string of digits:

12345678910111213141 5...

What is the 200th digit in that string?

**METHOD 1 *Strategy*:** *Draw a table and count the number of digits in an organised way.*

We can start by drawing a table to record the number of digits that single-digit counting numbers, 2 digit counting numbers, and 3 digit counting numbers require.

| Counting numbers | 1 → 9 | 10 → 99 | 100 → 999 |
|---|---|---|---|
| Number of digits | 9 | 180 | 900 |
| Cumulative number of digits | 9 | 189 | 2700 |

200th

The numbers 1 through to 9 require 9 digits.

The numbers 10 through to 99 require 180 digits.

Therefore the numbers 1 through to 99 require 189 digits.

We need another 11 digits to find the 200th digit in the string of digits.

We continue our string starting with 100, the first 3 digit counting number.

....10010110210....

**The 200th digit is 0.**