**Suggested Time: *30 Minutes***

1A. In a game show, if a player completes a challenge, they gain 10 points.

If they do not complete a challenge, they lose 10 points.

Mila's final score was 50 points.

If there were 9 challenges in total, how many did Mila complete?

Hint: You could guess a number of completed challenges, and see if it works.

Write your answers in the boxes on the back.

←

Keep your answers hidden by folding backwards on this line.

1B. Mr Harrison has between 10 and 30 students in his class.

If he arranges his students into groups of 5, there are 2 students left over.

If he arranges his students into groups of 4, there are 3 students left over.

How many students are in Mr Harrison's class?

Hint: Try using just one of the groupings. What possibilities are there for the number of students?

1C. I am making a pattern using a rhombus-shaped tile. Each tile has a perimeter of 40 cm.

The diagrams show patterns using 1, 3, 5 and 7 tiles.

A dark line marks the perimeter of each pattern.
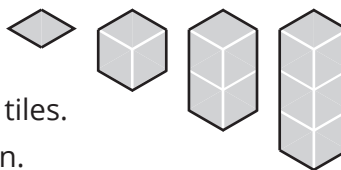
How many tiles are used if the pattern has a perimeter of 2 metres?

Hint: You could build a table.

1D. A packet of white paper costs $9.20.

A packet of coloured paper costs $14.50.

Aadarsh paid $75 in total for paper to use in his office.

How many packets of white paper did Aadarsh buy?

Hint: You could guess a number of packets of white paper, and see if it works.

1E. Julia has written a computer program that takes an input number and prints an output number.

- If the input number has one digit, it multiplies the number by 3.

- If the input number has two digits, it removes the tens digit, and increases the result by 1.

2 → Julia's Program → 6

37 → Julia's Program → 8
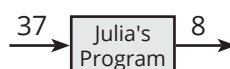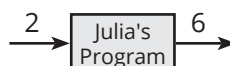
Julia's first input number is 1. She then continuously takes the output number that the program prints, and uses it as the next input number.

What would be Julia's 100th input number?

Hint: Look for a pattern.

APSMO
WEDNESDAY 8 MAY 2024

MATHS GAMES
SENIOR

1A.

Student Name:

1B.

Fold here. Keep your answers hidden.

1C.

1D.

1E.

Solutions and Answers
(Items in parentheses are not required)

1A: 7	1B: 27	1C: 17	1D: 5	1E: 27

1A. The question is, How many challenges did Mila complete?

Strategy 1: Build a Table

Suppose Mila completed all **9** challenges.
If so, she would have gained **9 × 10 = 90** points.

Points gained	Points lost	Points total
9 × 10 = 90	0 × 10 = 0	90 – 0 = 90

If Mila completed **8** challenges, she would have gained **8 × 10 = 80** points.

She would also have lost **1 × 10 = 10** points because she did not complete **1** of the **9** challenges.

Her total score would have been **80 – 10 = 70** points.

Points gained	Points lost	Points total
9 × 10 = 90	0 × 10 = 0	90 – 0 = 90
8 × 10 = 80	1 × 10 = 10	80 – 10 = 70

If Mila completed **7** challenges, she would have:
- gained **7 × 10 = 70** points, and
- lost **2 × 10 = 20** points.

Her total score would have been **70 – 20 = 50** points.

This matches the question.

Mila completed **7** challenges.

Points gained	Points lost	Points total
9 × 10 = 90	0 × 10 = 0	90 – 0 = 90
8 × 10 = 80	1 × 10 = 10	80 – 10 = 70
7 × 10 = 70	2 × 10 = 20	70 – 20 = 50

Strategy 2: Find a Pattern

Every incomplete challenge reduces Mila's final score by **20**.

If she does not complete a challenge she loses **10** points, and she also misses out on gaining **10** points.

Mila completed **7** challenges.

Complete	0	1	2	3	4	5	6	7	8	9
Incomplete	9	8	7	6	5	4	3	2	1	0
Points	-90	-70	-50	-30	-10	10	30	50	70	90

Strategy 3: Reason Algebraically

Let x represent the number of challenges that Mila completed.

She did not complete $(9 - x)$ challenges.

Her score is **50** points.

We can set up the following equation: $10x - 10(9 - x) = 50$

Expand brackets (*note the negative*): $10x - 90 + 10x = 50$

Collect like terms: $20x - 90 = 50$

Add **90** to both sides: $20x = 140$

Divide both sides by **20**: $x = 7$

Mila completed **7** challenges.

Follow-Up: *Suppose there were 12 challenges, and Mila scored 60 points. How many challenges did she complete?* [9]

1B. The question is, How many students are in Mr Harrison's class?

Strategy 1: **Draw a Diagram, and Build a Table**

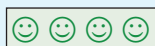If Mr Harrison arranges his students into groups of **5**, there are **2** students left over.

The diagrams show some options for how his class might look.

☺☺☺☺☺ ☺☺ **7** is too few. There are at least **10** students.

☺☺☺☺☺ ☺☺☺☺☺ ☺☺

☺☺☺☺☺ ☺☺☺☺☺ ☺☺☺☺☺ ☺☺

If Mr Harrison arranges his students into groups of **4**, there are **3** students left over.

☺☺☺☺ ☺☺☺ **7** is too few.

☺☺☺☺ ☺☺☺☺ ☺☺☺
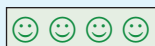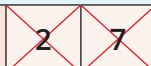
Listing possible numbers of students in Mr Harrison's class, we can see that both groupings would occur for a class of **27** students.

| Groups of **5** students + 2 | ~~2~~ | ~~7~~ | 12 | 17 | 22 | (27) | ~~32~~ |
| Groups of **4** students + 3 | ~~3~~ | ~~7~~ | 11 | 15 | 19 | 23 | (27) |

Strategy 2: **Build a Table, and Eliminate All But One Possibility**

Mr Harrison has between **10** and **30** students in his class.

Arranging them in groups of **5** leaves a remainder of **2**.

No. of students	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
Remainder after making groups of 5	1	(2)	3	4	0	1	(2)	3	4	0	1	(2)	3	4	0	1	(2)	3	4

Arranging them in groups of **4** leaves a remainder of **3**.

No. of students	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
Remainder after making groups of 5	1	(2)	3	4	0	1	(2)	3	4	0	1	(2)	3	4	0	1	(2)	3	4
Remainder after making groups of 4	(3)	0	1	2	(3)	0	1	2	(3)	0	1	2	(3)	0	1	2	(3)	0	1

If there are **27** students in Mr Harrison's class, then he will have:

- **2** students left over if he creates groups of **5**, and
- **3** students left over if he creates groups of **4**.

Follow-Up: There are between 60 and 80 students in the school band. As the band conductor, Mr Harrison was surprised to find that the same situation occurs as for his class: if he arranges the band members in groups of 5, there are 2 left over, and if he arranges them in groups of 4, there are 3 left over. How many students are there in the band? [67]
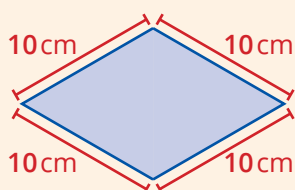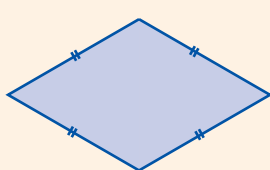
1C. The question is, How many tiles would there be in a pattern with a perimeter of 2 metres?

Each tile has a perimeter of **40** cm.

Since a rhombus has four equal sides, each side is **40 ÷ 4 = 10** cm long.

10 cm   10 cm
10 cm   10 cm

With side lengths of **10** cm, it would be useful to have all of the measurements expressed in centimetres.
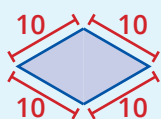
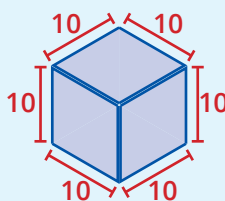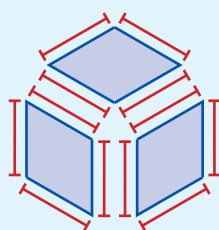The perimeter of the target pattern is **2 metres**.

Since **1** metre is **100** cm,

2 metres is 2 × 100 = 200 cm.
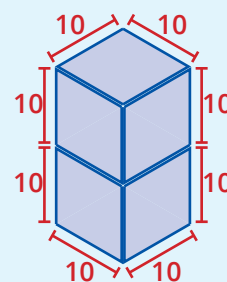
Strategy 1: **Find a Pattern, and Build a Table**

1 tile has a perimeter of **4 × 10 = 40** cm.

10   10
10   10

When we put **3** tiles together, the perimeter will be **6 × 10 = 60** cm.

10   10
10   10

When we put **5** tiles together, the perimeter will be **8 × 10 = 80** cm.

10   10
10   10
10   10

Every time we add **2** tiles, the total perimeter increases by **20** cm.

| No. of tiles	1	3	5	7	9	11	13	15	17
Perimeter (cm)	40	60	80	100	120	140	160	180	200

+2 across tiles; +20 across perimeter

There would be **17** tiles in a pattern with a perimeter of **200** cm = **2** metres.

Strategy 2: **Find a Pattern, and Reason Algebraically**

The side length of a tile is **10** cm.

For a total perimeter of **200** cm, the perimeter must comprise **200 ÷ 10 = 20** side lengths.

Every time we increase the number of tiles by **2**, we also increase the number of sides in the perimeter by **2**.
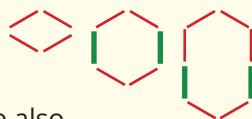
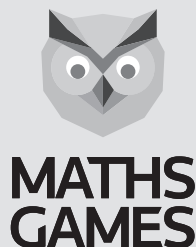| No. of tiles	1	3	5	...	x
Side lengths	4	6	8	...	$x + 3$

Using x to represent the number of tiles, we want to know how many tiles there would be in a pattern with a perimeter comprising **20** side lengths.

$$x + 3 = 20$$

Subtract **3** from both sides: $x = 17$

The pattern has **17** tiles.

Follow-Up: *How many tiles would there be in a pattern with a perimeter of 10 metres?* [97]

1D. The question is, How many packets of white paper did Aadarsh buy?

Strategy 1: Build a Table

A packet of white paper costs **$9.20**.

We can build a table to show how much Aadarsh might have spent on white paper.

No. of Packets	Total Cost ($)
1	9.20
2	18.40
3	27.60
4	36.80
5	46.00
6	55.20
7	64.40
8	73.60

A packet of coloured paper costs **$14.50**.

We can likewise show how much Aadarsh might have spent on coloured paper.

No. of Packets	Total Cost ($)
1	14.50
2	29.00
3	43.50
4	58.00
5	72.50

Aadarsh spent **$75.00** in total.

Using a table, we can work out possible combinations where the cost is close to **$75.00**.

Cost of white paper ($)	Cost of coloured paper ($)				
	14.50	29.00	43.50	58.00	72.50
9.20				67.20	81.70
18.40			61.90	76.40	
27.60			71.10	85.60	
36.80		65.80	80.30		
46.00		75.00			
55.20	69.70	84.20			
64.40	78.90				
73.60	88.10				

The total is **$75.00** if Aadarsh spends **$46.00** on white paper.

He can buy **5** packets of white paper for **$46.00**.

Aadarsh bought **5** packets of white paper.

Strategy 2: Use Number Sense

A packet of white paper costs **$9.20**, or **9** dollars and **20** cents.

A packet of coloured paper costs **$14.50**, or **14** dollars and **50** cents.

Aadarsh spent **75** dollars exactly.

For the total to be a whole number of dollars, the number of cents that Aadarsh spent must be a multiple of **100**.

To create **100** from combinations of **20** and **50**, we can have:

• **5 × 20 = 100**
• **2 × 50 = 100**

There are no combinations that use both **20** and **50**.

Since white paper costs **9** dollars and **20** cents, the number of packets of white paper must be a multiple of **5**.

Packets of white paper	5	10	15
Cost of white paper ($)	46	~~92~~	~~138~~

Since coloured paper costs **14** dollars and **50** cents, the number of packets of coloured paper must be a multiple of **2**.

Packets of coloured paper	2	4	6
Cost of white paper ($)	29	58	~~87~~

Aadarsh spent **$75** in total, so he could not have bought more than **5** packets of white paper.

Aadarsh's total is **$75** if he buys **5** packets of white paper, and **2** packets of coloured paper.

> ***Follow-Up:*** *Cynthia spent $138.30 on 11 packets of paper. How many packets of white paper did she buy?* [4]
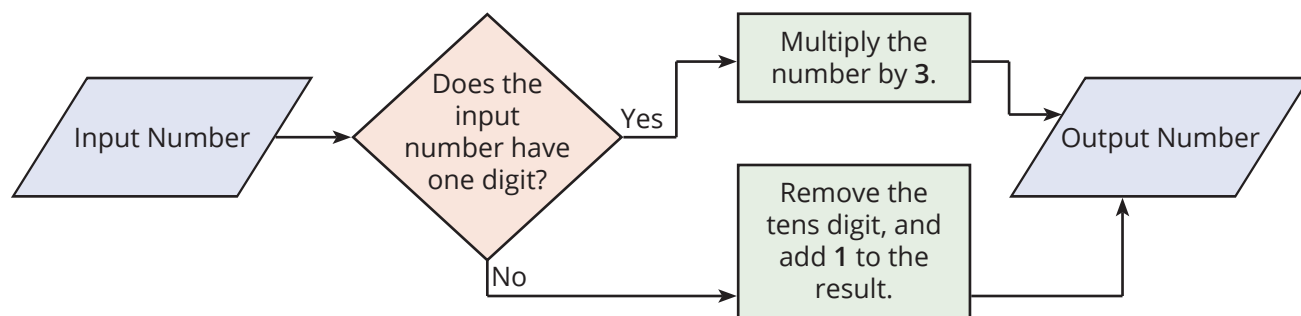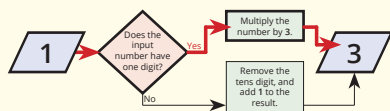
1E. The question is, What would be Julia's 100th input number?

***Strategy:* Find a Pattern, and Build a Table**

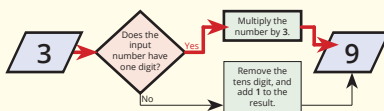Julia's computer program takes an input number, and performs an operation on it.

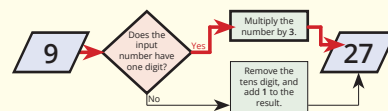

Julia's first input number is **1**.

The first output number is **3**.

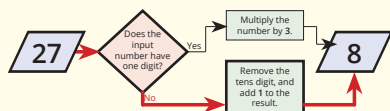Julia enters the output number **3** as her next input number.

The second output number is **9**.

Julia enters the output number **9** as her third input number.

The third output number is **27**.

Julia enters the output number **27** as her fourth input number.

The fourth output number is **8**.

The sequence of numbers continues, as follows.

Position in sequence	1	2	3	4	5	6	7	8	9	10	11	12	13	14	...
Input number	1	3	9	27	8	24	5	15	6	18	9	27	8	24	...

We notice a repeating pattern for the input number:

Input number	1	3	9	27	8	24	5	15	6	18	9	27	8	24	...

Starting from the **3rd** input number, the sequence repeats for every **8th** input number.

Position in sequence	1	2	3	4	5	6	7	8	9	10
Input number	1	3	9	27	8	24	5	15	6	18
Position in sequence			11	12	13	14	15	16	17	18
Input number			9	27	8	24	5	15	6	18
Position in sequence			19	20	21	22	23	24	25	26
Input number			9	27	8	24	5	15	6	18

We notice that the input number is **15** when the position is a multiple of **8**.

A multiple of **8** that is close to **100** is **88**.
The **88**th input number must be **15**.

Position in sequence					88			
Input number	9	27	8	24	5	15	6	18
Position in sequence					96	97	98	
Input number	9	27	8	24	5	15	6	18
Position in sequence	99	100						
Input number	9	27	8	24	5	15	6	18

Both the **88**th and **96**th input numbers are **15**.
Continuing the pattern from the **96**th number, we see that the **100**th input number would be **27**.

***Follow-Up:* What would be the 1000th input number?** [15]