APSMO

MATHS GAMES

WEDNESDAY 3 MAY 2023

MATHS GAMES
SENIOR
1

Suggested Time: ***30 Minutes***

1A. At an ice-cream stall, there are five flavours of ice-cream: chocolate, vanilla, strawberry, pistachio and banana.

You can choose to have your ice-cream in either a cup or a cone.

You can have fudge, nuts, or nothing on top.

The stall only sells ice-creams with 1 scoop, and no more than 1 topping.

How many different combinations of ice-cream does the stall sell?

Hint: How many different combinations of ice-cream does the stall sell, that have no toppings?

1B. Buses have 6 wheels.

Semi-trailers have 18 wheels.

At the truck and bus service station, there were 7 vehicles and a total of 78 wheels.

How many buses were there at the service station?

Hint: How many wheels would there be, if all 7 vehicles were buses?

1C. Olga has been asked to work for 25 days in a row.

She gets paid more for working on weekends, either Saturday or Sunday, so she wants to include as many of those days as possible.

On what days of the week could Olga start working, in order to maximise the number of weekend days she works?

Hint: How might you set out Olga's options in an organised way?

1D. $3^1 = 3.$ $\quad$ $3^2 = \underbrace{3 \times 3}_{\text{2 times}} = 9.$ $\quad$ $3^3 = \underbrace{3 \times 3 \times 3}_{\text{3 times}} = 27.$

Suppose we multiplied out $3^{100} = \underbrace{3 \times 3 \times 3 \times \ldots \times 3}_{\text{100 times}}.$

What would the ones digit of the product be?

Hint: What is the ones digit of 3^1? 3^2? 3^3? 3^4? Look for a pattern.

1E. Lamingtons come in boxes of 6, and boxes of 15.

In how many different ways can you buy exactly 96 lamingtons?

Hint: Is it possible to buy exactly 96 lamingtons if you buy just boxes of 15?

Write your answers in the boxes on the back.

←

Keep your answers hidden by folding backwards on this line.

MATHS GAMES

APSMO

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MATHS GAMES SENIOR 1

Student Name:

1A.

1B.

1C.

1D.

1E.

Fold here. Keep your answers hidden.

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WEDNESDAY 3 MAY 2023

MATHS GAMES SENIOR 1

Solutions and Answers

(Items in parentheses are not required)

1A: 30	**1B:** 4	**1C:** Thu, Fri, Sat	**1D:** 1	**1E:** 4

1A. The question is, How many different combinations of ice-cream does the stall sell in total?

Strategy 1: **Build a Table**

The stall has **5** flavours of ice-cream: chocolate, vanilla, strawberry, pistachio, and banana.

C	V	S	P	B
Chocolate	Vanilla	Strawberry	Pistachio	Banana

You can have your ice-cream in either a cup or a cone.

So far, there are **2 × 5 = 10** different combinations.

We now need to consider the toppings that we might put on each of these **10** different combinations of ice-cream.

	Chocolate	Vanilla	Strawberry	Pistachio	Banana
Cup	C	V	S	P	B
Cone	C	V	S	P	B

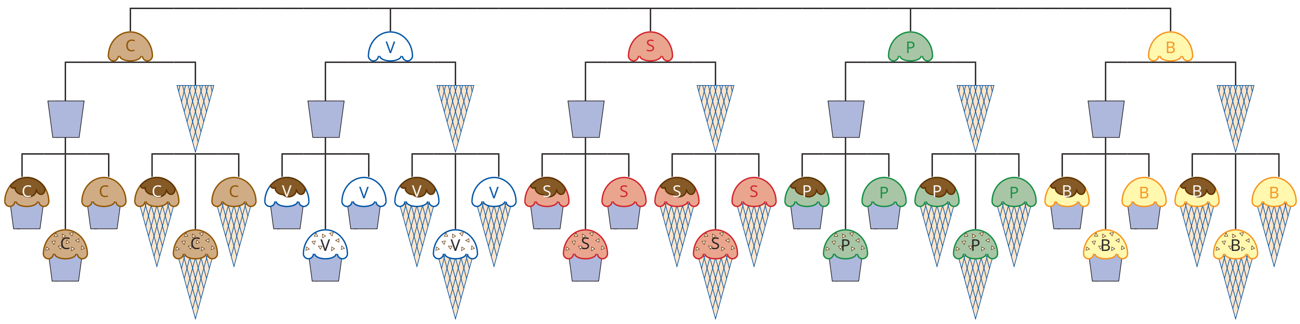
Each ice-cream can have one of **3** choices of topping: fudge, nuts, or nothing (no topping).

The stall sells **3 × 2 × 5 = 30** different combinations of ice-cream.

	C	V	S	P	B	C	V	S	P	B
Fudge	C	V	S	P	B	C	V	S	P	B
Nuts	C	V	S	P	B	C	V	S	P	B
No topping	C	V	S	P	B	C	V	S	P	B

Strategy 2: **Draw a Diagram**

By drawing a tree diagram, we can see that there are **5 × 2 × 3 = 30** different combinations of ice-cream.



Follow-Up: *If customers can buy two scoops, how many different combinations of two-scoop ice-cream does the stall sell?* [90]

1B. The question is, How many buses were there at the service station?

Strategy 1: **Build a Table, and Find a Pattern**

A bus has 6 wheels, and a semi-trailer has 18 wheels.

We can build a table that shows how many wheels there would be, with different combinations of **7** vehicles.

Notice that, when we exchange a bus for a semi-trailer, we increase the number of wheels by **12**.

Exchanging a semi-trailer for a bus decreases the number of wheels by **12**.

Why does this happen?

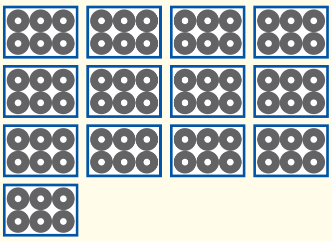
Buses (6 wheels)	Semi-trailers (18 wheels)	Total wheels	
7	0	7 × 6 + 0 × 18 = 42 + 0 = 42	
6	1	6 × 6 + 1 × 18 = 36 + 18 = 54	+12
5	2	5 × 6 + 2 × 18 = 30 + 36 = 66	+12
4	3	4 × 6 + 3 × 18 = 24 + 54 = 78	+12
3	4	3 × 6 + 4 × 18 = 18 + 72 = 90	−12
2	5	2 × 6 + 5 × 18 = 12 + 90 = 102	−12
1	6	1 × 6 + 6 × 18 = 6 + 108 = 114	−12
0	7	0 × 6 + 7 × 18 = 0 + 126 = 126	−12

There are **78** wheels in total if there are **4** buses and **3** semi-trailers.

Strategy 2: **Draw a Diagram, and Find a Pattern**

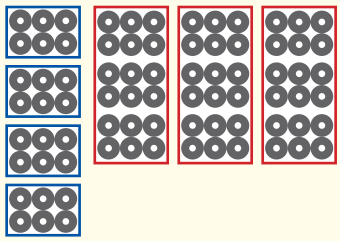
Suppose every one of the **78** wheels was on a bus.

If so, there would be 78 ÷ 6 = 13 buses.



By grouping **3** sets of **6** wheels, we can create one semi-trailer.

We can keep doing this until we have exactly **7** vehicles.



When there are **4** buses and **3** semi-trailers, there are **78** wheels in total.

Strategy 3: **Use Simultaneous Equations**

Let there be x buses with **6** wheels each, and y semi-trailers with **18** wheels each.

There are **7** vehicles in total.	$x + y = 7$	(1)
There are **78** wheels in total.	$6x + 18y = 78$	(2)

Method 1: Substitution

From (1):	$y = 7 - x$	(3)
Subst (3) into (2):	$6x + 18(7 - x) = 78$	
	$6x + 126 - 18x = 78$	
	$-12x = 78 - 126$	
	$= -48$	
	$x = 4$	

Method 2: Elimination

From (1):	$6x + 6y = 42$	(3)
Subtract (3) from (2):	$12y = 78 - 42$	
	$= 36$	
	$y = 3$	(4)
Subst (4) into (1):	$x + 3 = 7$	
	$x = 4$	

Since x = **4**, there were **4** buses at the service station.

Follow-Up: *How many buses are there, if there are 10 vehicles and 168 wheels?* [1]

1C. The question is, What days of the week should Olga choose from, for her first working day?

Strategy 1: **Build a Table**

Olga wants to work as many Saturdays and Sundays as possible.

Sun	Mon	Tue	Wed	Thu	Fri	Sat

If Olga's first day is a Sunday, then, **25** days later, she will have worked **7** weekend days.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25			

If Olga's first day is a Monday, then she will only work **6** weekend days.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25		

If Olga's first day is a Saturday, she will have two straight weekend days at the **beginning** of her **25** consecutive days.

If so, Olga will work **8** weekend days.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25				

Starting on Thursday will result in two straight weekend days at the **end** of Olga's **25** consecutive days.

In this case, Olga will also work **8** weekend days.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25						

Starting on Friday will also result in Olga working **8** weekend days.

Olga should choose **Thursday, Friday or Saturday** for her first day.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25					

Strategy 2: **Find a Pattern, and Reason Logically**

Since the days of the week cycle every **7** days, there must be exactly **2** weekend days in the first **7** days.

There will be another two weekend days in the next **7** days,

and another two in the **7** days from the **15**th day to the **21**st.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

To maximise her weekend days, Olga should try to arrange to have **2** weekend days in the last **4** days.

This can be achieved by starting on a:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Saturday,	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T
Friday,	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M
or Thursday.	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S

Follow-Up: *Suppose Olga has to work 30 days. On which days should she start working, to maximise her weekend days?* [Saturday only]

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1D. The question is, What would the ones digit of the product $3^{100} = \underbrace{3 \times 3 \times 3 \times \ldots \times 3}_{100\text{ times}}$ be?

Strategy: **Find a Pattern, and Build a Table**

We can begin by considering simpler problems.

A pattern begins to emerge after listing a few of the products.

The ones digit of the product repeats for every **4**th value: **3**, **9**, **7**, **1**, then the cycle begins again at **3**.

				Power of 3	Ones digit of product
$3^1 =$	3	=	3	1	3
$3^2 =$	3×3	=	9	2	9
$3^3 =$	$3 \times 3 \times 3$	=	27	3	7
$3^4 =$	$3 \times 3 \times 3 \times 3$	=	81	4	1
$3^5 =$	$3 \times 3 \times 3 \times 3 \times 3$	=	243	5	3
$3^6 =$	$3 \times 3 \times 3 \times 3 \times 3 \times 3$	=	729	6	9
$3^7 =$	$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$	=	2187	7	7
$3^8 =$	$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$	=	6561	8	1
$3^9 =$	$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$	=	19683	9	3
$3^{10} =$	$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$	=	59049	10	9

We note that, since we are only interested in the ones digit, there is no need to multiply out the entire value of the product in each case.

Option 1: Written Algorithm

$$\begin{array}{r} 3 \\ \times\ 3 \\ \hline 9 \end{array} \quad \begin{array}{r} 9 \\ \times\ 3 \\ \hline 27 \end{array} \quad \begin{array}{r} 27 \\ \times\ 3 \\ \hline 81 \end{array} \quad \begin{array}{r} 81 \\ \times\ 3 \\ \hline ..3 \end{array} \quad \begin{array}{r} ..3 \\ \times\ 3 \\ \hline ..9 \end{array} \quad \begin{array}{r} ..9 \\ \times\ 3 \\ \hline ..7 \end{array} \quad \begin{array}{r} ..7 \\ \times\ 3 \\ \hline ..1 \end{array}$$

To find the ones digit of the next product, we only need to multiply the ones digit of the previous result, by **3**.

Option 2: Area Model

To evaluate **81** × **3**:

	80	+	1
3	240	+	3

To evaluate **243** × **3**:

	240	+	3
3	720	+	9

Only the ones digit of each multiplicand is required, to find the ones digit of the product.

Alternatively, we can recognise that the ones digit of 3^4 = **81** is **1**.

Option 3: Multiplying by 81

Multiplying any number by 3^4 = **81** will have no effect on the ones digit of that number.

	720	+	9
80	80 × 720	+	80 × 9
+ 1	1 × 720	+	1 × 9

We can see that the ones digit of the product repeats for every **4**th power of **3**.

Power of 3	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ones digit of product	3	9	7	1	3	9	7	1	3	9	7	1	3	9	7	1	3	9	7	1

Powers that are multiples of **4** all have **1** as the ones digit.

100 = **25** × **4**, so **100** is also a multiple of **4**.

Since we can see why the pattern works, we can conclude that 3^{100} has a ones digit of **1**.

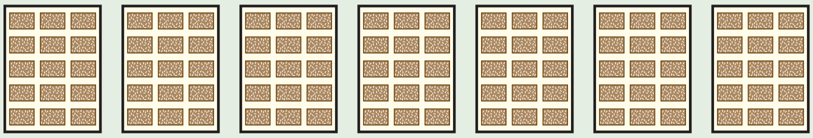
Follow-Up: *What is the ones digit of 9^{999}? [9]*

1E. The question is, In how many different ways can you buy exactly **96** lamingtons?

***Strategy 1:* Build a Table**

Suppose we just buy boxes that each contain **15** lamingtons.

The quantities of lamingtons we might have are:



15 **30** **45** **60** **75** **90** **105**, etc.

To end up with exactly **96** lamingtons, we could have:

No. of boxes of **15** lamingtons	**0**	**1**	**2**	**3**	**4**	**5**	**6**
No. of lamingtons in boxes of **15**	**0**	**15**	**30**	**45**	**60**	**75**	**90**
Remaining lamingtons, bought in boxes of **6**	**96**	**81**	**66**	**51**	**36**	**21**	**6**
No. of boxes of **6** lamingtons	**16**		**11**		**6**		**1**

There are some quantities of "remaining lamingtons", that are not multiples of **6**.

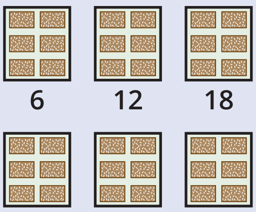
For example, **21** is not a multiple of **6**, and so we cannot buy exactly **21** lamingtons in boxes of **6**.

There are **4** ways to buy exactly **96** lamingtons.

***Strategy 2:* Find the Lowest Common Multiple of 15 and 6, and Build a Table**

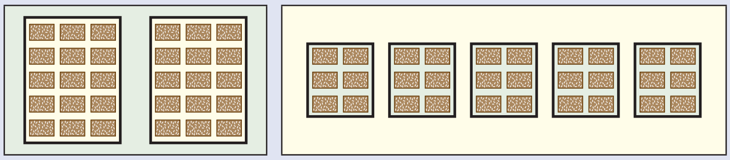
From Strategy 1, we can see that **15, 30, 45, 60, 75** and **90** are all multiples of **15**.

Multiples of **6** include:



6 **12** **18** **24** **30** **36**, etc.

The lowest common multiple of **15** and **6** is **30**.
There are **2** different ways to buy **30** lamingtons.



To buy **96** lamingtons, we can buy **3 lots of 30**, and then another **6**.

We can now list the **4** different ways to buy exactly **96** lamingtons.

	30	**30**	**30**	**6**
Method 1	5 boxes of 6	5 boxes of 6	5 boxes of 6	1 × 6
Method 2	2 boxes of 15	5 boxes of 6	5 boxes of 6	1 × 6
Method 3	2 boxes of 15	2 boxes of 15	5 boxes of 6	1 × 6
Method 4	2 boxes of 15	2 boxes of 15	2 boxes of 15	1 × 6

***Strategy 3:* Construct an Algebraic Equation**

Let there be x boxes of **15**, and y boxes of **6** lamingtons.

All together, there are **96** lamingtons. $15x + 6y = 96$

Subtract $15x$ from both sides. $6y = 96 - 15x$

Divide both sides by **6**. $y = 16 - \frac{5}{2}x$

For this to work, x must be even, and y must be non-negative.

x	0	2	4	6
y	16	11	6	1

There are **4** possible solutions.

Follow-Up: *In how many ways can you buy exactly 150 lamingtons?* [6]