APSMO

2023 : DIVISION S

WEDNESDAY 3 MAY 2023

OLYMPIAD 1

MATHS OLYMPIAD

Total Time Allowed: ***30 Minutes***

Write your answers in the boxes on the back.

Keep your answers hidden by folding backwards on this line. ←

1A. The letters in the word "*Algebra*" are cycled, so that the first three letters become the last three letters.

This action is continuously repeated, with the results placed in a numbered list, as shown.

For what value of N will the word "*Algebra*" next appear?

1. Algebra
2. ebraAlg
3. aAlgebr
...
N. Algebra

1B. When the edges of a cube are increased in length by 10%, its surface area will increase by N%.

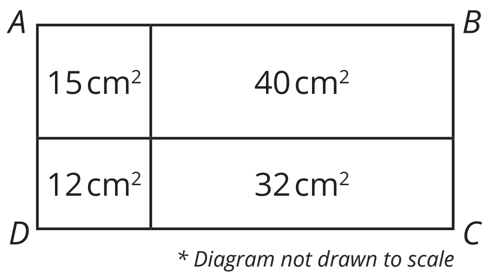
Find N.

1C. Three integers, when added together two at a time, have the sums +5, −31, and −2.

Find the lowest of the three integers.

1D. Rectangle *ABCD* is partitioned into four smaller rectangles with areas 15 cm², 40 cm², 32 cm², and 12 cm², as shown.

The side lengths of each rectangle are a whole number of centimetres.



** Diagram not drawn to scale*

Find the <u>perimeter</u> of rectangle *ABCD*, in centimetres.

1E. How many more 4-digit even numbers than 4-digit odd numbers can be formed using only digits from the set {0, 1, 6}?

Note: A 4-digit number begins with a non-zero digit. 0161 is not a 4-digit number.

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OLYMPIAD

1

MATHS OLYMPIAD

1A.

Student Name:

Fold here. Keep your answers hidden.

1B.

1C.

1D.

1E.

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OLYMPIAD 1

MATHS OLYMPIAD

Solutions and Answers

(Items in parentheses are not required)

For teacher use only. Not for Distribution.

1A: 8	1B: 21 (%)	1C: –19	1D: 40 (cm)	1E: 18 OR "twice as many"

1A. The question is: For what value of *N* will the word "*Algebra*" next appear?

1. *Algebra*
2. *ebraAlg*
3. *aAlgebr*

...

N. *Algebra*

METHOD 1: *Continue the pattern.*

Letters are being cycled:

- Front to back,
- Three letters at a time.

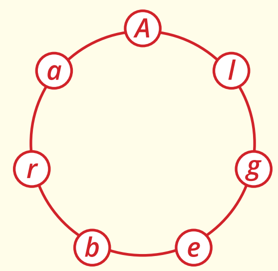
We can continue this pattern until the word *Algebra* appears once again.

1.	*Algebra*	→	*ebraAlg*
2.	*ebraAlg*	→	*aAlgebr*
3.	*aAlgebr*	→	*gebraAl*
4.	*gebraAl*	→	*raAlgeb*
5.	*raAlgeb*	→	*lgebraA*
6.	*lgebraA*	→	*braAlge*
7.	*braAlge*	→	*Algebra*
8.	*Algebra*	→	

The word "*Algebra*" will next appear at **8** in the numbered list.

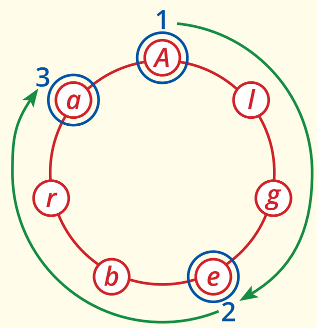
METHOD 2: *Draw a diagram, and find a pattern.*

Since the letters are being cycled, we might begin by arranging the letters in a circle.



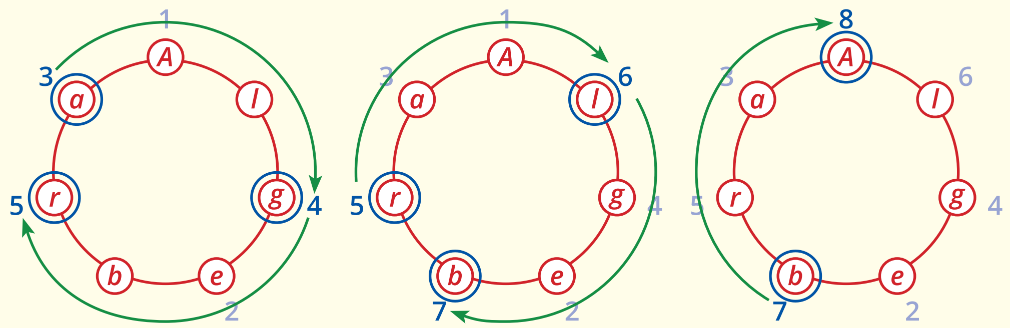
The first "word" in the list begins with *A*.

The second "word" begins with *e*, and the third begins with *a*.



We continue this pattern, indicating every third letter, until we end up back at the *A*.

Since 3 and 7 are mutually prime, every possible "word" will have been formed.



After the 1st word, the next word to read "*Algebra*" will be the **8**th word in the numbered list.

Follow-Up: *The letters in each of the words SOCIAL and STUDIES are cycled, front to back, one letter at a time. The cycles look like this: (1) SOCIAL STUDIES, (2) OCIALS TUDIESS, (3) CIALSO UDIESST, and so on until (N) SOCIAL STUDIES. Find the lowest value of N greater than 1.* [43]

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2023 : DIVISION S

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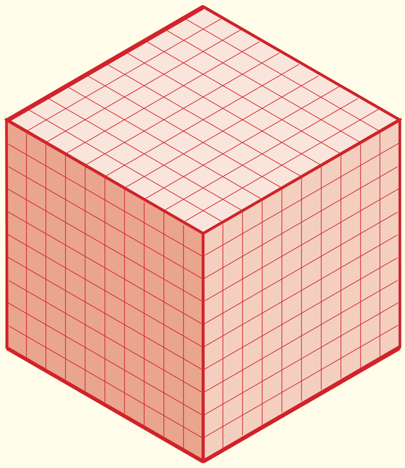
OLYMPIAD 1

1B. The question is: Find *N*, if the surface area of a cube increases by *N*% when the edges are increased by 10%.

METHOD 1: *Use a convenient example.*

We can use a cube that has convenient dimensions.

Since we want to increase the edges by 10% or one-tenth, then a convenient starting cube might have 10 cm edges.



When the lengths of the edges are increased by 10%, each face goes from 10 cm × 10 cm = 100 cm^2, to 11 cm × 11 cm = 121 cm^2.

Each face increases by 121 cm^2 – 100 cm^2 = 21 cm^2.

This happens to each of the six faces.

The surface area of the original cube was 6 × 100 cm^2.

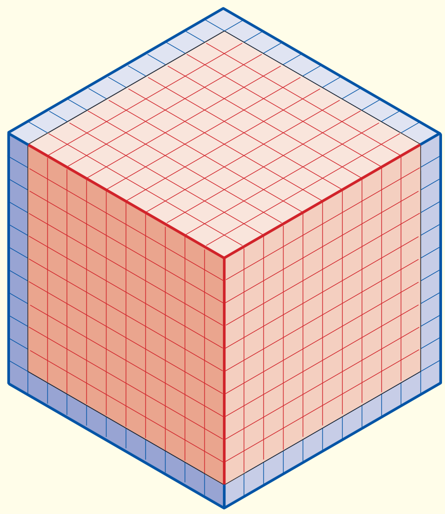
After the lengths of the edges are increased, the surface area increases by 6 × 21 cm^2.

The surface area of the cube increases by

$\frac{6 \times 21}{6 \times 100} \times 100\%$

$= \frac{6}{6} \times \frac{21}{100} \times 100\%$

= **21%**.

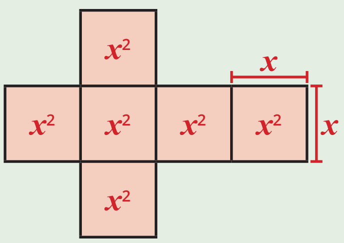


METHOD 2: *Construct an algebraic expression.*

To find the surface area of a cube, we might begin by finding the area of one face.

Since the cube has **6** identical faces, the surface area is **6** times the area of one face.

So if the edge of the cube is x units long, then the surface area would be $6x^2$ square units.



When the edges are increased by 10%, the edge length will be

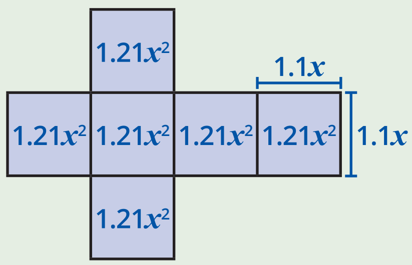
$(100\% \times x) + (10\% \times x)$ units

$= 110\% \times x$ units

$= 1.1x$ units.

After the increase, the surface area of each face will be

$1.1x \times 1.1x = 1.21x^2$ square units.



After the edges are increased, the surface area of the cube is $6 \times 1.21x^2$ square units.

The new surface area exceeds the original surface area by

$6 \times 1.21x^2 - 6x^2$

$= 6x^2(1.21 - 1)$

$= 6x^2(0.21)$ square units.

The percentage increase in surface area is therefore

$\frac{6x^2(0.21)}{6x^2} \times 100\%$

$= 0.21 \times 100\%$

= **21%**.

Follow-Up: When the lengths of the edges of a cube are decreased by 10%, its surface area will decrease by N%. Find N. [19]

1C. The question is: Find the lowest of the three integers.

Let the three integers be represented by a, b, and c.

When added two at a time, the sums are +5, −31, and −2.

$a + b = +5$ (1)

$a + c = -31$ (2)

$b + c = -2$ (3)

METHOD 1: *Add the three equations together.*

The sum of all of the expressions on the left side of the equations, will equal the sum of all of the values on the right side.

(1) + (2) + (3): $a + b + a + c + b + c = +5 - 31 - 2$

$2a + 2b + 2c = -28$

We can halve both sides of the equation, and the statement will still be true.

$a + b + c = -14$ (4)

By finding the difference between equation 4 and each of the other three equations, we can find the values of a, b, and c.

(4) − (1): $a + b + c - (a + b) = -14 - (+5)$

$c = -19$

(4) − (2): $a + b + c - (a + c) = -14 - (-31)$

$b = 17$

(4) − (3): $a + b + c - (b + c) = -14 - (-2)$

$a = -12$

Having worked out that a, b, and c are −12, 17 and −19, we can see that the least of the three integers is **−19**.

METHOD 2: *Find the difference between pairs of equations.*

We begin by finding the difference between equation 1 and equation 2.

(1) − (2): $a + b - (a + c) = +5 - (-31)$

$b - c = 36$ (5)

By adding equation 3 to equation 5, we can find the value of b.

(3) + (5): $b + c + (b - c) = -2 + 36$

$2b = 34$

$b = 17$

Substituting the value of b back into equations 1 and 3, we find that the values of a, b, and c are −12, 17 and −19.

The least of the integers is **−19**.

(1): $a + 17 = +5$

$a = -12$

(3): $17 + c = -2$

$c = -19$

FOLLOW-UP: *Three integers, when multiplied two at a time, have products –21, –15, and +35. What is the sum of the three integers?* [9 or –9]

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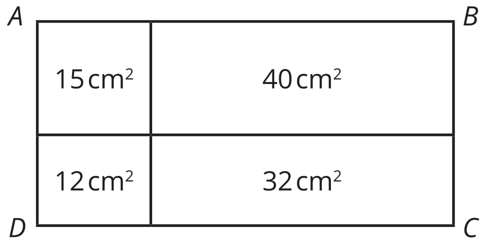
OLYMPIAD 1

1D. The question is: Find the perimeter of rectangle *ABCD*, in centimetres.



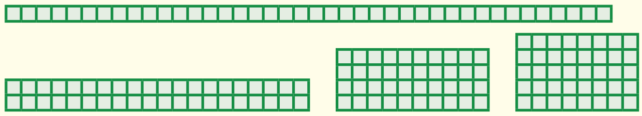
METHOD: *Eliminate all but one possibility.*

We can begin by considering all of the possible dimensions for the different rectangles.

On the top edge, we have rectangles with areas 15 cm² and 40 cm².

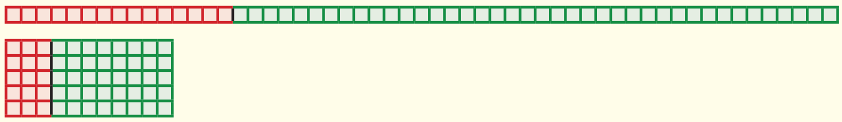
The dimensions of the 15 cm² rectangle could be 1 cm × 15 cm, or 3 cm × 5 cm.

The dimensions of the 40 cm² rectangle could be 1 cm × 40 cm, 2 cm × 20 cm, 4 cm × 10 cm, or 5 cm × 8 cm.



Since these rectangles share a common side, they must have one side length in common.

The possible dimensions are therefore 1 cm × 15 cm and 1 cm × 40 cm, or 5 cm × 3 cm and 5 cm × 8 cm.

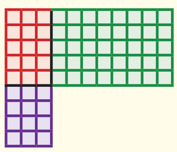


On the left side, we have rectangles with areas 15 cm² and 12 cm².

If the 15 cm² rectangle had dimensions 1 cm × 15 cm, then the 12 cm² rectangle would need to have a 15 cm side.

12 is not divisible by 15, so that possibility does not work.

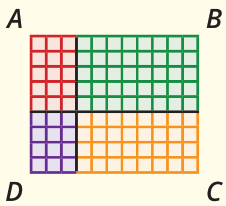
If the 15 cm² rectangle had dimensions 5 cm × 3 cm, then the 12 cm² rectangle would need to have a 3 cm side.



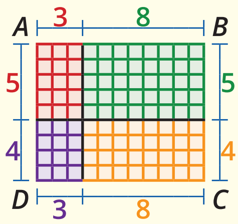
12 ÷ 3 = 4, so the 12 cm² rectangle could have dimensions 4 cm × 3 cm.

The remaining rectangle must have dimensions 4 cm × 8 cm, and an area of 32 cm².

That matches the question.



Rectangle ***ABCD*** must be 3 cm + 8 cm = 11 cm wide, and 5 cm + 4 cm = 9 cm high.



The perimeter of rectangle ***ABCD*** is **11 + 9 + 11 + 9 = 40 cm**.

Follow-Up: *Rectangle EFGH is partitioned into four smaller rectangles with perimeters 27 cm, 20 cm, 17 cm, and 24 cm, as shown. Find the perimeter of rectangle EFGH, in centimetres.* [44]

APSMO

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OLYMPIAD

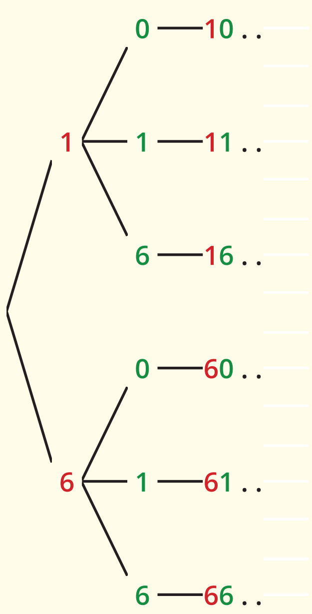
1

1E. The question is: How many more 4-digit even numbers than 4-digit odd numbers can be formed using only digits from the set {0, 1, 6}?

METHOD 1: *Draw a Tree Diagram, and Make an Organised List.*

The first digit of a number can be any digit except for **0**, so all of the possible 4-digit numbers must begin with either a 1 or a 6.

The second digit can then be any one of 0, 1, or 6.



Likewise, the third digit can then be any one of 0, 1, or 6.

Each of the resulting sequences of three digits will be the first digits of three four-digit numbers.

With:

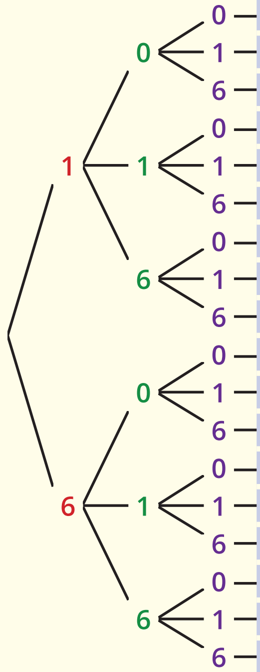
- 2 possible values for the first digit,
- 3 possible values for the 2nd digit, and
- 3 possible values for the third digit,

there are 2 × 3 × 3 = 18 possible sequences for the first three digits.

Each of these sequences creates:

- 2 even four-digit numbers that end in **0** or **6**, and
- 1 odd four digit number that ends in **1**.

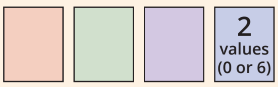
With 18 × 2 = 36 even four-digit numbers, and 18 × 1 = 18 odd four-digit numbers, there are 36 – 18 = 18 more 4-digit even numbers than 4-digit odd numbers.



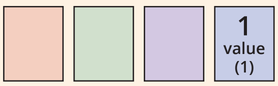
Even	Odd	Even
1000,	1001,	1006
1010,	1011,	1016
1060,	1061,	1066
1100,	1101,	1106
1110,	1111,	1116
1160,	1161,	1166
1600,	1601,	1606
1610,	1611,	1616
1660,	1661,	1666
6000,	6001,	6006
6010,	6011,	6016
6060,	6061,	6066
6100,	6101,	6106
6110,	6111,	6116
6160,	6161,	6166
6600,	6601,	6606
6610,	6611,	6616
6660,	6661,	6666

METHOD 2: *Reason Logically.*

There are 2 units digits that will result in an even number,

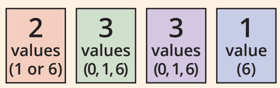


and 1 units digit that will result in an odd number.



Every 4-digit even number ending in **0** will be followed by a 4-digit odd number ending in **1**.

The difference between the number of even numbers and the number of odd numbers, can therefore be found by just counting all of the possible 4-digit numbers that end in the digit **6**.



With 2 × 3 × 3 × 1 = 18 4-digit numbers ending in **6**, there are **18** more 4-digit even numbers than 4-digit odd numbers.

Follow-Up: *How many 3-digit prime numbers can be formed using only the digits in the set {0, 1, 6} ?*
[3: 101, 601, 661]