APSMO

2023 : DIVISION J

WEDNESDAY 3 MAY 2023

OLYMPIAD 1

Total Time Allowed: ***30 Minutes***

Write your answers in the boxes on the back.

1A. What whole number is equal to 2023 + 4046 + 6069 + 8092?

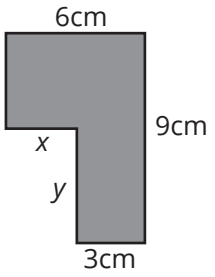
1B. A group of 8 equally skilled workers assembles 1000 toys in 1 hour.

When 2 workers of the same ability are added to the group, how many toys will all 10 workers assemble in an 8-hour day?

Keep your answers hidden by folding backwards on this line.

1C. The polygon shown has area 39 cm^2. Each pair of sides in the polygon intersect at right angles.

What is the total length of the two segments marked *x* and *y*?



1D. A palindrome is a whole number that reads the same forwards and backwards.
For example, 30 903 is a palindrome.
Jill has worked out the greatest palindrome that is less than 30 903.
Kyle found the smallest palindrome that is greater than 30 903.

What is the sum of Jill's palindrome and Kyle's palindrome?

1E. What's the least possible sum in the given cryptarithm if the digit 3 is NOT used anywhere?

```
  A R M
+ A R M
-------
L E G S
```

[Note: In a cryptarithm, different letters represent different digits, and no leading digit can be zero.]

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OLYMPIAD 1

1A.

1B.

1C.

1D.

1E.

Student Name:

Fold here. Keep your answers hidden.

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WEDNESDAY 3 MAY 2023

OLYMPIAD 1

Solutions and Answers

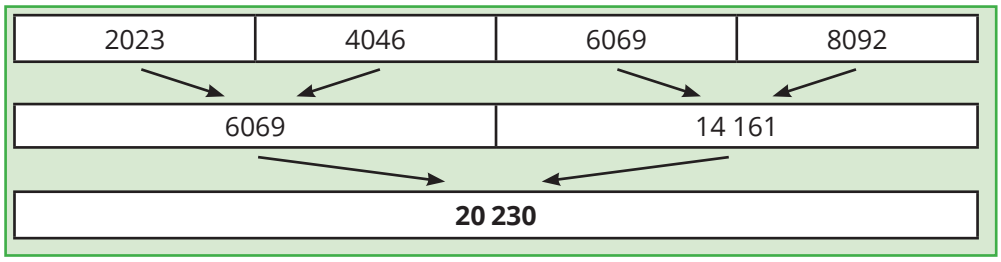
For teacher use only. Not for Distribution.

1A: 20 230	**1B:** 10 000	**1C:** 8cm	**1D:** 61 816	**1E:** 1092

1A. The question is: What whole number is equal to 2023 + 4046 + 6069 + 8092?

METHOD 1 ***Strategy:*** *Perform as indicated.*

The diagrams could also be mirror images or rotations of the ones drawn here.



METHOD 2 ***Strategy:*** *Find a Pattern.*

Notice that each number increases by 2023, and so the equation can be shown as:

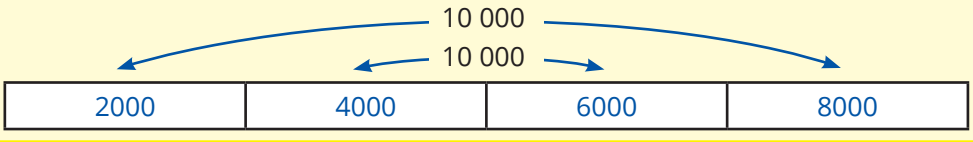
$$\begin{aligned} 2023 + 4046 + 6060 + 8092 &= 2023 \times 1 + 2023 \times 2 + 2023 \times 3 + 2023 \times 4 \\ &= 2023 \times (1 + 2 + 3 + 4) \\ &= 2023 \times 10 \\ &= \mathbf{20\,230} \end{aligned}$$

METHOD 3 ***Strategy:*** *Convert to a More Convenient Form by Grouping Numbers.*

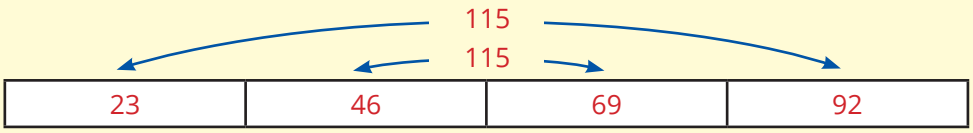
Let's break the numbers into thousands and tens, and then group them so they are easier to add.

$$\begin{aligned} & 2023 + 4046 + 6069 + 8092 \\ = \; & 2000 + 23 + 4000 + 46 + 6000 + 69 + 8000 + 92 \end{aligned}$$

Group and add the thousands:



Group and add the tens:



$$\begin{aligned} & (10\,000 + 10\,000) + (115 + 115) \\ = \; & 20\,000 + 230 \\ = \; & \mathbf{20\,230} \end{aligned}$$

Follow up: *Compute the sum of the first 6 odd multiples of 2023.* [72 828]

1B. The question is: When 2 workers of the same ability are added to the group, how many toys will all 10 workers assemble in an 8-hour day?.

METHOD 1 ***Strategy:*** *Draw a Diagram.*

We start with 8 equally skilled workers assembling 1000 toys in an hour.

1 Hour							
1W	1W	1W	1W	1W	1W	1W	1W
1000 Toys							

Dividing the toys between the workers, 1000 ÷ 8, shows us that each worker assembled 125 toys.

1 Hour							
1W	1W	1W	1W	1W	1W	1W	1W
125 toys	125 toys	125 toys	125 toys	125 toys	125 toys	125 toys	125 toys
1000 Toys							

Add two workers of the same ability to the group.

This will **not** increase the time, but it will increase the number of workers and the number of toys

1 Hour									
1W	1W	1W	1W	1W	1W	1W	1W	1W	1W
125 toys	125 toys	125 toys	125 toys	125 toys	125 toys	125 toys	125 toys	125 toys	125 toys
1000 Toys								250 Toys	

10 workers assemble 10 × 125 toys in 1 hour = 1250 toys.
Therefore, in 8 hours, 10 workers assemble 8 × 1250 toys = **10 000 toys**.

METHOD 2 ***Strategy:*** *Reason Logically*

We know that 8 equally skilled workers assemble 1000 toys in 1 hour.

We use division (1000 ÷ 8 = 125) to determine that 1 worker assembles 125 toys in 1 hour.

Therefore, 10 workers assemble 10 × 125 toys in 1 hour = 1250 toys

In 8 hours, 10 workers assemble 1250 toys × 8 = **10 000 toys.**

Follow-Up*: Ace can paint a room in 3 hours, and Speedy can paint the same room in 2 hours. Working together, they require 1 hour and M minutes to paint the room. Find M.* [12]

1C. The question is: What is the total length of the two segments marked x and y?

METHOD 1 ***Strategy:*** *Complete the rectangle.*

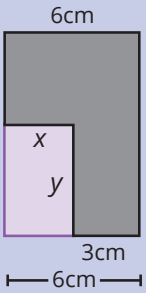
We begin by adding two lines to make a rectangle.

We now know the area of the polygon + the shaded area is $6\,\text{cm} \times 9\,\text{cm} = 54\,\text{cm}^2$.

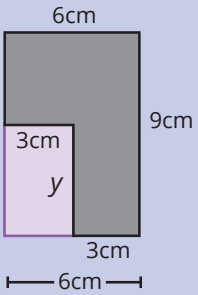
We know the area of the polygon is $39\,\text{cm}^2$.

The shaded area is then $54\,\text{cm}^2 - 39\,\text{cm}^2 = 15\,\text{cm}^2$.

x multiplied by $y = 15\,\text{cm}^2$

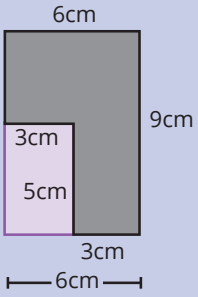


We can see that x must be 3 cm. So $3y = 15\,\text{cm}^2$



Therefore $y = 5$ cm.

The total length of $x + y =$ **8 cm.**



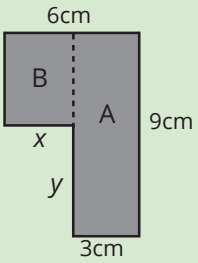
METHOD 2 ***Strategy:*** *Divide the Polygon.*

We begin by dividing the polygon into two rectangles.

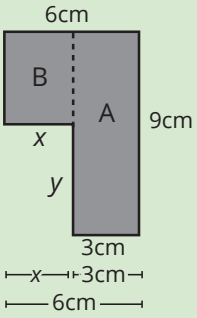
Let's name the smaller one B and the larger one A.

We can calculate the area of A as $27\,\text{cm}^2$ ($3 \times 9 = 27$)

We know the area of the polygon is $39\,\text{cm}^2$. If we subtract A from the polygon, we find the area of B is $12\,\text{cm}^2$.



We find x is 3 cm by subtracting 3 from 6.



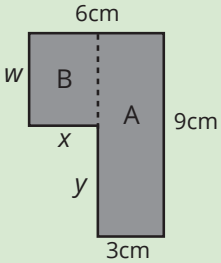
Let's label the unknown height of B as w, noting that $y = 9\,\text{cm} - w$.

We can also write the area of B as $x \times w = 12\,\text{cm}^2$.

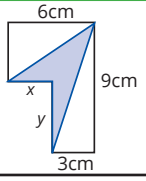
We know that x is 3 cm so $3\,\text{cm} \times w = 12\,\text{cm}^2$ so therefore w is 4 cm.

This tells us that $y = 9\,\text{cm} - 4\,\text{cm} = 5\,\text{cm}$.

The total length of $x + y =$ **8 cm**.

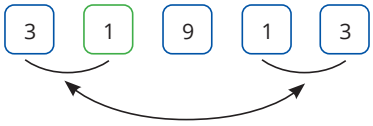
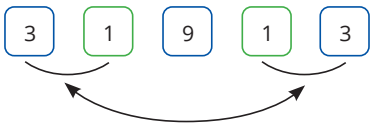
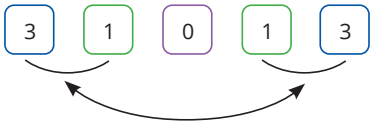


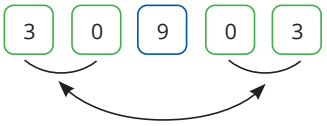
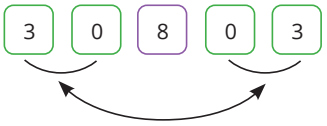
Follow-Up: *The figure has been partitioned into two triangles and a non-convex quadrilateral. What is the area of the quadrilateral?* [13.5 cm^2]



1D. The question is: What is the sum of Jill's palindrome and Kyle's palindrome?

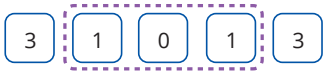
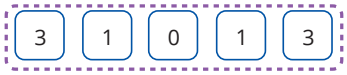
METHOD 1 ***Strategy:*** *Work from the outside in.*

No larger palindrome starting with 30 thousand is possible.	
The next palindrome after 30 903 is greater than 31 000	
The last two digits will be 1 and 3	
The smallest middle digit possible is 0	
Kyle's palindrome is 31 013	

The palindrome less than 30 903 will have 3 and 0 as its first two digits	
It will have 0 and 3 as its last two digits	
To be just under 30 903, the middle digit will be 8	
Jill's palindrome is 30 803	

The sum of the two palindromes is 31 013 + 30 803 = **61 816.**

METHOD 2 ***Strategy:*** *Work from the inside out*

Consider the number formed by the middle 3 digits of the palindrome	
The next greater 3-digit palindrome is 101	
The outer digits need not change.	
Kyle's palindrome is 31 013	

Consider the number formed by the middle 3 digits of the palindrome	
The palindrome just under 30 903 will have 080 as its middle digits	
The outer digit need not change.	
Jill's palindrome is 30 803	

The sum of the two palindromes is 31 013 + 30 803 = **61 816.**

Follow-Up: *How many 4-digit palindromes are multiples of 5?* [10]*; Of 6?* [13]*; Of 11?* [90]

1E. The question is, What's the least possible sum in the given cryptarithm if the digit 3 is NOT used anywhere?

$$\begin{array}{r} A\ R\ M \\ +\ A\ R\ M \\ \hline L\ E\ G\ S \end{array}$$

METHOD 1 ***Strategy:*** *Use number properties to reduce possible guesses*

The digits we can use in this cryptarithm are 0 1 2 4 5 6 7 8 9.

As our leading number can't be 0, the smallest possibility for **L** =1.

The smallest possibility for **E** = 0.

Let's try to see if we can find a solution with these in place - if we can, then we don't need to consider other values for L and E.
A must equal 5 to satisfy A + A = 10.

$$\begin{array}{r} 5\ R\ M \\ +\ 5\ R\ M \\ \hline 1\ 0\ G\ S \end{array}$$

Let's mark the digits we have used: **0 1 2 4 5 6 7 8 9**

We want **G** to be as small as possible.

It can't be 0 or 1, as they have been used.

$$\begin{array}{r} 5\ R\ M \\ +\ 5\ R\ M \\ \hline 1\ 0\ G\ S \end{array}$$

R can't be greater than or equal to 5 as if it were, **R** + **R** would carry into the hundreds column.

R also can't be 1 (as it is assigned) so **G** can't be 2.

We can't use 3.

Can **G** be 4? Let's try. That would make **R** = 2

Let's mark the digits we have used: **0 1 2 4 5 6 7 8 9**

This can't work, as any of the remaining digits to assign to **M** would result in a carry to the tens column.

$$\begin{array}{r} 5\ 2\ M \\ +\ 5\ 2\ M \\ \hline 1\ 0\ 4\ S \end{array}$$ ✗

Put 2 and 4 back in play: **0 1 2 4 5 6 7 8 9**

We need to look for the next smallest number to assign to **G**.

G can't = 5 as it has been assigned to **A**.

G can't = 6 as 3 is not used in the puzzle and if **R** were 8 there would be a carry into the hundreds column.

G can't = 7 as that would rely on **R** = 3 plus 1 ten carried from the tens column and 3 is not used in the puzzle.

Can **G** = 8? Then **R** = 4. **0 1 2 4 5 6 7 8 9**

$$\begin{array}{r} 5\ R\ M \\ +\ 5\ R\ M \\ \hline 1\ 0\ G\ S \end{array}$$

This can't work as there is no solution for **M** and **S** with the remaining digits except for ones that would involve carrying into the tens column.

Since we have eliminated all other possibilities for G, it must be 9

$$\begin{array}{r} 5\ 4\ M \\ +\ 5\ 4\ M \\ \hline 1\ 0\ 8\ S \end{array}$$ ✗

This means **R** = 4 and **M** + **M** is greater than or equal to 10 to carry a ten into the tens column. **0 1 2 4 5 6 7 8 9**

The smallest way to make **M** + **M** greater than ten is to assign 6 to **M**.

$$\begin{array}{r} 5\ 4\ M \\ +\ 5\ 4\ M \\ \hline 1\ 0\ 9\ S \end{array}$$

M + **M** = 12, leaving the 2 in the ones column and providing the extra ten needed in the tens column.

Now we know it is possible to find a solution with LE=10.

$$\begin{array}{r} 5\ 4^{1}\ 6 \\ +\ 5\ 4\ 6 \\ \hline 1\ 0\ 9\ 2 \end{array}$$

The least possible sum is **1092.**

1E. The question is, What's the least possible sum in the given cryptarithm if the digit 3 is NOT used anywhere?

METHOD 2 ***Strategy:*** *Use a table to search in a targeted way.*

Let's start the same way for the same reasons, where **L** = 1, **E** = 0 and **A** = 5.

```
  5 R M
+ 5 R M
1 0 G S
```

We can now just try to solve for the tens and units columns, remembering that there is no carrying into the hundreds.

```
  R M
+ R M
  G S
```

We can make a table relating these letters (**R M G S**) to the remaining digits (2 4 6 7 8 9).

	2	4	6	7	8	9
R			x	x	x	x
M						
G						
S						

R must be less than 5 to avoid a carry.

```
  R M
+ R M
  G S
```

	2	4	6	7	8	9
R	x		x	x	x	x
M		x				
G		x				
S		x				

R can't be 2 because **G** would be 4 and all remaining choices for **M** would carry. **R** = 4 by elimination.

```
  4 M
+ 4 M
  G S
```

	2	4	6	7	8	9
R	x		x	x	x	x
M		x				
G	x	x	x	x		
S		x				

As **R** = 4, **G** must = 8 or 9.

	2	4	6	7	8	9
R	x		x	x	x	x
M		x				x
G	x	x	x	x	x	
S		x				x

G must = 9 as there are no digits to assign to **M** and **S** that don't carry.

```
  4 M
+ 4 M
  9 S
```

	2	4	6	7	8	9
R	x		x	x	x	x
M	x	x				x
G	x	x	x	x	x	
S		x				x

M can't be 2 as it must carry for **G** to be 9.

	2	4	6	7	8	9
R	x		x	x	x	x
M	x	x		x	x	x
G	x	x	x	x	x	
S		x	x	x	x	x

Try the smallest remaining digit for **S**. This is 2. This can be satisfied with **M** = 6.

```
 1 4 6
+ 4 6
  9 2
```

Now we can combine the two parts of the cryptarithm for the solution:

```
  5 4 1 6
+ 5 4 6
1 0 9 2
```

Follow-Up: *What is the greatest possible sum if the digit 3 is used?* [1872 = 936 + 936]