2023 Maths Games Senior - Years 7 & 8
Resource Kit 3
Teaching Problem Solving

Problem Solving Strategies

This resource kit focuses on the following problem solving strategies:

1. Solve a Simpler Related Problem

Many hard problems are actually simpler problems that have been extended to larger numbers.

Patterns can sometimes be identified by trying the problem with smaller numbers.

2. Eliminate All But One Possibility

Deciding what a quantity is not, can narrow the field to a very small number of possibilities.

These can then be tested against the conditions of the original problem.

It follows on from strategies introduced in the Preparation Resource Kit and Resource Kits 1 and 2:

Guess, Check and Refine

Draw a Diagram

Find a Pattern

Build a Table

Work Backwards

Make an Organised List

Resource Kit 3 focuses on:

Solve a Simpler Related Problem

Eliminate All But One Possibility

Set Yellow

Example problems for which full worked solutions are included.

Set Green

Problems that are designed to be similar to Set Yellow, but with fewer difficult elements.

Set Orange

Problems that are similar in mathematical structure to the corresponding Yellow problems.

Further questions and solution methods can be found in the APSMO resource book "Building Confidence in Maths Problem Solving", available from www.apsmo.edu.au.

How to use these problems

At the start of the lesson, present the problem and ask the students to think about it. Encourage students to try to solve it in any way they like. When the students have had enough time to consider their solutions, ask them to describe or present their methods, taking particular note of different ways of arriving at the same solution.

Each question includes at least one solution method that the majority of students should be able to follow. By participating in lessons that demonstrate achievable problem solving techniques, students may gain increased confidence in their own ability to address unfamiliar problems.

Finally, the consideration of different solution methods is fundamental to the students' development as effective and sophisticated problem solvers. Even when students have solved a problem to their own satisfaction, it is important to expose them to other methods and encourage them to judge whether or not the other methods are more efficient.

Preparation Kit

Guess, Check and Refine

This involves making a reasonable guess of the answer, and checking it against the conditions of the problem. An incorrect guess may provide more information that may lead to the answer.

Draw a Diagram

A diagram may reveal information that may not be obvious just by reading the problem.

It is also useful for keeping track of where the student is up to in a multi-step problem.

Resource Kit 1

Find a Pattern

A frequently used problem solving strategy is that of recognising and extending a pattern.

Students can often simplify a difficult problem by identifying a pattern in the problem.

Build a Table

A table displays information so that it is easily located and understood.

A table is an excellent way to record data so the student doesn't have to repeat their efforts.

Resource Kit 2

Work Backwards

If a problem describes a procedure and then specifies the final result, this method usually makes the problem much easier to solve.

Make an Organised List

Listing every possibility in an organised way is an important tool.

How students organise the data often reveals additional information.

Resource Kit 3

Solve a Simpler Related Problem

Many hard problems are actually simpler problems that have been extended to larger numbers.

Patterns can sometimes be identified by trying the problem with smaller numbers.

Eliminate All But One Possibility

Deciding what a quantity is not, can narrow the field to a very small number of possibilities.

These can then be tested against the conditions of the original problem.

Resource Kit 4

Convert to a More Convenient Form

There are times when changing some of the conditions of a problem makes a solution clearer or more convenient.

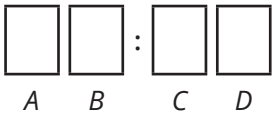
Divide a Complex Shape

Sometimes it is possible to divide an unusual shape into two or more common shapes that are easier to work with.

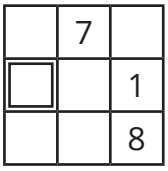
Set Yellow

3.1) Archie had 5 shots for goal and scored 3 of them.
Charlotte had 6 shots for goal and scored 4 of them.
George had 7 shots for goal and scored 5 of them.
Louis had 8 shots for goal and scored 5 of them.
Who had the best scoring rate?

3.2) There is a 12-hour digital clock on my oven.
At five minutes past midnight it shows 12:05.
At twenty-three minutes past 7 o'clock it shows 07:23.
The clock always shows four digits.
We will call those digits *A*, *B*, *C*, and *D*.
What time is it if *C* is four more than *B*, and *D* is five more than *C*?



3.3) The numbers from 1 to 9 can be placed in these boxes so that every row, column and diagonal add up to give the answer 15.
What number goes in the box that looks like this: ☐ ?



3.4) The house numbers on my side of the street are consecutive odd numbers.
When I add the numbers of the houses of my two immediate neighbours (the house to the right and the house to the left of my house) I get 18.
What is the number of my house?

Set Yellow

3.5) Scarlet, Jade and Violet each play a different instrument: clarinet, drums, and guitar, although not necessarily in that order.

The guitarist is Jade's sister.

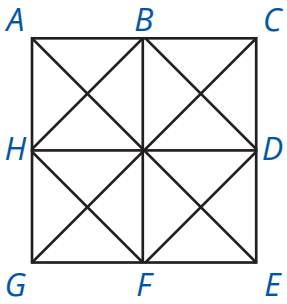
The drummer helped Violet and the guitarist pack up their music stands.

Who plays the drums?

3.6) Square *ACEG* is drawn at the right.

Points *B*, *D*, *F*, and *H* are halfway along the sides of the square.

What is the total number of squares of all sizes which can be traced using only the lines drawn?



3.7) Charlotte, Declan, Faye and Thomas each have a different pet: a canary, a dog, a fish and a turtle.

Neither Charlotte nor Thomas has a fish.

Faye does not have a canary or a turtle.

Nobody has a pet that starts with the same letter as their own name.

What is Charlotte's pet?

3.8) Kerry is making signs.

Each sign would say either ***ENTRY*** or ***EXIT***.

He has 6 of the letter ***E***, 3 ***'I'***s, 3 ***'N'***s, 4 ***'R'***s, 5 ***'T'***s, 4 ***'X'***s, and 5 ***'Y'***s.

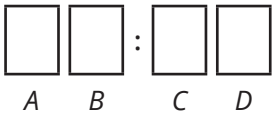
How many complete signs can he make?

E	*E*	*E*	*E*	*E*	*E*
I	*I*	*I*	*N*	*N*	*N*
R	*R*	*R*	*R*	*T*	*T*
T	*T*	*T*	*X*	*X*	*X*
X	*Y*	*Y*	*Y*	*Y*	*Y*

Set Green

3.1) Archie had 4 shots for goal and scored 0 of them.
Charlotte had 6 shots for goal and scored 3 of them.
George had 7 shots for goal and scored 3 of them.
Who had the best scoring rate?

3.2) There is a 12-hour digital clock in my classroom.
At five minutes past twelve it shows 12:05.
At twenty-three minutes past 9 o'clock it shows 09:23.
The clock always shows four digits.
We will call those digits *A*, *B*, *C*, and *D*.
At about morning tea time, *B* was 1 less than A, C was 3 more than B, and D was 2 more than C.
What time was it?



3.3) The numbers from 1 to 9 can be placed in these boxes so that every row, column and diagonal add up to give the answer 15.
What number goes in the box that looks like this: ☐ ?

8		4
1		
6	☐	

3.4) The house numbers on my side of the street are consecutive odd numbers.
When I add the numbers of the houses of my two next door neighbours (the house to the right and the house to the left of my house) I get 6.
What is the number of my house?

Set Green

3.5) Scarlet, Jade and Violet each play a different instrument: clarinet, drums, and guitar, although not necessarily in that order.

Scarlet plays the guitar.

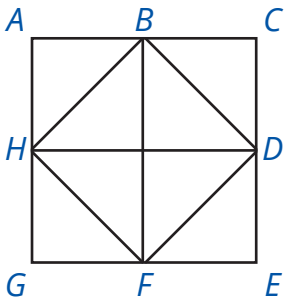
The drummer helped Violet and the guitarist pack up their music stands.

Who plays the drums?

3.6) Square *ACEG* is drawn at the right.

Points *B*, *D*, *F*, and *H* are halfway along the sides of the square.

What is the total number of squares of all sizes which can be traced using only the lines drawn?



3.7) Charlotte, Declan, Faye and Thomas each have a different pet: a canary, a dog, a fish and a turtle.

Neither Charlotte nor Thomas has a fish.

Faye does not have a canary or a turtle.

Nobody has a pet that starts with the same letter as their own name.

What is Declan's pet?

3.8) Kerry is making signs.

Each sign would say either ***ENTRY*** or ***EXIT***.

He has 4 of the letter ***E***, 2 ***'I'***s, 2 ***'N'***s, 2 ***'R'***s, 3 ***'T'***s, 2 ***'X'***s, and 1 ***'Y'***.

How many complete signs can he make?

E	E	E	E
I	I	N	N
R	R	T	T
T	X	X	Y

Set Orange

3.1) A group consisted of 2 girls for every boy.

24 more girls joined the group.

There are now 5 girls for every boy.

How many boys are in the group?

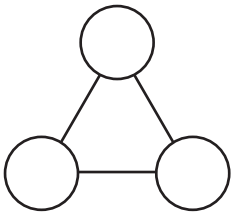
3.2) Anna drew three circles joined by three lines.

She wrote a number in each circle.

Then, she added the numbers from each pair of circles, and wrote the sum on the line joining them.

She found that she had all of the numbers 1, 2, 3, 4, 5 and 6 somewhere on her diagram.

What were the numbers in the circles, from smallest to largest?



3.3) Each row and column of this square contain all of the numbers 1, 2, 3 and 4, in some order.

What number goes in the box that looks like this: □ ?

2		3	
		2	
			4
	□		

3.4) In a set of natural counting numbers, all have different values.

Their sum is 350.

Their average is 50.

One of the numbers is 100.

What is the greatest number that can be in the set?

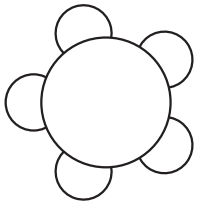
Set Orange

3.5) Gemma, Harry, Ivy, Jared and Kelly are sitting around a round table, facing the centre.

Kelly is next to Gemma, on Gemma's right side.

Harry is not next to Kelly or Ivy.

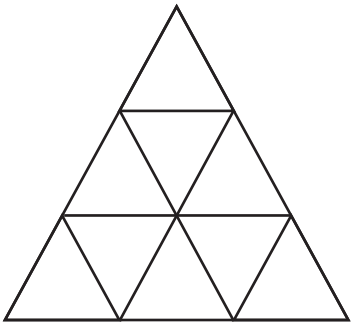
Name the two students who are sitting next to Jared.



3.6) The diagram shows one large triangle.

There are some straight lines drawn between the sides.

How many triangles, of any size, can be traced on the lines in the diagram?



3.7) Peter, Quinn, Rob and Stephen are all different ages: 9, 10, 11 and 12.

Peter is older than both Rob and Stephen.

Quinn is two years younger than Rob.

How old is Stephen?

3.8) Aiah lists all the counting numbers from 1 through 200.

How many times will the digit 4 appear on Aiah's list?

Maths Games – Example Problem 3.1

Example Problem 3.1 - Green

Archie had 4 shots for goal and scored 0 of them.
Charlotte had 6 shots for goal and scored 3 of them.
George had 7 shots for goal and scored 3 of them.
Who had the best scoring rate?

Example Problem 3.1 - Yellow

Archie had 5 shots for goal and scored 3 of them.
Charlotte had 6 shots for goal and scored 4 of them.
George had 7 shots for goal and scored 5 of them.
Louis had 8 shots for goal and scored 5 of them.
Who had the best scoring rate?

Example Problem 3.1 - Orange

A group consisted of 2 girls for every boy.
24 more girls joined the group.
There are now 5 girls for every boy.
How many boys are in the group?

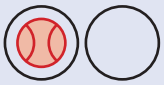
Maths Games Example Solution 3.1 - Yellow

Archie had 5 shots for goal and scored 3 of them. Charlotte had 6 shots for goal and scored 4 of them. George had 7 shots for goal and scored 5 of them. Louis had 8 shots for goal and scored 5 of them.
Who had the best scoring rate?

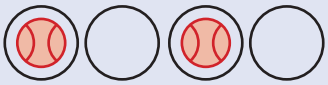
Strategy: Solve a Simpler Related Problem

Let's start by thinking about some simpler scoring rates.

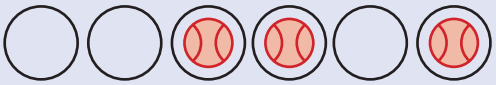
If I had **2** shots for goal and scored **1** of them,



that's the same scoring rate as if I had **4** shots for goal and scored **2** of them,

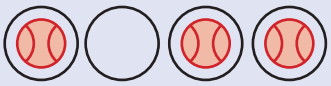


or if I had **6** shots for goal and scored **3** of them.



These three scenarios give scoring rates of $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$, all of which are equivalent.

Next, let's consider a scenario where Sam had **4** shots for goal, and scored **3** of them.



The scoring rate here is higher.
$\frac{3}{4}$ is greater than $\frac{2}{4}$.
Sam has a better scoring rate than I do.

We can see that Sam's **3** goals out of **4** attempts is better than my **3** goals out of **6** attempts.

- Both *Sam and I scored the same number of goals*, but *Sam got them in fewer attempts*.

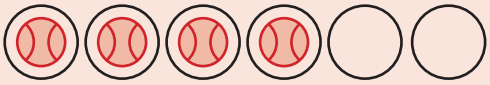
We can also see that Sam's **3** goals out of **4** attempts is better than my **1** goal out of **2** attempts.

- Both *Sam and I missed the same number of goals*, but *Sam had more attempts at goal* (and scored).

Now we can try comparing Archie and Charlotte.

Archie had **5** shots for goal and scored **3** of them.

Charlotte had **6** shots for goal and scored **4** of them.



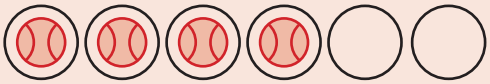
Both Archie and Charlotte *missed the same number of goals*.

- When Sam and I both *missed the same number of goals*, Sam had a better scoring rate, because he had *more attempts at goal* (and scored).

Charlotte had *more attempts at goal* than Archie.

So, Charlotte has a better scoring rate than Archie.

Charlotte had **6** shots for goal and scored **4** of them.



George had **7** shots for goal and scored **5** of them.

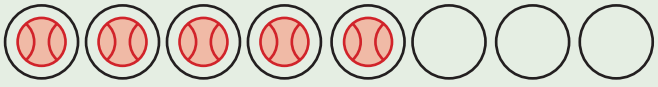
Both Charlotte and George *missed the same number of goals*.

George had *more attempts at goal* than Charlotte.

So, George has a better scoring rate than Charlotte.

George had **7** shots for goal and scored **5** of them.

Louis had **8** shots for goal and scored **5** of them.



Both George and Louis *scored the same number of goals*, but George *got them in fewer attempts* than Louis.

- Sam had a better scoring rate than me, because he *scored the same number of goals in fewer attempts*.

So, George has a better scoring rate than Louis.

Of the four students, **George** has the best scoring rate.

Answers

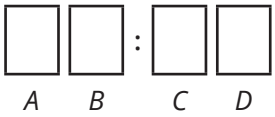
3.1 - Green: Charlotte

3.1 - Orange: 8

3.1 - Yellow: George

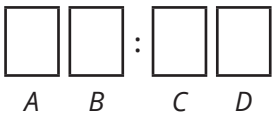
Maths Games – Example Problem 3.2

Example Problem 3.2 - Green



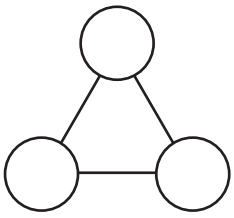
There is a 12-hour digital clock in my classroom.
At five minutes past twelve it shows 12:05.
At twenty-three minutes past 9 o'clock it shows 09:23.
The clock always shows four digits.
We will call those digits *A*, *B*, *C*, and *D*.
At about morning tea time, *B* was 1 less than *A*, *C* was 3 more than *B*, and *D* was 2 more than *C*.
What time was it?

Example Problem 3.2 - Yellow



There is a 12-hour digital clock on my oven.
At five minutes past midnight it shows 12:05.
At twenty-three minutes past 7 o'clock it shows 07:23.
The clock always shows four digits.
We will call those digits *A*, *B*, *C*, and *D*.
What time is it if *C* is four more than *B*, and *D* is five more than *C*?

Example Problem 3.2 - Orange



Anna drew three circles joined by three lines.
She wrote a number in each circle.
Then, she added the numbers from each pair of circles, and wrote the sum on the line joining them.
She found that she had all of the numbers 1, 2, 3, 4, 5 and 6 somewhere on her diagram.
What were the numbers in the circles, from smallest to largest?

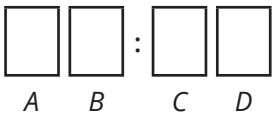
Maths Games Example Solution 3.2 - Yellow

There is a 12-hour digital clock on my oven. At five minutes past midnight it shows 12:05.

At twenty-three minutes past 7 o'clock it shows 07:23.

The clock always shows four digits. We will call those digits *A*, *B*, *C*, and *D*.

What time is it if *C* is four more than *B*, and *D* is five more than *C*?

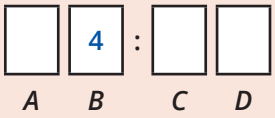


Strategy: Eliminate All But One Possibility

Let's guess a number and see what happens.

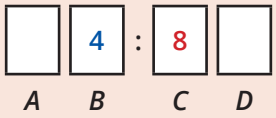
We'll guess ***B***, because the question doesn't say anything about ***A***.

Suppose ***B*** = 4.



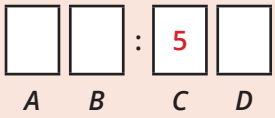
C is four more than ***B***. So ***C*** would be 4 + 4 = 8.

But that would mean that the clock is showing 80-something minutes past the hour, which is much too big.



With **60** minutes in one hour, we know that the greatest possible number of minutes is **59**.

Therefore the greatest possible value for ***C*** is 5.

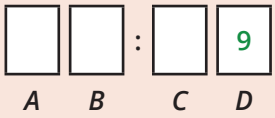


D is five more than ***C***.

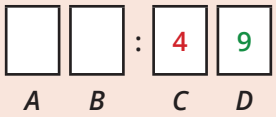
So ***D*** would be 5 + 5 = 10.

This is impossible, since ***D*** must be a single digit.

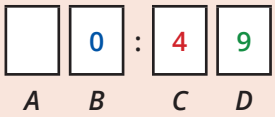
The greatest possible value for ***D*** is 9.



Since ***D*** is five more than ***C***, ***C*** would be 9 – 5 = 4.



Since ***C*** is four more than ***B***, ***B*** would be 4 – 4 = 0.

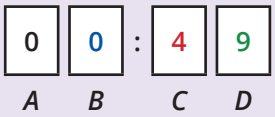


B cannot be less than 0, and ***D*** cannot be greater than 9.

So we can be certain that we have found the only possible set of values for ***B***, ***C*** and ***D***.

Now we need to find the number that is in position ***A***.

Could ***A*** be **0**?



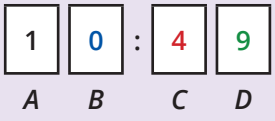
There are digital clocks that show a time like this, and it would mean **49** minutes past midnight.

However, my oven has a **12**-hour clock.

At five minutes past midnight, the oven clock shows **12:05**.

So at **49** minutes past midnight, the oven clock will show **12:49**, not **00:49**.

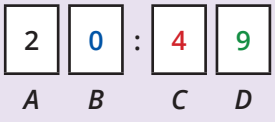
Could ***A*** be **1**?



This works.

It would mean **49** minutes past **10** o'clock.

Could ***A*** be **2**?



Since it's a **12**-hour clock, the hour can't be bigger than **12**.

Therefore ***A*** can't be any bigger than **1**.

Therefore the time must be **10:49**.

Answers

3.2 - Green: 10:35

3.2 - Yellow: 10:49

3.2 - Orange: 1, 2, 4

Maths Games – Example Problem 3.3

Example Problem 3.3 - Green

The numbers from 1 to 9 can be placed in these boxes so that every row, column and diagonal add up to give the answer 15.

What number goes in the box that looks like this: ☐ ?

8		4
1		
6	☐	

Example Problem 3.3 - Yellow

The numbers from 1 to 9 can be placed in these boxes so that every row, column and diagonal add up to give the answer 15.

What number goes in the box that looks like this: ☐ ?

	7	
☐		1
		8

Example Problem 3.3 - Orange

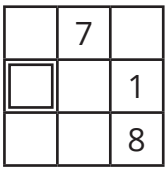
Each row and column of this square contain all of the numbers 1, 2, 3 and 4, in some order.

What number goes in the box that looks like this: ☐ ?

2		3	
		2	
			4
	☐		

Maths Games Example Solution 3.3 - Yellow

The numbers from 1 to 9 can be placed in these boxes so that every row, column and diagonal add up to give the answer 15.



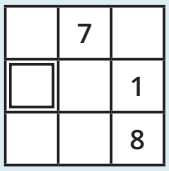
What number goes in the box that looks like this: □ ?

Strategy 1: Eliminate All But One Possibility (1)

We know that all together, the boxes contain each digit from 1 to 9.

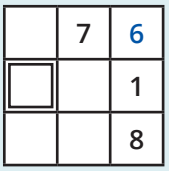
We also know that each row, column and diagonal adds up to 15.

Let's fill in any squares we can figure out.



The rightmost column has □ + 1 + 8 = 15.

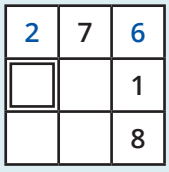
So □ + 9 = 15, and □ = 6.



The top row has □ + 7 + 6 = 15.

So □ + 13 = 15

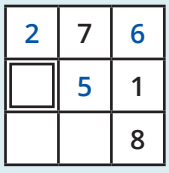
and □ = 2.



The diagonal from top left has 2 + □ + 8 = 15.

So □ + 10 = 15

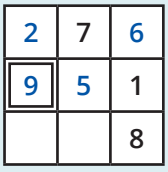
and □ = 5.



The middle row has □ + 5 + 1 = 15.

So □ + 6 = 15

and □ = 9.



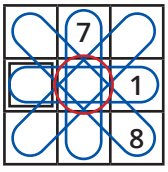
So the digit in the □ square is a **9**.

Strategy 2: Eliminate All But One Possibility (2)

The square in the centre is included in *one row*, *one column* and *both diagonals*.

So we need four different ways to make **15** using this centre digit, plus two more digits.

Let's find all of the different ways we can make **15** using three different digits:



We can't have, for example, 1 + 2 + 12 because 12 is not a one-digit number.

We also can't have 1 + 7 + 7 because the digits are not all different.

1 + 5 + 9	2 + 4 + 9	3 + 4 + 8	4 + 5 + 6
1 + 6 + 8	2 + 5 + 8	3 + 5 + 7	
	2 + 6 + 7		

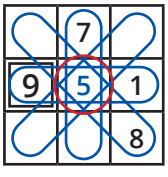
Looking at these possibilities, we can see that 5 is the only digit that appears four times.

1 + 5 + 9	2 + 4 + 9	3 + 4 + 8	4 + 5 + 6
1 + 6 + 8	2 + 5 + 8	3 + 5 + 7	
	2 + 6 + 7		

So the centre square must be a 5.

Therefore, the middle row has □ + 5 + 1 = 15, so, □ = 15 – 6.

Since 15 – 6 = 9, the digit in the □ square must be **9**.



Answers

3.3 - Green: 7

3.3 - Orange: 3

3.3 - Yellow: 9

Maths Games – Example Problem 3.4

Example Problem 3.4 - Green

The house numbers on my side of the street are consecutive odd numbers.

When I add the numbers of the houses of my two next door neighbours (the house to the right and the house to the left of my house) I get 6.

What is the number of my house?

Example Problem 3.4 - Yellow

The house numbers on my side of the street are consecutive odd numbers.

When I add the numbers of the houses of my two immediate neighbours (the house to the right and the house to the left of my house) I get 18.

What is the number of my house?

Example Problem 3.4 - Orange

In a set of natural counting numbers, all have different values.

Their sum is 350.

Their average is 50.

One of the numbers is 100.

What is the greatest number that can be in the set?

Maths Games Example Solution 3.4 - Yellow

The house numbers on my side of the street are consecutive odd numbers.

When I add the numbers of the houses of my two immediate neighbours (the house to the right and the house to the left of my house) I get 18.

What is the number of my house?

Strategy 3: Solve a Simpler Related Problem

Let's suppose my house number is 3.

Then my neighbours would have the house numbers

- 3 – 2 = 1, and
- 3 + 2 = 5.

The sum of their house numbers would be
3 – 2 + 3 + 2 = 6.

That's double the house number we're testing.

If my house number is 13, then my neighbours would have the house numbers

- 13 – 2 = 11, and
- 13 + 2 = 15.

The sum of their house numbers would be
13 – 2 + 13 + 2 = 200.

Again, this is double my house number.

The sum of my neighbours' house numbers is 18.

Therefore, 18 must be double my house number, so my house number is 18 ÷ 2 = 9.

Let's check:

- Neighbour 1 is at 9 – 2 = 7.
- Neighbour 2 is at 9 + 2 = 11.

The sum of their house numbers is 7 + 11 = 18.

This matches the question.

Strategy 1: Build a Table

We know that the house numbers on my side of the street are consecutive odd numbers.

Let's build a table to list the possible house numbers.

Neighbour 1's house number	1	3	5	7	
My house number	3	5	7	9	
Neighbour 2's house number	5	7	9	11	
Neighbour 1 + Neighbour 2	6	10	14	18	

We can see that, if the sum of my neighbours' house numbers is **18**, then my house must be number **9**.

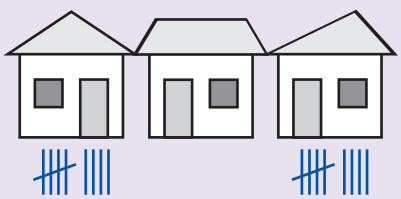
Strategy 2: Draw a Diagram

We can imagine that house numbers on my street are represented by tally marks.

If we collect all of the tally marks from my two neighbours, we'll have 18 sticks in total.

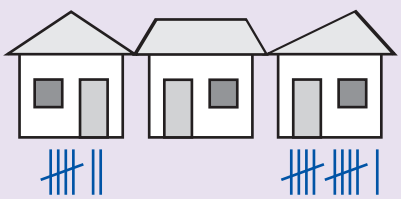
Let's begin by distributing the 18 sticks evenly between the two neighbours.

Each neighbour has 18 ÷ 2 = 9 sticks.



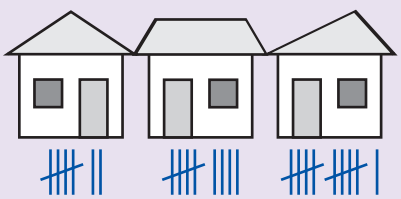
If we move two sticks from one house to the other, we'll have:

- the previous odd number (7) on one side, and
- the following odd number (11) on the other side.



This works, because there are still 18 sticks in total for the two neighbours.

With number **7** on one side, and number **11** on the other side, my house must be house number **9**.



Answers

3.4 - Green: 3

3.4 - Yellow: 9

3.4 - Orange: 235

Maths Games – Example Problem 3.5

Example Problem 3.5 - Green

Scarlet, Jade and Violet each play a different instrument: clarinet, drums, and guitar, although not necessarily in that order.

Scarlet plays the guitar.

The drummer helped Violet and the guitarist pack up their music stands.

Who plays the drums?

Example Problem 3.5 - Yellow

Scarlet, Jade and Violet each play a different instrument: clarinet, drums, and guitar, although not necessarily in that order.

The guitarist is Jade's sister.

The drummer helped Violet and the guitarist pack up their music stands.

Who plays the drums?

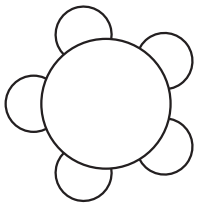
Example Problem 3.5 - Orange

Gemma, Harry, Ivy, Jared and Kelly are sitting around a round table, facing the centre.

Kelly is next to Gemma, on Gemma's right side.

Harry is not next to Kelly or Ivy.

Name the two students who are sitting next to Jared.



Maths Games Example Solution 3.5 - Yellow

Scarlet, Jade and Violet each play a different instrument: clarinet, drums, and guitar, although not necessarily in that order. The guitarist is Jade's sister.

The drummer helped Violet and the guitarist pack up their music stands. Who plays the drums?

Strategy 1: Eliminate All But One Possibility

Scarlet plays	clarinet?	drums?	guitar?
Jade plays	clarinet?	drums?	guitar?
Violet plays	clarinet?	drums?	guitar?

Scarlet plays	clarinet?	drums?	guitar?
Jade plays	clarinet?	drums?	guitar? x
Violet plays	clarinet?	drums?	guitar?

Scarlet plays	clarinet?	drums?	guitar?
Jade plays	clarinet?	drums?	guitar? x
Violet plays	clarinet?	drums? x	guitar? x

Scarlet plays	clarinet? x	drums?	guitar?
Jade plays	clarinet? x	drums?	guitar? x
Violet plays	clarinet? ✓	drums? x	guitar? x

Scarlet plays	clarinet? x	drums? x	guitar? ✓
Jade plays	clarinet? x	drums?	guitar? x
Violet plays	clarinet? ✓	drums? x	guitar? x

Scarlet plays	clarinet? x	drums? x	guitar? ✓
Jade plays	clarinet? x	drums? ✓	guitar? x
Violet plays	clarinet? ✓	drums? x	guitar? x

Strategy 2: Build a Table, and Eliminate All But One Possibility

Scarlet, Jade and Violet each play one of the three instruments: clarinet, drums and guitar.

	clarinet	drums	guitar
Scarlet			
Jade			
Violet			

The guitarist is Jade's sister.

So Jade can't be the guitarist.

	clarinet	drums	guitar
Scarlet			
Jade			x
Violet			

The drummer helped Violet and the guitarist pack up their music stands.

So Violet isn't the drummer or guitarist.

	clarinet	drums	guitar
Scarlet			
Jade			x
Violet		x	x

Therefore, Violet must play the clarinet.

So neither Scarlet nor Jade plays the clarinet.

	clarinet	drums	guitar
Scarlet	x		
Jade	x		x
Violet	✓	x	x

Since Jade and Violet don't play the guitar, the guitarist must be Scarlet.

	clarinet	drums	guitar
Scarlet	x	x	✓
Jade	x		x
Violet	✓	x	x

This means that the drummer must be **Jade**.

	clarinet	drums	guitar
Scarlet	x	x	✓
Jade	x	✓	x
Violet	✓	x	x

Answers

3.5 - Green: Jade

3.5 - Yellow: Jade

3.5 - Orange: Harry and Ivy

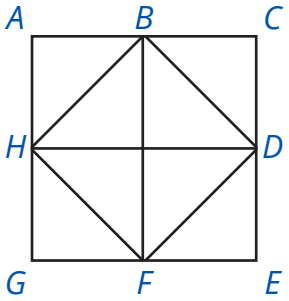
Maths Games – Example Problem 3.6

Example Problem 3.6 - Green

Square *ACEG* is drawn at the right.

Points *B*, *D*, *F*, and *H* are halfway along the sides of the square.

What is the total number of squares of all sizes which can be traced using only the lines drawn?

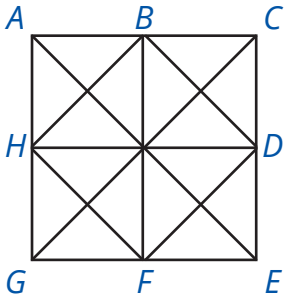


Example Problem 3.6 - Yellow

Square *ACEG* is drawn at the right.

Points *B*, *D*, *F*, and *H* are halfway along the sides of the square.

What is the total number of squares of all sizes which can be traced using only the lines drawn?

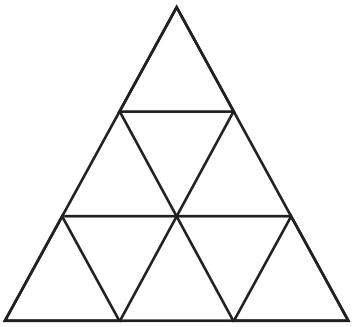


Example Problem 3.6 - Orange

The diagram shows one large triangle.

There are some straight lines drawn between the sides.

How many triangles, of any size, can be traced on the lines in the diagram?

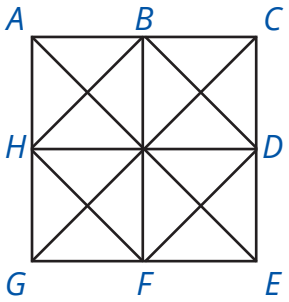


Maths Games Example Solution 3.6 - Yellow

Square *ACEG* is drawn at the right.

Points *B*, *D*, *F*, and *H* are halfway along the sides of the square.

What is the total number of squares of all sizes which can be traced using only the lines drawn?



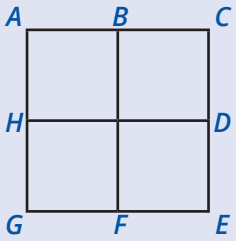
Strategy: Solve a Simpler Related Problem

This picture has a lot of lines. Let's simplify it by taking out some lines.

First let's think about squares where one of the sides is a horizontal (left-right) line.

- Can you use vertical lines in the same square? (Yes)
- Can you use other horizontal lines in the same square? (Yes)
- Can you use slanted lines in the same square? (No)

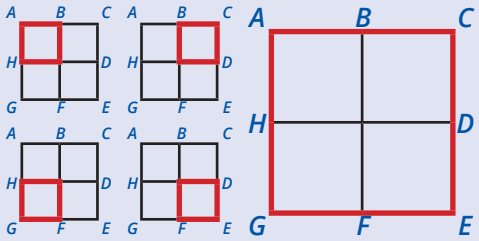
Let's take out the slanted lines for now.



We can now count the squares that are in our simplified diagram.

There's one big square around the outside, and four smaller squares inside.

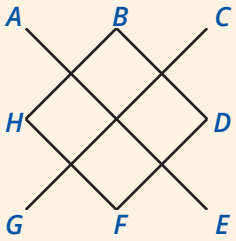
There are **5** horizontal-vertical squares.



Next let's think about squares where one of the sides is a slanted line.

- Can you use vertical lines in the same square? (No)
- Can you use horizontal lines in the same square? (No)

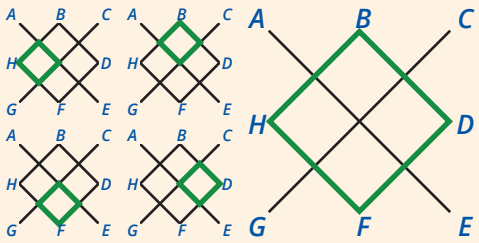
Let's take out the horizontal and vertical lines for now.



We can now count the squares that are in our simplified diagram.

There's one big square around the outside, and four smaller squares inside.

There are **5** slanted squares.



There are **5** horizontal-vertical squares, and **5** slanted squares.

There are **5** + **5** = **10** squares in the diagram.

Answers

3.6 - Green: 6

3.6 - Orange: 13

3.6 - Yellow: 10

Maths Games – Example Problem 3.7

Example Problem 3.7 - Green

Charlotte, Declan, Faye and Thomas each have a different pet: a canary, a dog, a fish and a turtle.
Neither Charlotte nor Thomas has a fish.
Faye does not have a canary or a turtle.
Nobody has a pet that starts with the same letter as their own name.
What is Declan’s pet?

Example Problem 3.7 - Yellow

Charlotte, Declan, Faye and Thomas each have a different pet: a canary, a dog, a fish and a turtle.
Neither Charlotte nor Thomas has a fish.
Faye does not have a canary or a turtle.
Nobody has a pet that starts with the same letter as their own name.
What is Charlotte's pet?

Example Problem 3.7 - Orange

Peter, Quinn, Rob and Stephen are all different ages: 9, 10, 11 and 12.
Peter is older than both Rob and Stephen.
Quinn is two years younger than Rob.
How old is Stephen?

Maths Games Example Solution 3.7 - Yellow

Charlotte, Declan, Faye and Thomas have a different pet each: a canary, a dog, a fish and a turtle.

Neither Charlotte nor Thomas has a fish. Faye does not have a canary or a turtle.

Nobody has a pet that starts with the same letter as their own name.

What is Charlotte's pet?

Strategy 1: Eliminate All But One Possibility (1)

Let's list all of the possible options.

Charlotte has a	canary?	dog?	fish?	turtle?
Declan has a	canary?	dog?	fish?	turtle?
Faye has a	canary?	dog?	fish?	turtle?
Thomas has a	canary?	dog?	fish?	turtle?

Neither Charlotte nor Thomas has a fish.

Charlotte has a	canary?	dog?	~~fish?~~	turtle?
Declan has a	canary?	dog?	fish?	turtle?
Faye has a	canary?	dog?	fish?	turtle?
Thomas has a	canary?	dog?	~~fish?~~	turtle?

Faye does not have a canary or a turtle.

Charlotte has a	canary?	dog?	~~fish?~~	turtle?
Declan has a	canary?	dog?	fish?	turtle?
Faye has a	~~canary?~~	dog?	fish?	~~turtle?~~
Thomas has a	canary?	dog?	~~fish?~~	turtle?

Nobody's pet starts with the same letter as their name.

Charlotte has a	~~canary?~~	dog?	~~fish?~~	turtle?
Declan has a	canary?	~~dog?~~	fish?	turtle?
Faye has a	~~canary?~~	dog?	~~fish?~~	~~turtle?~~
Thomas has a	canary?	dog?	~~fish?~~	~~turtle?~~

We can see that the fish can only belong to Declan, and the only option for Faye is a dog.

Charlotte has a	~~canary?~~	dog?	~~fish?~~	turtle?
Declan has a	canary?	~~dog?~~	fish? ✓	turtle?
Faye has a	~~canary?~~	dog? ✓	~~fish?~~	~~turtle?~~
Thomas has a	canary?	dog?	~~fish?~~	~~turtle?~~

Since the dog belongs to Faye, Charlotte isn't the owner of the dog.

Charlotte has a	~~canary?~~	~~dog?~~	~~fish?~~	turtle?
Declan has a	~~canary?~~	~~dog?~~	fish? ✓	~~turtle?~~
Faye has a	~~canary?~~	dog? ✓	~~fish?~~	~~turtle?~~
Thomas has a	canary?	~~dog?~~	~~fish?~~	~~turtle?~~

Charlotte doesn't have a canary, a dog or a fish, so Charlotte's pet must be a **turtle**.

Strategy 2: Build a Table, and Eliminate All But One Possibility

We can represent the options more efficiently by building a table.

	Charlotte	Declan	Faye	Thomas
canary				
dog				
fish				
turtle				

Neither Charlotte nor Thomas has a fish.

Faye does not have a canary or a turtle.

	Charlotte	Declan	Faye	Thomas
canary			✗	
dog				
fish	✗			✗
turtle			✗	

Nobody has a pet that starts with the same letter as their own name.

	Charlotte	Declan	Faye	Thomas
canary	✗		✗	
dog		✗		
fish	✗		✗	✗
turtle			✗	✗

We can now follow each of the pets to the correct owners.

	Charlotte	Declan	Faye	Thomas
canary	✗	✗	✗	
dog	✗	✗	✓	✗
fish	✗	✓	✗	✗
turtle		✗	✗	✗

Charlotte doesn't have a canary, a dog or a fish, so Charlotte's pet must be a **turtle**.

Answers

3.7 - Green: Fish

3.7 - Orange: 10

3.7 - Yellow: Turtle

Maths Games – Example Problem 3.8

Example Problem 3.8 - Green

Kerry is making signs.
Each sign would say either ***ENTRY*** or ***EXIT***.
He has 4 of the letter ***E***, 2 ***'I'***s, 2 ***'N'***s, 2 ***'R'***s, 3 ***'T'***s, 2 ***'X'***s, and 1 ***'Y'***.
How many complete signs can he make?

E	*E*	*E*	*E*
I	*I*	*N*	*N*
R	*R*	*T*	*T*
T	*X*	*X*	*Y*

Example Problem 3.8 - Yellow

Kerry is making signs.
Each sign would say either ***ENTRY*** or ***EXIT***.
He has 6 of the letter ***E***, 3 ***'I'***s, 3 ***'N'***s, 4 ***'R'***s, 5 ***'T'***s, 4 ***'X'***s, and 5 ***'Y'***s.
How many complete signs can he make?

E	*E*	*E*	*E*	*E*	*E*
I	*I*	*I*	*N*	*N*	*N*
R	*R*	*R*	*R*	*T*	*T*
T	*T*	*T*	*X*	*X*	*X*
X	*Y*	*Y*	*Y*	*Y*	*Y*

Example Problem 3.8 - Orange

Aiah lists all the counting numbers from 1 through 200.
How many times will the digit 4 appear on Aiah's list?

Maths Games Example Solution 3.8 - Yellow

Kerry is making signs.

Each sign would say either ***ENTRY*** or ***EXIT***.

He has 6 of the letter ***E***, 3 ***'I'***s, 3 ***'N'***s, 4 ***'R'***s, 5 ***'T'***s, 4 ***'X'***s, and 5 ***'Y'***s.

How many complete signs can he make?

E	E	E	E	E	E
I	I	I	N	N	N
R	R	R	R	T	T
T	T	T	X	X	X
X	Y	Y	Y	Y	Y

Strategy 1: Solve a Simpler Related Problem

Instead of keeping track of both kinds of sign, we can start by getting Kerry to make just one kind of sign.

Option 1: Try *ENTRY* signs first.

Since there are only 3 ***'N'***s, Kerry can only make **3** signs that read ***ENTRY***.

E	N	T	R	Y
6	3	5	4	5

After taking out the letters to make **3** ***ENTRY*** signs, we can work out what's still available to use to make ***EXIT*** signs.

~~E~~	~~E~~	~~E~~	E	E	E
~~I~~	~~I~~	~~I~~	~~N~~	~~N~~	~~N~~
~~R~~	~~R~~	~~R~~	R	T	T
~~T~~	~~T~~	~~T~~	X	X	X
X	Y	Y	~~Y~~	~~Y~~	~~Y~~

Since there are only **2** ***'T'***s left, Kerry can only make **2** signs that read ***EXIT***.

E	X	I	T
3	4	3	2

Option 2: Try *EXIT* signs first.

Since there are only 3 ***'I'***s, Kerry can only make **3** signs that read ***EXIT***.

E	X	I	T
6	4	3	5

After taking out the letters to make **3** ***EXIT*** signs, we can work out what's still available to use to make ***ENTRY*** signs.

~~E~~	~~E~~	~~E~~	E	E	E
~~I~~	~~I~~	~~I~~	N	N	N
R	R	R	R	T	T
~~T~~	~~T~~	~~T~~	~~X~~	~~X~~	~~X~~
X	Y	Y	Y	Y	Y

Since there are only **2** ***'T'***s left, Kerry can only make **2** signs that read ***ENTRY***.

E	N	T	R	Y
3	3	2	4	5

So Kerry can only make **3 + 2 = 5** complete signs.

Strategy 2: Reason Logically

For each letter, we can begin by checking whether it is used in one or both signs.

Since each sign needs one ***E*** and one ***T***, the number of signs is limited by the numbers of ***'E'***s and ***'T'***s.

With **6** ***'E'***s and **5** ***'T'***s, we know that there are enough ***'E'***s and ***'T'***s to make **5** signs.

	E	I	N	R	T	X	Y
Total number of letters	6	3	3	4	5	4	5
No. required for *ENTRY*	1		1	1	1		1
No. required for *EXIT*	1	1			1	1	

To check if **5** signs are, in fact, possible, we now need to consider the other letters required for those signs.

	N	R	Y
Other letters for *ENTRY*	3	4	5

With just **3** ***'N'***s, there are enough letters for up to **3** ***ENTRY*** signs.

	X	I
Other letters for *EXIT*	4	3

With just **3** ***'I'***s, there are enough letters for up to **3** ***EXIT*** signs.

Kerry can make up to **3** ***ENTRY*** signs, up to **3** ***EXIT*** signs, and up to **5** signs in total.

Since **3 + 3 > 5**, we can see that Kerry can make at most **5** complete signs.

Answers

3.8 - Green: 3

3.8 - Orange: 40

3.8 - Yellow: 5

Answers

Set Green	
3.1	Charlotte
3.2	10:35
3.3	7
3.4	3
3.5	Jade
3.6	6
3.7	Fish
3.8	3

Set Yellow	
3.1	George
3.2	10:49
3.3	9
3.4	9
3.5	Jade
3.6	10
3.7	Turtle
3.8	5

Set Orange	
3.1	8
3.2	1, 2, 4
3.3	3
3.4	235
3.5	Harry and Ivy
3.6	13
3.7	10
3.8	40