2023 Maths Games Senior - Years 7 & 8
Resource Kit 2
Teaching Problem Solving

MATHS GAMES

Problem Solving Strategies

This resource kit focuses on the following problem solving strategies:

1. Work Backwards

If a problem describes a procedure and then specifies the final result, this method usually makes the problem much easier to solve.

2. Make an Organised List

Listing every possibility in an organised way is an important tool.

How students organise the data often reveals additional information.

It follows on from strategies introduced in the preparation resource kit and resource kit 1:

Guess, Check and Refine

Draw a Diagram

Find a Pattern

Build a Table

Resource Kit 2 focuses on:

Work Backwards

Make an Organised List

Set Yellow

Example problems for which full worked solutions are included.

Set Green

Problems that are designed to be similar to Set Yellow, but with fewer difficult elements.

Set Orange

Problems that are similar in mathematical structure to the corresponding Yellow problems.

Further questions and solution methods can be found in the APSMO resource book "Building Confidence in Maths Problem Solving", available from www.apsmo.edu.au.

How to use these problems

At the start of the lesson, present the problem and ask the students to think about it. Encourage students to try to solve it in any way they like. When the students have had enough time to consider their solutions, ask them to describe or present their methods, taking particular note of different ways of arriving at the same solution.

Each question includes at least one solution method that the majority of students should be able to follow. By participating in lessons that demonstrate achievable problem solving techniques, students may gain increased confidence in their own ability to address unfamiliar problems.

Finally, the consideration of different solution methods is fundamental to the students' development as effective and sophisticated problem solvers. Even when students have solved a problem to their own satisfaction, it is important to expose them to other methods and encourage them to judge whether or not the other methods are more efficient.

Preparation Kit

Guess, Check and Refine

This involves making a reasonable guess of the answer, and checking it against the conditions of the problem. An incorrect guess may provide more information that may lead to the answer.

Draw a Diagram

A diagram may reveal information that may not be obvious just by reading the problem.

It is also useful for keeping track of where the student is up to in a multi-step problem.

Resource Kit 1

Find a Pattern

A frequently used problem solving strategy is that of recognising and extending a pattern.

Students can often simplify a difficult problem by identifying a pattern in the problem.

Build a Table

A table displays information so that it is easily located and understood.

A table is an excellent way to record data so the student doesn't have to repeat their efforts.

Resource Kit 2

Work Backwards

If a problem describes a procedure and then specifies the final result, this method usually makes the problem much easier to solve.

Make an Organised List

Listing every possibility in an organised way is an important tool.

How students organise the data often reveals additional information.

Resource Kit 3

Solve a Simpler Related Problem

Many hard problems are actually simpler problems that have been extended to larger numbers.

Patterns can sometimes be identified by trying the problem with smaller numbers.

Eliminate All But One Possibility

Deciding what a quantity is not, can narrow the field to a very small number of possibilities.

These can then be tested against the conditions of the original problem.

Resource Kit 4

Convert to a More Convenient Form

There are times when changing some of the conditions of a problem makes a solution clearer or more convenient.

Divide a Complex Shape

Sometimes it is possible to divide an unusual shape into two or more common shapes that are easier to work with.

Set Yellow

2.1) Alana had a lump of clay.

She gave half of it to Bernard.

She used a third of the clay she had left to make a bowl.

She then had 100 grams of clay left to make other things.

What was the weight, in grams, of the original lump of clay?

2.2) In the addition shown, different letters represent different digits.

What is the four-digit number represented by *ABCD*?

$$\begin{array}{cccccc}
 & & & & 8 & A \\
 & & & 2 & B & 9 \\
 & & 1 & C & 6 & 7 \\
+ & 1 & D & 7 & 5 & 4 \\
\hline
 & 1 & 2 & 3 & 4 & 5
\end{array}$$

2.3) Rachel, Stuart and Terry each have the same number of pencils.

When Will joined their group, he didn't have any pencils, so the students shared all of the pencils equally amongst the four of them.

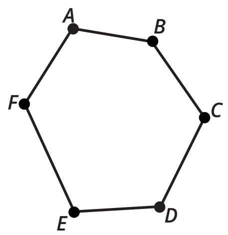
If Rachel ended up with two fewer pencils than she had at the beginning, how many pencils do the students have in total?

2.4) Teo's hexagon has six different side lengths.

Each vertex is marked with a black dot.

He cuts the hexagon into two pieces using a single straight cut from one dot to another.

In how many different ways could Teo cut his hexagon?



Set Yellow

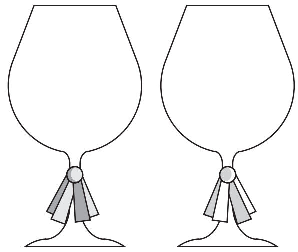
2.5) Miss Timms is having a party.

So that her guests don't accidentally pick up someone else's drink, she ties coloured ribbons in a single knot on the stem of each glass.

There are five different colours of ribbon.

Each glass has exactly two different colours.

How many different colour combinations can there be?

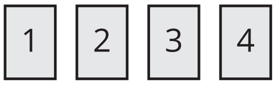


2.6) I have four cards on which are written the numbers 1, 2, 3 and 4.

I can use the cards to make the number 1234.

I can't use them to make 1224, because I only have one card with a 2 on it.

Using these four cards, what is the second-largest number I can make?



2.7) Angela, Ben, Caroline and David need to sit in a row on the stage to receive awards.

Angela and Caroline have to sit together.

Ben and David have to sit together.

In how many different ways can they sit?

2.8) I bought a large box of oranges at the farmers' market.

I gave half of the oranges to my brother, and then I gave him one more.

I gave half of the remaining oranges to my sister, and then I gave her two more.

I gave half of the remaining oranges to my neighbour, and then I gave him three more.

I had just two oranges left.

How many oranges were in the box I bought from the farmers' market?

Set Green

2.1) Alana had a lump of clay.

She gave half of it to Charlie.

She used half of the clay she had left to make a vase.

She then had 100 grams of clay left to make other things.

What was the weight, in grams, of the original lump of clay?

2.2) In this addition problem, some digits are missing.

What digit goes in the box that looks like this: ⬡

$$\begin{array}{r} \square\ 3\ \square\ 5 \\ +\ 8\ ⬡\ 9\ \square \\ \hline 1\ 1\ 4\ 6\ 3 \end{array}$$

2.3) Rachel and Stuart both have the same number of pencils.

When Terry joined their group, he didn't have any pencils, so the students shared all of the pencils equally amongst the three of them.

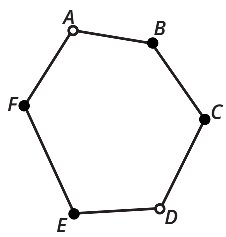
If Rachel ended up with two fewer pencils than she had at the beginning, how many pencils do the students have in total?

2.4) Teo's hexagon has six different side lengths.

Each vertex is marked with a dot.

He cuts the hexagon into two pieces using a single straight cut from one dot to another.

In how many different ways could Teo cut his hexagon, if he starts cutting from either vertex *A* or vertex *D*?



Set Green

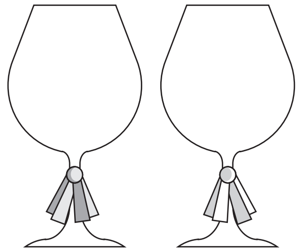
2.5) Miss Timms is having a party.

So that her guests don't accidentally pick up someone else's drink, she ties coloured ribbons in a single knot on the stem of each glass.

There are three different colours of ribbon.

Each glass has exactly two different colours.

How many different colour combinations can there be?



2.6) I have three cards on which are written the numbers 1, 2, and 3.

I can use the cards to make the number 123.

I can't use them to make 122, because I only have one card with a 2 on it.

Using these three cards, what is the second-largest number I can make?

2.7) Angela, Ben and Caroline need to sit in a row on the stage to receive awards.

Angela and Caroline have to sit together.

In how many different ways can they sit?

2.8) I bought a large box of oranges at the farmers' market.

I gave half of the oranges to my brother, and then I gave him one more.

I gave half of the remaining oranges to my sister, and then I gave her one more.

I had just one orange left.

How many oranges were in the box I bought from the farmers' market?

Set Orange

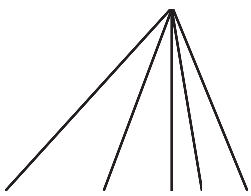
2.1) Mrs Allen spends $\frac{3}{5}$ of her money at the grocery store.
She spends $\frac{3}{5}$ of her remaining money at the service station.
She then has $8.00 left.
How many dollars did Mrs Allen have to begin with?

2.2) Let ***P***, ***Q*** and ***R*** represent the missing digits in the subtraction shown.
Find the sum ***P*** + ***Q*** + ***R***.

$$\begin{array}{r} 8\ 4\ P \\ -\ Q\ 6\ 8 \\ \hline 4\ R\ 3 \end{array}$$

2.3) The average (mean) of four consecutive even integers is 17.
Find the largest of the four integers.

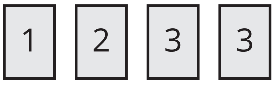
2.4) An acute angle is an angle that measures less than 90°.
Using the lines in the diagram, how many different acute angles can be formed?



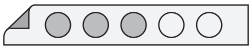
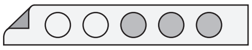
Set Orange

2.5) Two numbers represented by A and B are chosen from the following set of numbers: { 1, 2, 3, 4, 5, 6 }.
In how many different ways will $\frac{A}{B}$ have a value less than $\frac{1}{2}$?
Consider $\frac{1}{3}$ and $\frac{2}{6}$ as two different ways.

2.6) I have four cards on which are written the numbers 1, 2, 3 and 3.
I can use the cards to make the number 1233.
I can't use them to make 1223, because I only have one card with a 2 on it.
Using these four cards, what is the third-largest number I can make?



2.7) Five identical circles are arranged in a straight line on a strip of paper.
In how many different ways can exactly 3 of these circles be coloured grey?

Note: Consider the arrangements  and  as the same, because the paper can be turned around.

2.8) There is a plate of crackers on the kitchen table.
Sara takes half of the crackers, plus 4 more.
Then Nick takes 2.
Joe takes half of what is left and then takes 2 more.
Finally Selena takes 5.
Four crackers remain on the plate.
How many crackers were on the plate to begin with?

Maths Games – Example Problem 2.1

Example Problem 2.1 - Green

Alana had a lump of clay.

She gave half of it to Charlie.

She used half of the clay she had left to make a vase.

She then had 100 grams of clay left to make other things.

What was the weight, in grams, of the original lump of clay?

Example Problem 2.1 - Yellow

Alana had a lump of clay.

She gave half of it to Bernard.

She used a third of the clay she had left to make a bowl.

She then had 100 grams of clay left to make other things.

What was the weight, in grams, of the original lump of clay?

Example Problem 2.1 - Orange

Mrs Allen spends $\frac{3}{5}$ of her money at the grocery store.

She spends $\frac{3}{5}$ of her remaining money at the service station.

She then has \$8.00 left.

How many dollars did Mrs Allen have to begin with?

Maths Games Example Solution 2.1 - Yellow

Alana had a lump of clay. She gave half of it to Bernard.

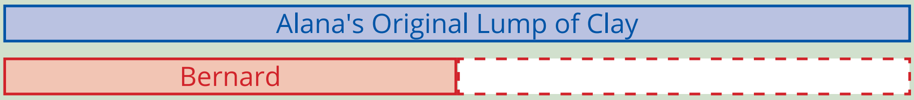
She used a third of the clay she had left to make a bowl.

She then had 100 grams of clay left to make other things.

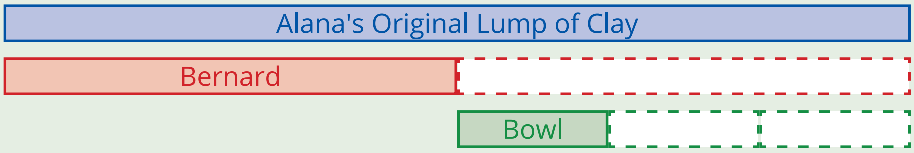
What was the weight, in grams, of the original lump of clay?

Strategy: Draw a Diagram, and Work Backwards

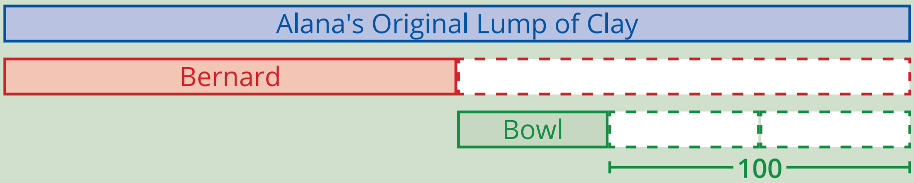
Alana had a lump of clay.
She gave half of it to Bernard.



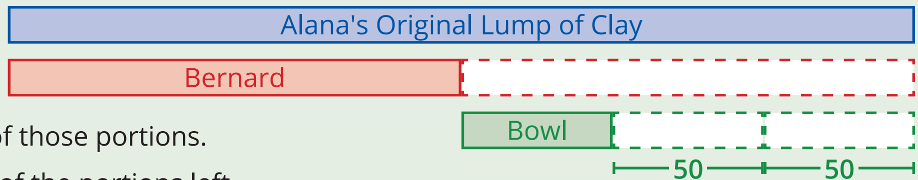
She used one third of the clay she had left to make a bowl.



She had 100 grams of clay left.



When Alana used one third of the clay to make the bowl, it was as though she broke the clay into 3 equal portions, and used 1 of those portions.

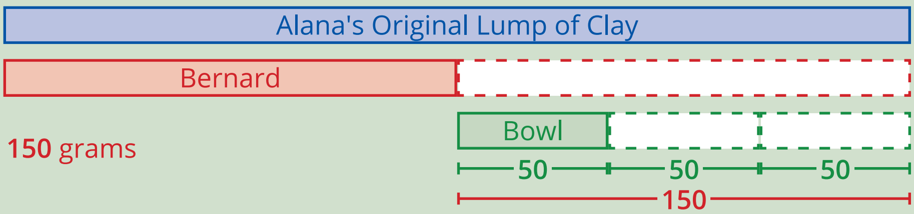


This means that she would have 2 of the portions left.

Together, those **two** portions weigh 100 grams, so **one** of the portions must weigh 100 ÷ 2 = 50 grams.

The other **one third** of the clay used to make the bowl must also have weighed 50 grams.

This means that Alana had 3 × 50 = 150 grams before she made the bowl.

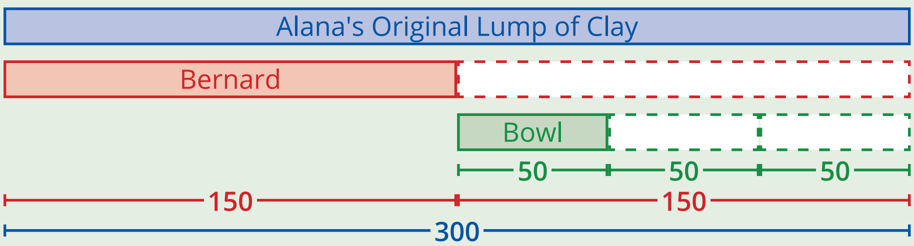


Before making the bowl, Alana gave half of her clay to Bernard. Alana kept 150 grams of clay for herself.

Because Alana gave Bernard half of the clay, both of them have the same amount of clay.

Bernard must have received 150 grams.

This means that Alana's original lump of clay weighed **2** × **150** = **300** grams.



Answers

2.1 - Green: 400

2.1 - Orange: $50

2.1 - Yellow: 300

Maths Games – Example Problem 2.2

Example Problem 2.2 - Green

In this addition problem, some digits are missing.

What digit goes in the box that looks like this: ⬡

$$
\begin{array}{rccccc}
 & & \square & 3 & \square & 5 \\
+ & 8 & \text{⬡} & 9 & \square & \\
\hline
 & 1 & 1 & 4 & 6 & 3
\end{array}
$$

Example Problem 2.2 - Yellow

In the addition shown, different letters represent different digits.

What is the four-digit number represented by *ABCD*?

$$
\begin{array}{rccccc}
 & & & & 8 & A \\
 & & & 2 & B & 9 \\
 & & 1 & C & 6 & 7 \\
+ & 1 & D & 7 & 5 & 4 \\
\hline
 & 1 & 2 & 3 & 4 & 5
\end{array}
$$

Example Problem 2.2 - Orange

Let *P*, *Q* and *R* represent the missing digits in the subtraction shown.

Find the sum $P + Q + R$.

$$
\begin{array}{rccc}
 & 8 & 4 & P \\
- & Q & 6 & 8 \\
\hline
 & 4 & R & 3
\end{array}
$$

Maths Games Example Solution 2.2 - Yellow

In the addition shown, different letters represent different digits.

What is the four-digit number represented by *ABCD*?

$$\begin{array}{r} 8\,A \\ 2\,B\,9 \\ 1\,C\,6\,7 \\ +\;1\,D\,7\,5\,4 \\ \hline 1\,2\,3\,4\,5 \end{array}$$

Strategy 1: Work Backwards

Working from the ones column, $A + 9 + 7 + 4$ must be a number that ends in **5**.

$A + 9 + 7 + 4 = A + 20$.

Since $A + 20$ ends in **5**, and A is a one-digit number, $A + 20 = 25$ and $A = 5$.

We'll carry the **2** from **25** to the tens column.

$$\begin{array}{r} {}^{2} \\ 8\,A \\ 2\,B\,9 \\ 1\,C\,6\,7 \\ +\;1\,D\,7\,5\,4 \\ \hline 1\,2\,3\,4\,5 \end{array}$$

In the tens column, $2 + 8 + B + 6 + 5$ must be a number that ends in **4**.

$2 + 8 + B + 6 + 5 = B + 21$.

Since $B + 21$ ends in **4**, and B is a one-digit number, $B + 21 = 24$ and $B = 3$.

We'll carry the **2** from **24** to the hundreds column.

$$\begin{array}{r} {}^{2} \\ 8\,5 \\ 2\,B\,9 \\ 1\,C\,6\,7 \\ +\;1\,D\,7\,5\,4 \\ \hline 1\,2\,3\,4\,5 \end{array}$$

In the hundreds column, $2 + 2 + C + 7$ must be a number that ends in **3**.

$2 + 2 + C + 7 = C + 11$.

Since $C + 11$ ends in **3**, and C is a one-digit number, $C + 11 = 13$ and $C = 2$.

We'll carry the **1** from **13** to the thousands column.

$$\begin{array}{r} {}^{2} \\ {}^{2}\;8\,5 \\ 2\,3\,9 \\ 1\,C\,6\,7 \\ +\;1\,D\,7\,5\,4 \\ \hline 1\,2\,3\,4\,5 \end{array}$$

In the thousands column, $1 + 1 + D$ must be a number that ends in **2**.

$1 + 1 + D = D + 2$.

Since $D + 2$ ends in **2**, and D is a one-digit number, $D + 2 = 2$ and $D = 0$.

There's nothing to carry to the ten thousand column.

$$\begin{array}{r} {}^{2} \\ {}^{2}\;8\,5 \\ {}^{1}\;2\,3\,9 \\ 1\,2\,6\,7 \\ +\;1\,D\,7\,5\,4 \\ \hline 1\,2\,3\,4\,5 \end{array}$$

If $A = 5$, $B = 3$, $C = 2$ and $D = 0$, then the four-digit number *ABCD* is **5320**.

Strategy 2: Use a Split Strategy

The number $8A = 80 + A$,

$2B9 = 209 + B0$,

$1C67 = 1067 + C00$,

and $1D754 = 10754 + D000$.

We can therefore rewrite the sum as

$80 + A + 209 + B0 + 1067 + C00 + 10754 + D000 = 12345$

which rearranges to become

$\underline{80 + 209 + 1067 + 10754} + \underline{A + B0 + C00 + D000} = 12345$.

Having worked out that

$80 + 209 + 1067 + 10754 = 12110$

and $A + B0 + C00 + D000 = DCBA$,

we have

$DCBA + 12110 = 12345$

and therefore

$DCBA = 12345 - 12110$.

$$\begin{array}{r} {}^{2} \\ {}^{2}\;8\,0 \\ {}^{1}\;2\,0\,9 \\ 1\,0\,6\,7 \\ +\;1\,0\,7\,5\,4 \\ \hline 1\,2\,1\,1\,0 \end{array}$$

By our calculations, $DCBA = (0)235$.

This means that $A = 5$, $B = 3$, $C = 2$, $D = 0$,

and so *ABCD* must represent **5320**.

$$\begin{array}{r} 1\,2\,3\,4\,5 \\ -\;1\,2\,1\,1\,0 \\ \hline 2\,3\,5 \end{array}$$

Answers

2.2 - Green: 0

2.2 - Orange: 11

2.2 - Yellow: 5320

Maths Games – Example Problem 2.3

Example Problem 2.3 - Green

Rachel and Stuart both have the same number of pencils.

When Terry joined their group, he didn't have any pencils, so the students shared all of the pencils equally amongst the three of them.

If Rachel ended up with two fewer pencils than she had at the beginning, how many pencils do the students have in total?

Example Problem 2.3 - Yellow

Rachel, Stuart and Terry each have the same number of pencils.

When Will joined their group, he didn't have any pencils, so the students shared all of the pencils equally amongst the four of them.

If Rachel ended up with two fewer pencils than she had at the beginning, how many pencils do the students have in total?

Example Problem 2.3 - Orange

The average (mean) of four consecutive even integers is 17.

Find the largest of the four integers.

Maths Games Example Solution 2.3 - Yellow

Rachel, Stuart and Terry each have the same number of pencils. When Will joined their group, he didn't have any pencils, so the students shared all of the pencils equally amongst the four of them. If Rachel ended up with two fewer pencils than she had at the beginning, how many pencils do the students have in total?

Strategy 1: Draw a Diagram, and Work Backwards

Rachel, Stuart and Terry each started with the same number of pencils.	[Rachel at beginning][Stuart at beginning][Terry at beginning]
When Will joined the group, they shared the pencils equally amongst the four of them.	[Rachel at beginning][Stuart at beginning][Terry at beginning] [Rachel at end][Stuart at end][Terry at end][Will at end]
Rachel ended up with two fewer pencils than she had at the beginning.	[Rachel at beginning] [Rachel at end] ⊢ 2 ⊣
Both Stuart and Terry started and ended up with the same number of pencils as Rachel.	[Rachel at beginning][Stuart at beginning][Terry at beginning] [Rachel at end] ⊢ 2 ⊣ [Stuart at end] ⊢ 2 ⊣ [Terry at end] ⊢ 2 ⊣
In effect, Rachel, Stuart and Terry each gave **2** pencils to Will. Will must have ended up with 2 + 2 + 2 = **6** pencils.	[Rachel at beginning][Stuart at beginning][Terry at beginning] [Rachel at end][Stuart at end][Terry at end] ⊢ 2 ⊣⊢ 2 ⊣⊢ 2 ⊣ [Rachel at end][Stuart at end][Terry at end][Will at end]

If Will ended up with **6** pencils, then Rachel, Stuart and Terry also ended up with **6** pencils each.

In total, there must have been **6 + 6 + 6 + 6 = 24** pencils.

Strategy 2: Construct an Algebraic Equation (1)

Let Rachel, Stuart and Terry each start with x pencils.

In total, the students have $3x$ pencils.

When Will joined the group, they shared the $3x$ pencils equally amongst the four of them.

After doing so, each member of the group had $\frac{3x}{4}$ pencils.

Since Rachel ended up with **2** fewer pencils than she had in the beginning, we can set up the equation at the right.

Having determined that $x = 8$, there must have been $3x$ **= 24** pencils in total.

$$x - 2 = \frac{3x}{4}$$

Multiply both sides by 4:

$$4x - 8 = 3x$$

Subtract $3x$ from both sides:

$$x - 8 = 0$$

Add 8 to both sides:

$$x = 8$$

Strategy 3: Construct an Algebraic Equation (2)

Let there be x pencils in total.

Prior to Will joining the group, Rachel had $\frac{x}{3}$ pencils.

After Will joined, Rachel had $\frac{x}{4}$ pencils.

This allows us to set up the equation at the right.

There were **24** pencils in total.

$$\frac{x}{3} - \frac{x}{4} = 2$$

Multiply both sides by 12:

$$4x - 3x = 24$$

$$x = 24$$

Answers

2.3 - Green: 12

2.3 - Orange: 20

2.3 - Yellow: 24

Maths Games – Example Problem 2.4

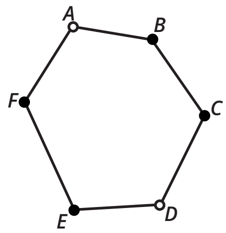
Example Problem 2.4 - Green

Teo's hexagon has six different side lengths.

Each vertex is marked with a dot.

He cuts the hexagon into two pieces using a single straight cut from one dot to another.

In how many different ways could Teo cut his hexagon, if he starts cutting from either vertex *A* or vertex *D*?



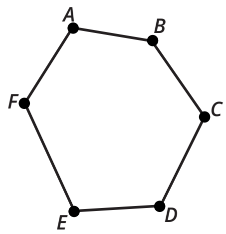
Example Problem 2.4 - Yellow

Teo's hexagon has six different side lengths.

Each vertex is marked with a black dot.

He cuts the hexagon into two pieces using a single straight cut from one dot to another.

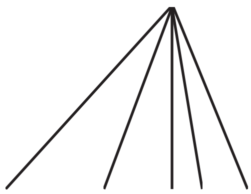
In how many different ways could Teo cut his hexagon?



Example Problem 2.4 - Orange

An acute angle is an angle that measures less than 90°.

Using the lines in the diagram, how many different acute angles can be formed?



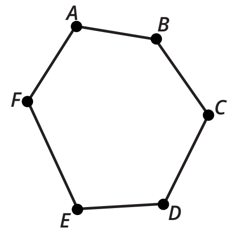
Maths Games Example Solution 2.4 - Yellow

Teo's hexagon has six different side lengths.

Each vertex is marked with a black dot.

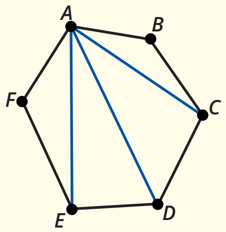
He cuts the hexagon into two pieces using a single straight cut from one dot to another.

In how many different ways could Teo cut his hexagon?

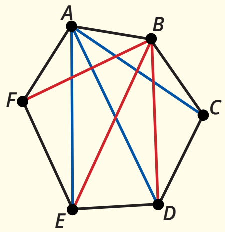


Strategy 1: Make an Organised List

If he starts from ***A***, Teo could cut his hexagon from ***A*** to ***C***, ***A*** to ***D***, or ***A*** to ***E***.



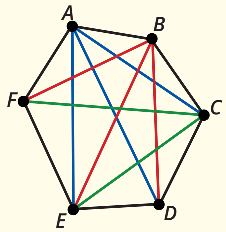
If he starts from ***B***, Teo could cut his hexagon from ***B*** to ***D***, ***B*** to ***E***, or ***B*** to ***F***.



If he starts from ***C***, Teo could cut his hexagon from ***C*** to ***E*** or ***C*** to ***F***.

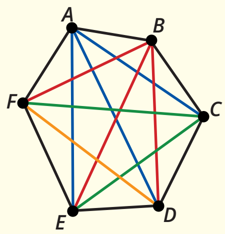
He could also cut from ***C*** to ***A***, but we've already counted that one - it's been counted as a cut from ***A*** to ***C***.

Since we've already included that cut on our list, we won't double-count it here.



If he starts from ***D***, Teo could cut his hexagon from ***D*** to ***F***.

He could also cut from ***D*** to ***A***, or from ***D*** to ***B***. However, as for the cut from ***C*** to ***A***, we've already counted those cuts - we just counted them in the opposite direction.



From the diagram, we can see that there are no further cuts.

Listing the cuts in an organised way, we can see that there are **9** possible ways for Teo to cut his hexagon.

AC	*AD*	*AE*	
	BD	*BE*	*BF*
		CE	*CF*
			DF

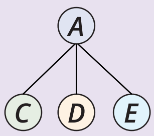
Strategy 2: Draw a Tree Diagram

Since a cut cannot be made from a vertex to itself, we cannot make a cut from ***A*** to ***A***, ***B*** to ***B***, etc.

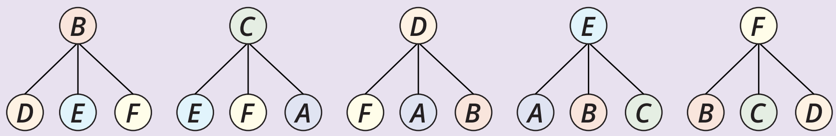
As we are cutting the hexagon into **2** pieces, a cut also cannot be made between **2** dots that are next to each other, such as from ***A*** to ***B*** or from ***A*** to ***F***.

C, ***D*** and ***E*** are the only points that are not adjacent to ***A***.

A cut can therefore be made from ***A*** to ***C***, ***A*** to ***D***, or ***A*** to ***E***.



Likewise, there are **3** possible cuts starting from any vertex on the hexagon.



This idea can also be represented in a table:

from \ to	A	B	C	D	E	F
A	*AA*	*AB*	*AC*	*AD*	*AE*	*AF*
B	*BA*	*BB*	*BC*	*BD*	*BE*	*BF*
C	*CA*	*CB*	*CC*	*CD*	*CE*	*CF*
D	*DA*	*DB*	*DC*	*DD*	*DE*	*DF*
E	*EA*	*EB*	*EC*	*ED*	*EE*	*EF*
F	*FA*	*FB*	*FC*	*FD*	*FE*	*FF*

We can see that there are **18** possible cuts.

However, each cut is listed twice. For example, cutting from ***A*** to ***C*** is the same as cutting from ***C*** to ***A***.

Therefore, the actual number of possible cuts is equal to **18 ÷ 2 = 9**.

Answers

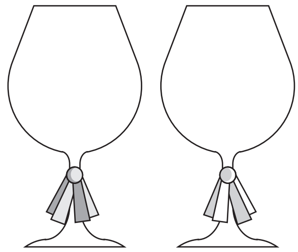
2.4 - Green: 5

2.4 - Orange: 10

2.4 - Yellow: 9

Maths Games – Example Problem 2.5

Example Problem 2.5 - Green



Miss Timms is having a party.

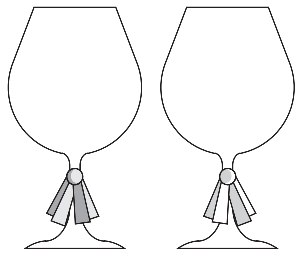
So that her guests don't accidentally pick up someone else's drink, she ties coloured ribbons in a single knot on the stem of each glass.

There are three different colours of ribbon.

Each glass has exactly two different colours.

How many different colour combinations can there be?

Example Problem 2.5 - Yellow



Miss Timms is having a party.

So that her guests don't accidentally pick up someone else's drink, she ties coloured ribbons in a single knot on the stem of each glass.

There are five different colours of ribbon.

Each glass has exactly two different colours.

How many different colour combinations can there be?

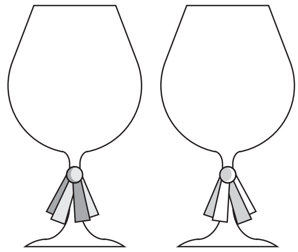
Example Problem 2.5 - Orange

Two numbers represented by A and B are chosen from the following set of numbers: { 1, 2, 3, 4, 5, 6 }.

In how many different ways will $\frac{A}{B}$ have a value less than $\frac{1}{2}$?

Consider $\frac{1}{3}$ and $\frac{2}{6}$ as two different ways.

Maths Games Example Solution 2.5 - Yellow



Miss Timms is having a party. So that her guests don't accidentally pick up someone else's drink, she ties coloured ribbons in a single knot on the stem of each glass.

There are five different colours of ribbon. Each glass has exactly two different colours.

How many different colour combinations can there be?

Strategy 1: Make an Organised List

Let's call the colours Blue (*B*), Red (*R*), Yellow (*Y*), Green (*G*) and Purple (*P*).

We will start by listing all of the combinations that include a Blue ribbon.

We'll take a blue ribbon, and pair it with each of the other colours.

B R *B Y* *B G* *B P*

Next, let's list all of the combinations that include a Red ribbon.

We already have a Blue-Red, so we don't need a Red-Blue, since it would be the same thing.

R Y *R G* *R P*

Next, let's list all of the combinations that include a Yellow ribbon.

Note that we don't need Yellow-Blue or Yellow-Red, as we have already listed Blue-Yellow and Red-Yellow.

Y G *Y P*

Next, let's list all of the combinations that include a Green ribbon.

Note that we don't need Green-Blue, Green-Red, or Green-Yellow.

G P

We've already listed Purple with all of the other four colours, so that's all of the combinations.

Our diagram shows that there are **10** different colour combinations.

Strategy 2: Build a Table

Let's build a table for the colour combinations.

We'll have the first colour on the horizontal axis, and the second colour on the vertical axis.

1st colour

2nd colour

	B	*R*	*Y*	*G*	*P*
B					
R					
Y					
G					
P					

There are always exactly two different colours, so we can't have Blue-Blue or Red-Red, etc.

	B	*R*	*Y*	*G*	*P*
B	X				
R		X			
Y			X		
G				X	
P					X

All of the other pairs are possible.

By counting the possible pairs, we can see that there are **20** pairs.

	B	*R*	*Y*	*G*	*P*
B	X	✓	✓	✓	✓
R	✓	X	✓	✓	✓
Y	✓	✓	X	✓	✓
G	✓	✓	✓	X	✓
P	✓	✓	✓	✓	X

We've counted **20** pairs, but each pair has been counted twice.

For example, we've got one pair that is Blue-Red, and another pair that is Red-Blue.

Since we have double-counted each pair, there are **20** ÷ **2** = **10** different colour combinations.

Answers

2.5 - Green: 3

2.5 - Yellow: 10

2.5 - Orange: 6
$(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{6}, \frac{2}{5})$

Maths Games – Example Problem 2.6

Example Problem 2.6 - Green

I have three cards on which are written the numbers 1, 2, and 3.
I can use the cards to make the number 123.
I can't use them to make 122, because I only have one card with a 2 on it.
Using these three cards, what is the second-largest number I can make?

1	2	3

Example Problem 2.6 - Yellow

I have four cards on which are written the numbers 1, 2, 3 and 4.
I can use the cards to make the number 1234.
I can't use them to make 1224, because I only have one card with a 2 on it.
Using these four cards, what is the second-largest number I can make?

1	2	3	4

Example Problem 2.6 - Orange

I have four cards on which are written the numbers 1, 2, 3 and 3.
I can use the cards to make the number 1233.
I can't use them to make 1223, because I only have one card with a 2 on it.
Using these four cards, what is the third-largest number I can make?

1	2	3	3

Maths Games Example Solution 2.6 - Yellow

I have four cards on which are written the numbers 1, 2, 3 and 4.

I can use the cards to make the number 1234.

I can't use them to make 1224, because I only have one card with a 2 on it.

Using these four cards, what is the second-largest number I can make?

1 2 3 4

Strategy 1: Make an Organised List (1)

Let's try listing the biggest numbers we can make, from smallest to largest.

There are 4 possible digits for the **thousands** place. Since we want the second-largest number, let's go for a big thousands digit. We will only consider 3 or 4 in the thousands place.	3	There are then 3 possible digits for the **hundreds** place.	31	There are then 2 possible digits for the **tens** place.	312	There is only 1 possible digit remaining for the **units** place.	3124
					314		3142
			32		321		3214
					324		3241
			34		341		3412
					342		3421
	4		41		412		4123
					413		4132
			42		421		4213
					423		4231
			43		431		4312
					432		4321

We've listed all of the numbers that use the digits 1, 2, 3 and 4, and are greater than 3000.

The second-largest number is **4312**.

Strategy 2: Make an Organised List (2)

Since thousands are bigger than hundreds, etc, the largest number we can make is 4321.

Let's count down and see how the numbers change.

4321, 4320	⟶	If we just change the units digit, the digits will no longer be 1, 2, 3 and 4. We must change more than one digit.
4319, 4318, ...	⟶	We have changed the tens digit to 1. If we keep counting down, we will get: 4317, 4316, 4315, 4314, 4313, 4312 The only digit out of 1, 2, 3, 4 that is not in use is the 2, so this number works.

The first number we reach that uses the digits 1, 2, 3, 4, when counting down from 4321, is 4312.

The second-largest number possible is **4312**.

Answers

2.6 - Green: 312

2.6 - Orange: 3231

2.6 - Yellow: 4312

Maths Games – Example Problem 2.7

Example Problem 2.7 - Green

Angela, Ben and Caroline need to sit in a row on the stage to receive awards.

Angela and Caroline have to sit together.

In how many different ways can they sit?

Example Problem 2.7 - Yellow

Angela, Ben, Caroline and David need to sit in a row on the stage to receive awards.

Angela and Caroline have to sit together.

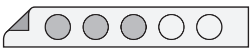
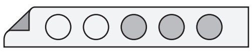
Ben and David have to sit together.

In how many different ways can they sit?

Example Problem 2.7 - Orange

Five identical circles are arranged in a straight line on a strip of paper.

In how many different ways can exactly 3 of these circles be coloured grey?

Note: Consider the arrangements  and  as the same, because the paper can be turned around.

Maths Games Example Solution 2.7 - Yellow

Angela, Ben, Caroline and David need to sit in a row on the stage to receive awards.

Angela and Caroline have to sit together.

Ben and David have to sit together.

In how many different ways can they sit?

Strategy 1: Make an Organised List (1)

We can place the students from left to right.

To choose students for each seat, we'll go alphabetically, remembering that:

- Angela and Caroline have to sit together, and
- Ben and David have to sit together.

If Angela is in the first seat, Caroline must be in the second seat. Going alphabetically, Ben is in the next seat, and then David. After that, the only possibility would be David, followed by Ben.	A C B D A C D B
If Ben is in the first seat, then David must be in the second seat. After that, we'd have Angela then Caroline, or Caroline then Angela.	B D A C B D C A
If Caroline is in the first seat, then Angela must be in the second seat. After that, we'd have Ben then David, or David then Ben.	C A B D C A D B
If David is in the first seat, then Ben must be in the second seat. After that, we'd have Angela then Caroline, or Caroline then Angela.	D B A C D B C A

That's all of the possible combinations.

The students can sit in **8** different ways.

Strategy 2: Make an Organised List (2)

Angela and Caroline need to sit together. Let's call them the Red Group.

Ben and David can be the Blue Group.

There are only two possible ways to arrange the groups:

- Red then Blue, or
- Blue then Red.

Red	Blue
Blue	Red

Within the Red Group, in how many ways can we arrange Angela and Caroline?

There are only two possibilities: Angela then Caroline, or Caroline then Angela.

We now have double the number of arrangements.

A C	Blue
Blue	A C
C A	Blue
Blue	C A

Within the Blue Group, in how many ways can we arrange Ben and David?

There are only two possibilities: Ben then David, or David then Ben.

We need to double the number of arrangements once again.

A C B D	A C D B
B D A C	D B A C
C A B D	C A D B
B D C A	D B C A

That's all of the possible combinations.

The students can sit in **8** different ways.

Answers

2.7 - Green: 4

2.7 - Orange: 6

2.7 - Yellow: 8

Maths Games – Example Problem 2.8

Example Problem 2.8 - Green

I bought a large box of oranges at the farmers' market.

I gave half of the oranges to my brother, and then I gave him one more.

I gave half of the remaining oranges to my sister, and then I gave her one more.

I had just one orange left.

How many oranges were in the box I bought from the farmers' market?

Example Problem 2.8 - Yellow

I bought a large box of oranges at the farmers' market.

I gave half of the oranges to my brother, and then I gave him one more.

I gave half of the remaining oranges to my sister, and then I gave her two more.

I gave half of the remaining oranges to my neighbour, and then I gave him three more.

I had just two oranges left.

How many oranges were in the box I bought from the farmers' market?

Example Problem 2.8 - Orange

There is a plate of crackers on the kitchen table.

Sara takes half of the crackers, plus 4 more.

Then Nick takes 2.

Joe takes half of what is left and then takes two more.

Finally Selena takes 5.

Four crackers remain on the plate.

How many crackers were on the plate to begin with?

Maths Games Example Solution 2.8 - Yellow

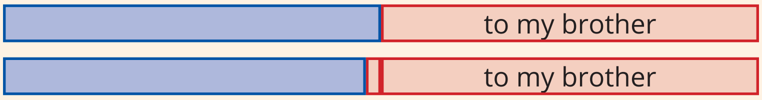
I bought a large box of oranges at the farmers' market. I gave half of the oranges to my brother, and then I gave him one more. I gave half of the remaining oranges to my sister, and then I gave her two more. I gave half of the remaining oranges to my neighbour, and then I gave him three more.

I had just two oranges left. How many oranges were in the box I bought from the farmers' market?

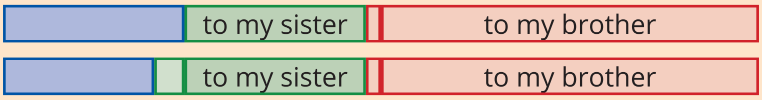
Strategy: Draw a Diagram, and Work Backwards

Let's use a bar to represent the number of oranges I bought:

I gave half of them to my brother, and then I gave him **1** more.

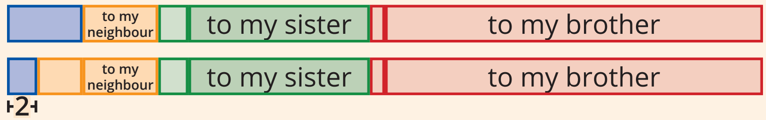


I gave half of the remaining oranges to my sister, and then I gave her **2** more.



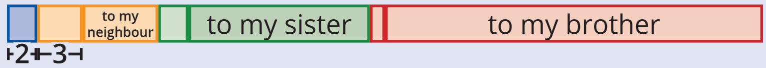
I gave half of the remaining oranges to my neighbour, and then I gave him **3** more.

I had just **2** oranges left.

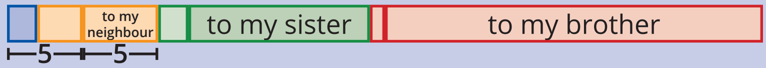


Let's work backwards to see how many oranges I had at the beginning.

Before I had just **2** oranges left, I gave **3** oranges to my neighbour.



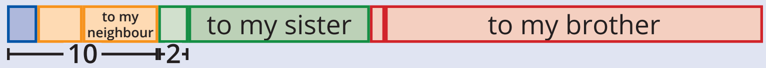
Before that, I gave him half of the oranges I had.



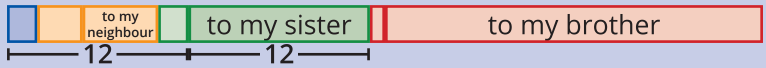
If I gave him half of what I had, then I must have been left with the other half.

Since I was left with **3 + 2 = 5** oranges, I must have given him **5** oranges.

I had **5 + 5 = 10** oranges before giving **2** oranges to my sister.

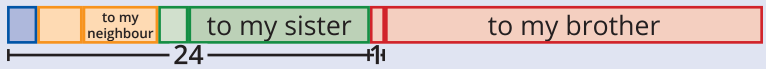


Before that, I gave my sister half of the oranges I had.

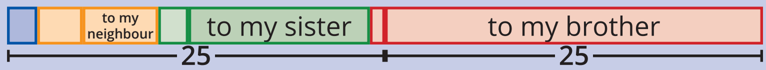


Since I was left with **10 + 2 = 12** oranges, I must have given her **12** oranges.

I had **12 + 12 = 24** oranges before giving **1** orange to my brother.

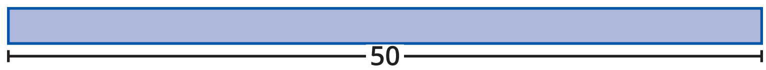


Before that, I gave my brother half of the oranges I had.



Since I was left with **24 + 1 = 25** oranges, I must have given him **25** oranges.

I must have bought **25 + 25 = 50** oranges at the farmers' market.



Answers

2.8 - Green: 10

2.8 - Orange: 56

2.8 - Yellow: 50

Answers

Set Green

2.1	400
2.2	0
2.3	12
2.4	5
2.5	3
2.6	312
2.7	4
2.8	10

Set Yellow

2.1	300
2.2	5320
2.3	24
2.4	9
2.5	10
2.6	4312
2.7	8
2.8	50

Set Orange

2.1	$50
2.2	11
2.3	20
2.4	10
2.5	6
2.6	3231
2.7	6
2.8	56