Important

The information contained in this file is ONLY for the use of registered participants of the 2023 APSMO Maths Games.

Organisation and Procedures

For full details, see the Members' Area

- Maths Games papers are to be conducted under test conditions.

DO	DO NOT
• Supervise students at all times. • Maintain silence. • Provide blank working paper. • Collect, mark and retain the papers.	• Print the papers prior to the scheduled date. • Read the questions aloud to the students. • Interpret the questions for students. • Permit any discussion or movement around the room. • Permit the use of calculators or other electronic devices.

- Papers should be scored by the PICO using the *Solutions and Answers* sheet provided.
- Original student answer sheets should be retained by the PICO until the end of the year.

Absent Students

- A student who is legitimately absent on the date of the Maths Games paper, may sit the paper on their return to school.
- If an absent student does not sit the paper on their return to school they should be marked as 'absent'.
- *Note: This policy differs from the Maths Olympiads Absent Student Policy which has additional requirements*.

APSMO

WEDNESDAY 26 JULY 2023

MATHS GAMES

MATHS GAMES SENIOR 3

Suggested Time: ***30 Minutes***

Write your answers in the boxes on the back.

←

Keep your answers hidden by folding backwards on this line.

3A. Alex, Paul and David were waiting in the school office.

One was a driver delivering a package, one needed to leave school early, and one was in trouble.

Alex helped the delivery driver carry the package to the front desk.

Paul had just attended the same class as the student who was in trouble.

Who was the delivery driver?

Hint: Could Paul be the student who was in trouble?

3B. Six cards are chosen from 50 cards, numbered from 1 to 50.

- The numbers on the six cards add to 50.
- One of the cards has 10 written on it.
- The largest two numbers on the cards differ by 4.

What is the largest possible number on one of the six cards?

Hint: If the sum of the values is 50, how can you maximise the largest two numbers?

3C. How many of the whole numbers from 1 to 1000 inclusive contain the digit 7?

Hint: How many of the whole numbers from 1 to 10 contain the digit 7? What about the whole numbers from 1 to 100?

3D. Alvin's Bakery puts jam in two-fifths of their doughnuts.

Brenda's Bakery puts jam in three-sevenths of their doughnuts.

Colm's Bakery puts jam in three-eighths of their doughnuts.

Dora's Bakery puts jam in four-ninths of their doughnuts.

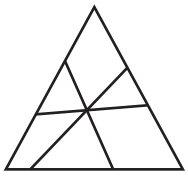
Each bakery produces the same number of doughnuts every week.

Which one of them makes the greatest number of jam doughnuts?

Hint: How might you solve this if there were only two bakeries?

3E. How many quadrilaterals can be traced on the lines in this diagram?

Note that the three lines within the triangle intersect at a single point.



Hint: How many lines are there in the diagram? How many lines are required to form a quadrilateral?

APSMO

WEDNESDAY 26 JULY 2023

MATHS GAMES

MATHS GAMES
SENIOR
3

3A.

3B.

3C.

3D.

3E.

Student Name:

Fold here. Keep your answers hidden.

APSMO

WEDNESDAY 26 JULY 2023

MATHS GAMES

MATHS GAMES SENIOR 3

Solutions and Answers

(Items in parentheses are not required)

3A: David	**3B:** 19	**3C:** 271	**3D:** Dora's Bakery	**3E:** 12

3A. The question is, Who was delivering a package?

Strategy: **Eliminate All But One Possibility**

1. Alex, Paul and David were either delivering a package, leaving school early, or in trouble.

Alex is...	Delivering a package?	Leaving school early?	In trouble?
Paul is...	Delivering a package?	Leaving school early?	In trouble?
David is...	Delivering a package?	Leaving school early?	In trouble?

2. Alex helped the delivery driver carry the package, so Alex cannot be the delivery driver.

Alex is...	~~Delivering a package?~~	Leaving school early?	In trouble?
Paul is...	Delivering a package?	Leaving school early?	In trouble?
David is...	Delivering a package?	Leaving school early?	In trouble?

3. Paul had been attending class with the student who was in trouble, so Paul can't be the delivery driver, and he isn't in trouble.

Alex is...	~~Delivering a package?~~	Leaving school early?	In trouble?
Paul is...	~~Delivering a package?~~	Leaving school early?	~~In trouble?~~
David is...	Delivering a package?	Leaving school early?	In trouble?

4. Paul only has one remaining option, he must be leaving school early. That means that neither Alex nor David are leaving school early.

Alex is...	~~Delivering a package?~~	~~Leaving school early?~~	In trouble?
Paul is...	~~Delivering a package?~~	Leaving school early?	~~In trouble?~~
David is...	Delivering a package?	~~Leaving school early?~~	In trouble?

5. Alex has only one option left - he is in trouble.

Alex is...	~~Delivering a package?~~	~~Leaving school early?~~	In trouble?
Paul is...	~~Delivering a package?~~	Leaving school early?	~~In trouble?~~
David is...	Delivering a package?	~~Leaving school early?~~	~~In trouble?~~

6. Therefore, **David** must be delivering a package.

Alex is...	~~Delivering a package?~~	~~Leaving school early?~~	In trouble?
Paul is...	~~Delivering a package?~~	Leaving school early?	~~In trouble?~~
David is...	Delivering a package?	~~Leaving school early?~~	~~In trouble?~~

A table can also be used to complete the question. For example:

	Delivering a package	Leaving school early	In trouble
Alex	x	x	✓
Paul	x	✓	x
David	✓	x	x

	Alex	Paul	David
Delivering a package	x	x	✓
Leaving school early	x	✓	x
In trouble	✓	x	x

Follow-Up: *Michael, who works in the school office, watched Paul get picked up before he left for his lunch break. David left between Michael and Alex. Paul was the second to leave. Who was the first to leave?* [Alex]

3B. The question is, What is the largest possible number on one of the six cards?

***Strategy:* Eliminate All But One Possibility**

Let's try to narrow down the range of values that the largest number could be.

10 is one of the cards that was chosen.

Since all of the numbers added together make **50**, and one of the numbers is 10, the sum of the other five numbers must be **50 – 10 = 40**.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

To maximise the two largest numbers, let's assume that the three other cards have the numbers 1, 2 and 3.

The sum of the two largest numbers would then be no more than **40 – 1 – 2 – 3 = 34**.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Method 1: Build a Table, and Guess, Check and Refine

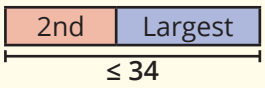
If we guess the largest number for a chosen card, we can check the guess against the parameters of the question, and use that information to make a better guess.

The largest possible number for a chosen card is **19**.

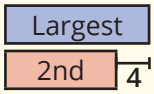
Largest number	Second-largest number	Sum of two largest numbers
25	25 – 4 = 21	25 + 21 = 46
20	20 – 4 = 16	20 + 16 = 36
19	19 – 4 = 15	19 + 15 = 34

Method 2: Reason Logically

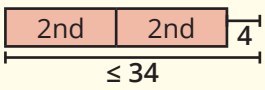
The sum of the two largest numbers is at most **34**.



The largest number is **4** more than the second-largest number.



Then, the greatest possible value for the second-largest number must be **½(34 – 4) = 15**.



The largest possible number on a chosen card is **15 + 4 = 19**.

Method 3: Reason Algebraically

If x is the largest value, the second-largest value would be $(x - 4)$.

$$x + (x - 4) \le 34$$
$$2x - 4 \le 34$$
$$2x \le 38$$
$$x \le 19$$

The largest possible value for x is **19**.

Follow-Up: *Suppose the question had specified five cards, not six. What would be the greatest possible value for a chosen card?* [20]

3C. The question is, How many of the whole numbers from 1 to 1000 inclusive contain the digit 7?

Strategy 1: **Solve a Simpler Related Problem**

We could start by working out how many numbers have a 7 in the hundreds place.

These are the numbers **700** to **799** inclusive.

7 □ □ There are 100 numbers with 7 in the hundreds place.

There are, likewise:

□ 7 □ 100 numbers that have 7 in the tens place, and

□ □ 7 100 numbers that have 7 in the ones place.

With 100 + 100 + 100 = 300 7s in the numbers from **1** to **999**, there are 300 7s in the numbers from **1** to **1000**.

While this list contains 300 7s, there are numbers that contain more than one 7.

We now need to remove numbers that we've counted more than once.

We begin by considering numbers that contain <u>exactly two</u> 7s. There are:

7 7 □ 9 numbers that have 7**s** in the hundreds place and the tens place (but not the ones place),

7 □ 7 9 numbers that have 7**s** in the hundreds place and the ones place (but not the tens place),

□ 7 7 9 numbers that have 7**s** in the tens place and the ones place (but not the hundreds place).

Each of these 9 + 9 + 9 = 27 numbers has been double-counted.

7 7 7 There is also 1 number that has been triple-counted.

We've double-counted 27 numbers, so we'll remove the duplicates: 300 – 27 = 273.

We've triple-counted 1 number, so we'll remove two of those instances: 273 – 2 = 271.

There are **271** numbers that include the digit 7.

Strategy 2: **Make an Organised List**

There are 10 numbers that have 7 in the ones place.	7, 17, 27, 37, 47, 57, 67, 77, 87, 97
There are 10 numbers that have 7 in the tens place.	70, 71, 72, 73, 74, 75, 76, 77, 78, 79

With 77 included in both lists, there are 10 + 10 – 1 = 19 whole numbers containing the digit 7.

Every hundred up to **1000**, excluding the 7**00**s, must have 19 numbers that contain the digit 7.

Range	1-99	100s	200s	300s	400s	500s	600s	700s	800s	900s
Numbers containing 7	19	19	19	19	19	19	19	?	19	19

Every number in the **700**s (**700** - **799**) contains the digit **7**. There are 100 numbers in that range.

In total, there are (9 × 19) + 100 = 171 + 100 = **271** numbers from 1 to 1000, that contain the digit 7.

Follow-Up: *How many whole numbers from 1 to 1000 contain the digit 1?* [272]

3D. The question is, Which bakery makes the greatest number of jam doughnuts?

***Strategy:* Solve a Simpler Related Problem**

Let's assume that every bakery makes doughnuts in batches.

Alvin's Bakery makes batches of **5** doughnuts, and puts jam in **2** of them, leaving **3** of them plain.

Brenda's Bakery makes batches of **7** doughnuts, and then puts jam in **3** of them, leaving **4** of them plain; and so on.

	Alvin's	Brenda's	Colm's	Dora's
	2	3	3	4
	3	4	5	5
Total	5	7	8	9

Method 1: Compare two bakeries at a time.

With either **3** or **4** plain doughnuts per batch, we can buy batches from **Alvin's Bakery** and **Brenda's Bakery** that each include **3 × 4 = 12** plain doughnuts.

Brenda's Bakery makes more jam doughnuts than **Alvin's Bakery**.

	Alvin's	Brenda's
	4 × 2 = 8	3 × 3 = 9
	4 × 3 = 12	3 × 4 = 12

With either **3** jam doughnuts per batch, we can compare **Brenda's Bakery** and **Colm's Bakery** directly.

Colm's Bakery makes more plain doughnuts than **Brenda's Bakery**, so **Brenda's Bakery** makes more jam doughnuts than **Colm's**.

	Brenda's	Colm's
	3	3
	4	5

With either **4** or **5** plain doughnuts per batch, we can buy batches from **Brenda's Bakery** and **Dora's Bakery** that each include **4 × 5 = 20** plain doughnuts.

Dora's Bakery makes more jam doughnuts than **Brenda's Bakery**.

	Brenda's	Dora's
	5 × 3 = 15	4 × 4 = 16
	5 × 4 = 20	4 × 5 = 20

Comparisons between two bakeries can likewise be performed by buying the same total number of doughnuts from both bakeries.

Dora's Bakery makes the greatest number of jam doughnuts.

Method 2: All bakeries make the same number of jam doughnuts.

If all bakers make the same number of jam doughnuts, the quantity must be a multiple of **2**, **3**, **3** and **4**. The LCM of **(2, 3, 3, 4)** is **12**.

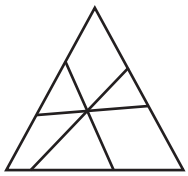
With **12** jam doughnuts each, it is clear that **Dora's Bakery** makes the smallest number of plain doughnuts, and therefore the greatest number of jam doughnuts.

	Alvin's	Brenda's	Colm's	Dora's
	6 × 2 = 12	4 × 3 = 12	4 × 3 = 12	3 × 4 = 12
	6 × 3 = 18	4 × 4 = 16	4 × 5 = 20	3 × 5 = 15

Dora's Bakery makes the greatest number of jam doughnuts.

Follow-Up: *Eva's Bakery makes batches of 10 doughnuts. They make a higher proportion of jam doughnuts than Brenda's Bakery, and a lower proportion of jam doughnuts than Colm's Bakery. How many jam doughnuts are in each batch?* [4]

3E. The question is, How many quadrilaterals can be traced on the lines in this diagram?



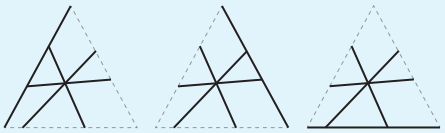
Strategy 1: **Make an Organised List, and Solve a Simpler Related Problem**

There are six lines in this diagram.

Since there are four lines in a quadrilateral, we could remove two lines at a time, and then see if the remaining lines form a quadrilateral.

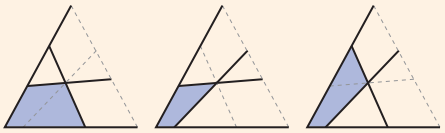
Suppose we remove two of the lines from the large triangle. Of the four lines that remain, three of them intersect at a point.

Since three of the lines meet at a single point, it isn't possible to use all three of those lines in the construction of a quadrilateral.



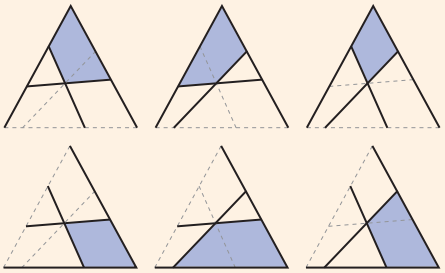
Suppose we remove just one of the lines from the large triangle.

If we remove one of the lines through the centre, the remaining four lines will define a quadrilateral.



This idea works irrespective of which of the lines is removed from the large triangle.

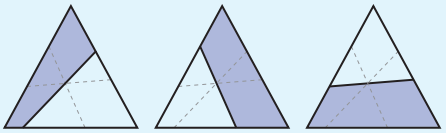
With three quadrilaterals defined every time we remove one line from the large triangle, there are **3 × 3 = 9** quadrilaterals that can be formed in this way.



Suppose we remove two of the lines from the centre.

The four lines that remain form a quadrilateral.

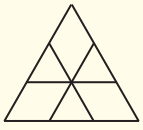
There are three quadrilaterals that can be formed in this way.



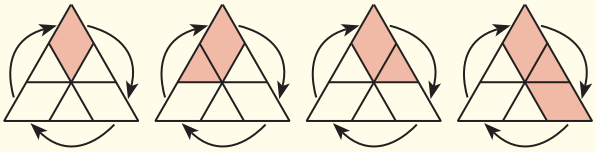
9 + 3 = 12 different quadrilaterals can be traced on the lines in the diagram.

Strategy 2: **Find a Pattern, and Solve a Simpler Related Problem**

We can redraw the diagram to have rotational symmetry.



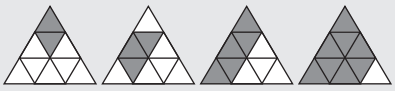
Each quadrilateral we find in this diagram also exists in two other orientations.



There are **3 × 4 = 12** different quadrilaterals that can be traced on the lines in the diagram.

Follow-Up: *How many quadrilaterals are there in this diagram?*

Rotations only (3 × 4 = 12)



Flips and Rotations (6 × 3 = 18)

[There are 12 + 18 = 30 quadrilaterals.]