APSMO

2023 MATHS GAMES

IMPORTANT

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Organisation and Procedures

For full details, see the Members' Area

- Maths Games papers are to be conducted under test conditions.

DO	DO NOT
• Supervise students at all times. • Maintain silence. • Provide blank working paper. • Collect, mark and retain the papers.	• Print the papers prior to the scheduled date. • Read the questions aloud to the students. • Interpret the questions for students. • Permit any discussion or movement around the room. • Permit the use of calculators or other electronic devices.

- Papers should be scored by the PICO using the *Solutions and Answers* sheet provided.
- Original student answer sheets should be retained by the PICO until the end of the year.

Absent Students

- A student who is legitimately absent on the date of the Maths Games paper, may sit the paper on their return to school.
- If an absent student does not sit the paper on their return to school they should be marked as 'absent'.
- *Note: This policy differs from the Maths Olympiads Absent Student Policy which has additional requirements*.

APSMO

MATHS GAMES

WEDNESDAY 14 JUNE 2023

MATHS GAMES
SENIOR
2

Suggested Time: ***30 Minutes***

Write your answers in the boxes on the back.

←

Keep your answers hidden by folding backwards on this line.

2A. Five people sit around a table in a noisy restaurant one evening.

Each person is only able to talk to the people sitting immediately beside them, on either side.

When they leave, each person shakes hands once with the people they had not yet spoken to that evening.

How many handshakes took place?

Hint: You could draw a diagram showing where each person sits at the table.

2B. Linus, Jake and Kelly each have some apples.

If Linus and Jake share their apples equally, they will have 5 apples each.

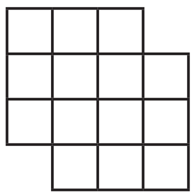
If Jake and Kelly share their apples equally, they will have 8 apples each.

If Linus and Kelly share their apples equally, they will have 6 apples each.

How many apples does Kelly have?

Hint: How many apples do Linus and Jake have in total, if they combined their apples together?

2C. How many squares, of any size, can be traced on the lines in this diagram?



Hint: How many 1×1 squares (☐) can be traced on these lines?
How many 2×2 squares (⊞) can be traced on these lines?
What other sizes might there be?

2D. In the subtraction shown, letters are used to represent digits.

$$\begin{array}{cccc} & A & 1 & 2 & B \\ - & 3 & C & D & 4 \\ \hline & 5 & 6 & 7 & 8 \end{array}$$

What is the four-digit number represented by ***ABCD***?

Hint: Different letters do NOT necessarily represent different digits.

2E. Tom went shopping.

He spent one-third of his money on swimmers.

He spent one-quarter of his remaining money on a pair of goggles.

He spent one-fifth of what remained on a hat.

If he had $60 remaining, how much did he have to start with, in dollars?

Hint: How much did Tom have before buying the hat?

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Student Name:

Fold here. Keep your answers hidden.

2A.

2B.

2C.

2D.

2E.

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Solutions and Answers

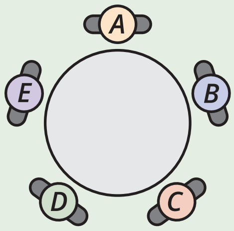
(Items in parentheses are not required)

2A: 5	2B: 9	2C: 23	2D: 9244	2E: ($)150

2A. The question is, How many handshakes took place?

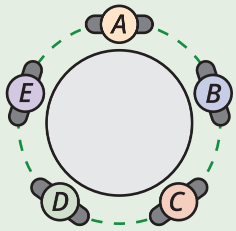
Strategy 1: **Draw a Diagram, and Make an Organised List**

We can draw the seating arrangement of the five people to get a better idea of the situation.

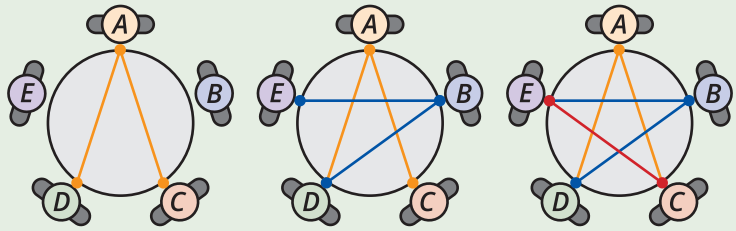


Each person is only able to talk to the people sitting next to them on either side.

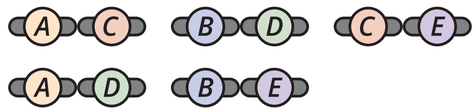
At the end of the evening, they don't shake hands with the people sitting immediately next to them.



They do shake hands with everyone else.



The diagram shows the following handshakes:



We can see that **5** different handshakes took place.

Strategy 2: **Build a Table**

Since each handshake occurs between two people, it may make sense to represent them in a two-way table.

	A	B	C	D	E
A					
B					
C					
D					
E					

People do not shake hands with either themselves, or their immediate neighbours.

	A	B	C	D	E
A	X	X			X
B	X	X	X		
C		X	X	X	
D			X	X	X
E	X			X	X

They do shake hands with everyone else.

From the table, we can see **10** possible handshakes.

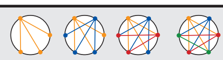
	A	B	C	D	E
A	X	X	✓	✓	X
B	X	X	X	✓	✓
C	✓	X	X	X	✓
D	✓	✓	X	X	X
E	X	✓	✓	X	X

However, each pair of handshake participants is listed twice in the table.

For example, A shaking hands with C is the same as C shaking hands with A.

Since each handshake is being listed twice, we can see that **10 ÷ 2 = 5** handshakes actually took place.

Follow-Up: *How many handshakes would there have been if there were 6 people around the table?* [9]



2B. The question is, How many apples does Kelly have?

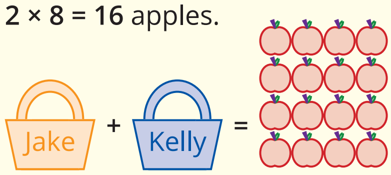
Strategy: **Work Backwards**

If Linus and Jake share their apples equally, they will each have **5** apples.

Together, they must have **2 × 5 = 10** apples.

If Jake and Kelly share their apples equally, they will each have **8** apples.

Together, they must have **2 × 8 = 16** apples.



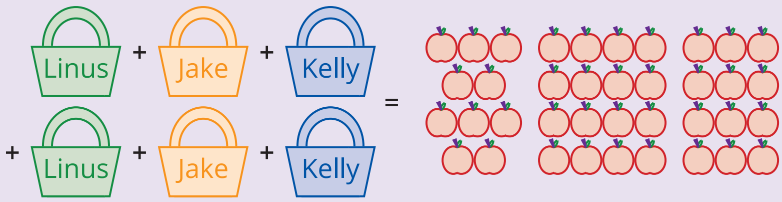
If Linus and Kelly share their apples equally, they will each have **6** apples.

Together, they must have **2 × 6 = 12** apples.

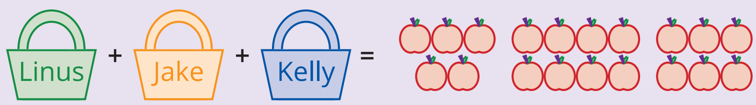
Method 1: Add all of the combinations together

If we add all of the different combinations together, there will be **10** + **16** + **12** = **38** apples.

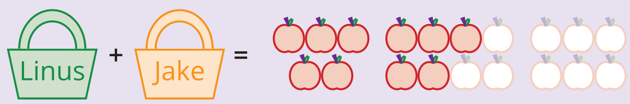
Each person's apples will have been counted twice.



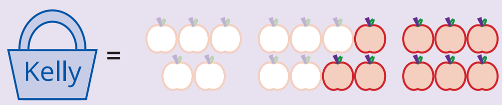
All together, the three people will have **38** ÷ **2** = **19** apples.



We saw earlier that Linus and Jake have **10** apples all together.



This means that Kelly must have **19** – **10** = **9** apples.



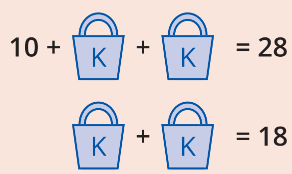
Method 2: Add together combinations that include Kelly's apples.

Jake and Kelly have **16** apples.

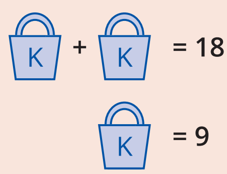
Linus and Kelly have **12** apples.

In total,

Since Linus and Jake have **10** apples all together,



We can see that Kelly must have **18** ÷ **2** = **9** apples.



Follow-Up: *Kelly gives one apple to Linus. How many more apples does she now have than Linus?* [4: Before Kelly gives an apple to Linus, he has 12 – 9 = 3 apples. After she gives him one apple, Linus will have 4, and Kelly will have 8.]

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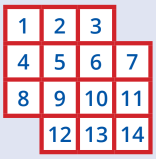
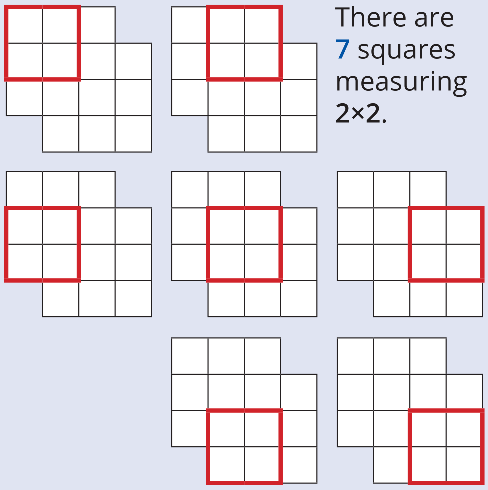
2C. The question is, How many squares of any size can be traced on the lines in the diagram?

Strategy 1: **Make an Organised List (1)**

We can count squares in order of size.

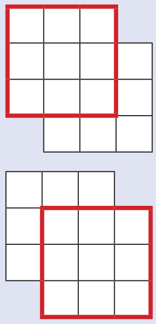


There are 14 1×1 squares.



There are 7 squares measuring 2×2.

There are 2 squares measuring 3×3.



We can trace 14 + 7 + 2 = **23** different squares on the lines in the diagram.

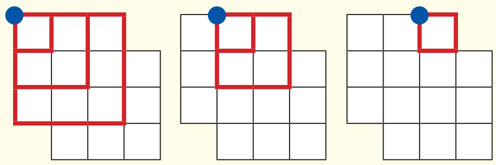
Strategy 2: **Make an Organised List (2)**

We can count each square according to its position in the diagram.

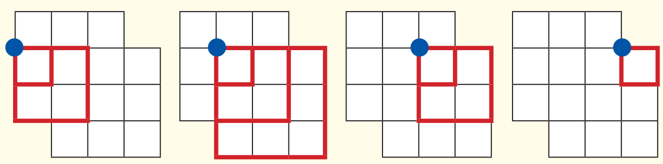
One way we might do this is by noting the position of each square's top left corner.

From the diagrams at the right, we can see that there are 6 + 8 + 6 + 3 = **23** squares.

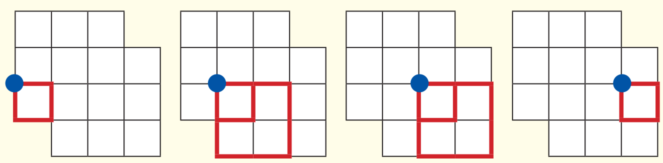
3 + 2 + 1 = 6 squares



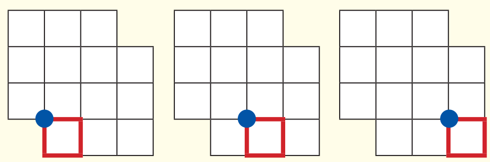
2 + 3 + 2 + 1 = 8 squares



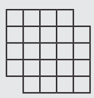
1 + 2 + 2 + 1 = 6 squares

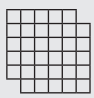


1 + 1 + 1 = 3 squares



Follow-Up: *How many squares can be drawn on the lines in these diagrams:*

 [46]

 [80]

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2D. The question is, What is the four-digit number represented by *ABCD*?

$$\begin{array}{r} A\ 1\ 2\ B \\ -\ 3\ C\ D\ 4 \\ \hline 5\ 6\ 7\ 8 \end{array}$$

Strategy 1: **Work Backwards (1)**

Working from the ones column, B – 4 must be a number that ends in 8.

If $B - 4 = 8$

then $B = 8 + 4$

$= 12$.

Since B is a one-digit number, B must equal 2, with trading from the tens column.

$$\begin{array}{r} A\ 1\ 2\ B \\ -\ 3\ C\ D\ 4 \\ \hline 5\ 6\ 7\ 8 \end{array} \qquad \begin{array}{r} A\ 1\ \overset{1}{\cancel{2}}\ \overset{1}{B} \\ -\ 3\ C\ D\ 4 \\ \hline 5\ 6\ 7\ 8 \end{array}$$

In the tens column, 1 – D must be a number that ends in 7.

While $1 - D = 7$ does not make sense for a subtraction algorithm, we can have $11 - D = 7$.

Then, $D = 11 - 7$, with trading from the hundreds column.

$D = 4$.

$$\begin{array}{r} A\ 1\ \overset{1}{\cancel{2}}\ \overset{1}{2} \\ -\ 3\ C\ D\ 4 \\ \hline 5\ 6\ 7\ 8 \end{array} \qquad \begin{array}{r} A\ \overset{0}{\cancel{1}}\ \overset{11}{\cancel{2}}\ \overset{1}{2} \\ -\ 3\ C\ D\ 4 \\ \hline 5\ 6\ 7\ 8 \end{array}$$

In the hundreds column, 0 – C must end in 6.

While $0 - C = 6$ does not make sense for a subtraction algorithm, we can have $10 - C = 6$, with trading from the thousands column.

Then, $C = 10 - 6$

$= 4$.

$$\begin{array}{r} A\ \overset{0}{\cancel{1}}\ \overset{11}{\cancel{2}}\ \overset{1}{2} \\ -\ 3\ C\ 4\ 4 \\ \hline 5\ 6\ 7\ 8 \end{array} \qquad \begin{array}{r} \overset{A-1}{\cancel{A}}\ \overset{10}{\cancel{1}}\ \overset{11}{\cancel{2}}\ \overset{1}{2} \\ -\ 3\ C\ 4\ 4 \\ \hline 5\ 6\ 7\ 8 \end{array}$$

In the thousands column, $(A - 1) - 3$ must end in 5.

If $(A - 1) - 3 = 5$

then $A - 1 = 5 + 3$

$A = 5 + 3 + 1$

$= 9$.

$$\begin{array}{r} \overset{A-1}{\cancel{A}}\ \overset{0}{\cancel{1}}\ \overset{11}{\cancel{2}}\ \overset{1}{2} \\ -\ 3\ 4\ 4\ 4 \\ \hline 5\ 6\ 7\ 8 \end{array} \qquad \begin{array}{r} \overset{9-1}{\cancel{A}}\ \overset{10}{\cancel{1}}\ \overset{11}{\cancel{2}}\ \overset{1}{2} \\ -\ 3\ 4\ 4\ 4 \\ \hline 5\ 6\ 7\ 8 \end{array}$$

We have: $A = 9$, $B = 2$, $C = 4$, $D = 4$

Let's check:

$$\begin{array}{r} A\ 1\ 2\ B \\ -\ 3\ C\ D\ 4 \\ \hline 5\ 6\ 7\ 8 \end{array} \qquad \begin{array}{r} 9\ 1\ 2\ 2 \\ -\ 3\ 4\ 4\ 4 \\ \hline 5\ 6\ 7\ 8 \end{array} \qquad \begin{array}{r} \overset{8}{\cancel{9}}\ \overset{10}{\cancel{1}}\ \overset{11}{\cancel{2}}\ \overset{1}{2} \\ -\ 3\ 4\ 4\ 4 \\ \hline 5\ 6\ 7\ 8 \end{array}$$

The four-digit number represented by *ABCD* is **9244**.

Strategy 2: **Work Backwards (2)**

If $A12B - 3CD4 = 5678$, then $3CD4 + 5678 = A12B$.

$$\begin{array}{r} 3\ C\ D\ 4 \\ +\ 5\ 6\ 7\ 8 \\ \hline A\ 1\ 2\ B \end{array}$$

In the ones column, $4 + 8 = 12$.

We can see that $B = 2$, and the remaining 10 is added to the tens column.

$$\begin{array}{r} 3\ C\ \overset{1}{D}\ 4 \\ +\ 5\ 6\ 7\ 8 \\ \hline A\ 1\ 2\ 2 \end{array}$$

In the tens column, $1 + D + 7 = 2$ won't work, but we can have

$1 + D + 7 = 12$

$D = 4$.

$$\begin{array}{r} 3\ \overset{1}{C}\ \overset{1}{4}\ 4 \\ +\ 5\ 6\ 7\ 8 \\ \hline A\ 1\ 2\ 2 \end{array}$$

In the hundreds column, $1 + C + 6 = 1$ won't work, so

$1 + C + 6 = 11$

$C = 4$.

$$\begin{array}{r} \overset{1}{3}\ \overset{1}{4}\ \overset{1}{4}\ 4 \\ +\ 5\ 6\ 7\ 8 \\ \hline A\ 1\ 2\ 2 \end{array}$$

In the thousands column, $1 + 3 + 5 = A$ and so $A = 9$.

The four-digit number represented by *ABCD* is **9244**.

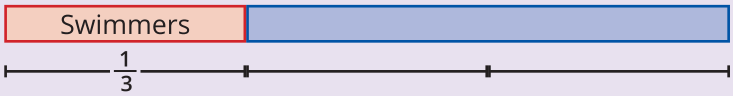
$$\begin{array}{r} \overset{1}{3}\ \overset{1}{4}\ \overset{1}{4}\ 4 \\ +\ 5\ 6\ 7\ 8 \\ \hline 9\ 1\ 2\ 2 \end{array}$$

Follow-Up: *What is the 5-digit number represented by ABCDE if A123B - 3CDE4 = 45678 ?* [82555]

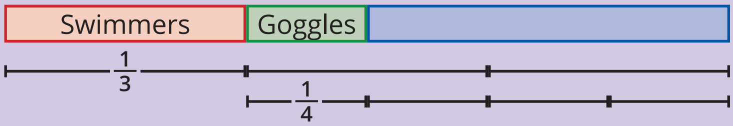
2E. The question is, How much money did Tom have to start with?

***Strategy 1:* Draw a Diagram, and Work Backwards**

Tom spent a third of his money on swimmers.

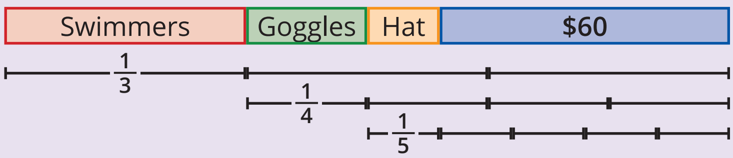


He spent a quarter of what remained on a pair of goggles.



He spent a fifth of the remainder on a hat.

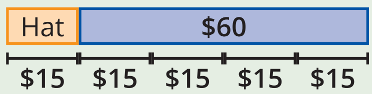
He has just **$60** left over.



Let's work backwards to see how much he started with.

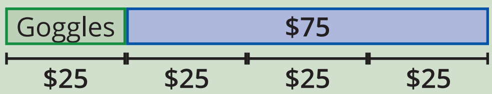
Before buying the hat, **$60** was four fifths of the money Tom had.

One fifth would be **60 ÷ 4 = $15**, so all five fifths would be **$15 × 5 = $75**.



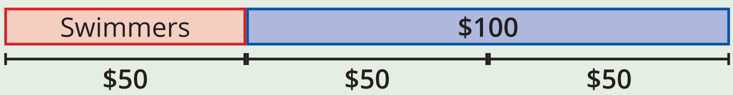
Before buying the goggles, **$75** was three quarters of the money Tom had.

One quarter would be **75 ÷ 3 = $25**, so all four quarters would be **$25 × 4 = $100**.



Before buying the swimmers, **$100** was two-thirds of the money Tom had.

One third would be **100 ÷ 2 = $50**, so all three thirds would be **$50 × 3 = $150**.



Tom must have started with **$150**.

***Strategy 2:* Reason Algebraically**

Let x represent the amount of money that Tom had in the beginning.

After spending $\frac{1}{3}$ of his money on swimmers, Tom would have $\frac{2}{3}x$ remaining afterwards.

After spending $\frac{1}{4}$ of the remaining money on goggles, Tom would have $\frac{3}{4} \times \frac{2}{3}x = \frac{3\times2}{4\times3}x = \frac{1}{2}x$ remaining.

After spending $\frac{1}{5}$ of the remaining money on a hat, Tom would have $\frac{4}{5} \times \frac{1}{2}x = \frac{4\times1}{5\times2}x = \frac{2}{5}x$ remaining.

If Tom's remaining money, $\frac{2}{5}x$, is equal to **$60**,

then Tom must have started with **$150**.

$$\frac{2}{5}x = \$60$$
$$2x = \$60 \times 5$$
$$= \$300$$
$$x = \$300 \div 2$$
$$= \$150$$

Follow-Up: *Suppose Tom bought the goggles first, and then the swimmers. Using the price of the goggles and swimmers in the question, what fractions of his remaining money were spent on each item?* [For $25 goggles: spent one-sixth of $150. For $50 swimmers: spent two-fifths of the remaining $125.]