2023 Maths Games Junior - Years 5 & 6
Resource Kit 1
Teaching Problem Solving

Problem Solving Strategies

This resource kit follows on from the Preparation Kit and its emphasis on:

Guess, Check and Refine

Draw a Diagram

The problems are sourced from previous Junior (Division J) Maths Olympiads and Maths Games papers.

They introduce two new problem solving strategies:

1. Find a Pattern

One of the most frequently used problem solving strategies is that of recognising and extending a pattern.

Students can often simplify a difficult problem by identifying a pattern in it, and then applying that pattern to the problem situation.

2. Build a Table

A table displays information so that it is easily located and understood, and missing information becomes obvious.

If students are not given the data for a problem, and must generate it themselves, a table is an excellent way to record what they have done so they don't have to repeat their efforts.

A table can also be invaluable for detecting significant patterns.

Resource Kit 1 focuses on:

Find a Pattern

Build a Table

Set Yellow

Example problems for which full worked solutions are included.

Set Green

Problems that are designed to be similar to Set Yellow, but with fewer difficult elements.

Set Orange

Problems that are similar in mathematical structure to the corresponding Yellow problems.

Further questions and solution methods can be found in the APSMO resource book "Building Confidence in Maths Problem Solving", available from www.apsmo.edu.au.

How to use these problems

At the start of the lesson, present the problem and ask the students to think about it. Encourage students to try to solve it in any way they like. When the students have had enough time to consider their solutions, ask them to describe or present their methods, taking particular note of different ways of arriving at the same solution.

Each question includes at least one solution method that the majority of students should be able to follow. By participating in lessons that demonstrate achievable problem solving techniques, students may gain increased confidence in their own ability to address unfamiliar problems.

Finally, the consideration of different solution methods is fundamental to the students' development as effective and sophisticated problem solvers. Even when students have solved a problem to their own satisfaction, it is important to expose them to other methods and encourage them to judge whether or not the other methods are more efficient.

Preparation Kit

Guess, Check and Refine

This involves making a reasonable guess of the answer, and checking it against the conditions of the problem. An incorrect guess may provide more information that may lead to the answer.

Draw a Diagram

A diagram may reveal information that may not be obvious just by reading the problem.

It is also useful for keeping track of where the student is up to in a multi-step problem.

Resource Kit 1

Find a Pattern

A frequently used problem solving strategy is that of recognising and extending a pattern.

Students can often simplify a difficult problem by identifying a pattern in the problem situation.

Build a Table

A table displays information so that it is easily located and understood.

A table is an excellent way to record data so the student doesn't have to repeat their efforts.

Resource Kit 2

Work Backwards

If a problem describes a procedure and then specifies the final result, this method usually makes the problem much easier to solve.

Make an Organised List

Listing every possibility in an organised way is an important tool.

How students organise the data often reveals additional information.

Resource Kit 3

Solve a Simpler Related Problem

Many hard problems are actually simpler problems that have been extended to larger numbers.

Patterns can sometimes be identified by trying the problem with smaller numbers.

Eliminate All But One Possibility

Deciding what a quantity is not, can narrow the field to a very small number of possibilities.

These can then be tested against the conditions of the original problem.

Resource Kit 4

Convert to a More Convenient Form

There are times when changing some of the conditions of a problem makes a solution clearer or more convenient.

Divide a Complex Shape

Sometimes it is possible to divide an unusual shape into two or more common shapes that are easier to work with.

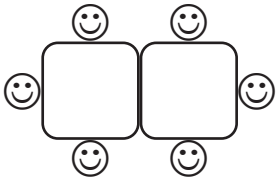
Set Yellow

1.1) Lily and Billy each have the same number of jellybeans.
Billy doesn't like black jellybeans though, so he gives his three black jellybeans to Lily.
How many more jellybeans does Lily now have than Billy?

1.2) Jon has interlocking blocks to build a tower.
He has blue, yellow, green, and red blocks.
He repeats the pattern "blue, yellow, green, red" over and over again.
What colour block would be at the 39th position?

1.3) Suppose today is Tuesday.
In all, how many Fridays are there in the next 53 days?

1.4) Four people can be seated at a single table.
If two tables are placed end to end, six people can be seated as shown in the diagram.
How many tables must be placed end to end to seat 22 people?



Set Yellow

1.5) There are 140 students at a music camp.
For lunch one day, they buy bread rolls from a bakery.
The bakery sells the rolls in bags of 9.
They buy the smallest number of bags of rolls so that each student can have one roll for lunch.
After each student gets a roll, how many rolls will they have left over?

1.6) There are 50 steps in a staircase.
Archer is on step number 5. He runs up the steps two at a time.
Joey is on step number 50. He runs down the steps three at a time.
If both boys land on a new step every second, on which number step do they meet?

1.7) One hat and two shirts cost $21.
Two hats and one shirt cost $18.
Megan has exactly enough money to buy one hat and one shirt.
How much money does Megan have?

1.8) The product of $1 \times 3 \times 5 \times 7 \times 9 \times \ldots \times 99$ is written as a counting number.
What is the last digit of that counting number?

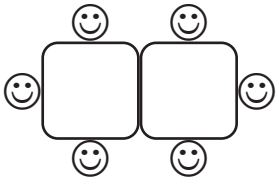
Set Green

1.1) Lily and Billy each have 10 jellybeans.
Billy doesn't like black jellybeans though, so he gives his three black jellybeans to Lily.
How many more jellybeans does Lily now have than Billy?

1.2) Jon has interlocking blocks to build a tower.
He has blue, yellow, and green blocks.
He repeats the pattern "blue, yellow, green" over and over again.
What colour block would be at the 17th position?

1.3) Suppose today is Tuesday.
In all, how many Fridays are there in the next 18 days?

1.4) Four people can be seated at a single table.
If two tables are placed end to end, six people can be seated as shown in the diagram.
How many tables must be placed end to end to seat 10 people?



Set Green

1.5) There are 30 students at a music camp.
For lunch one day, they buy bread rolls from a bakery.
The bakery sells the rolls in bags of 9.
They buy the smallest number of bags of rolls so that each student can have one roll for lunch.
After each student gets a roll, how many rolls will they have left over?

1.6) There are 20 steps in a staircase.
Archer is on step number 5. He runs up the steps two at a time.
Joey is on step number 20. He runs down the steps three at a time.
If both boys land on a new step every second, on which number step do they meet?

1.7) One hat and two shirts cost $8.
Two hats and one shirt cost $7.
Megan has exactly enough money to buy one hat and one shirt.
How much money does Megan have?

1.8) The product of $1 \times 3 \times 5 \times 7 \times 9$ is written as a counting number.
What is the last digit of that counting number?

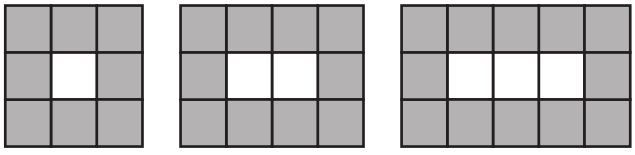
Set Orange

1.1) Marty has 6 more magazines than Jen has.
After he gives 10 magazines to Jen, how many more magazines will Jen have than Marty?

1.2) A list of numbers beginning with 28 is 28, 34, 40, 46, ...
Notice that each number after the first number is 6 more than the previous number.
How many numbers less than 100 are in this list?
Include the four numbers already listed.

1.3) The 25th of May, 2025, will occur on a Sunday.
On which day of the week will the 1st of May, 2025 occur?

1.4) I am making a pattern with square tiles.
I surround one row of white tiles with grey tiles.
The diagram shows what it looks like when I have 1, 2, and 3 white tiles.
How many grey tiles will I need when I have a row of 20 white tiles?



Set Orange

1.5) What is the greatest number of Mondays that can occur in 45 consecutive days?

1.6) Barry and Cecilia are both driving to Barry's house, 20 km away.
Barry drives at an average rate of 50 km per hour.
Cecilia drives at an average rate of 40 km per hour.
They both start at the same time, and neither person stops.
When Barry arrives home, how many minutes does he have to wait before Cecilia arrives?

1.7) In a stationery store, pencils have one price and pens have another price.
Two pencils and three pens cost 78c.
Three pencils and two pens cost 72c.
How much does one pencil cost?

1.8) Michelle's Number Recycling Machine obeys exactly two rules:

1. *If an inserted number has exactly 1 digit, double the number.*
2. *If an inserted number has exactly 2 digits, compute the sum of the digits.*

The first number Michelle inserts is 1.
Then every answer she gets is inserted back into the machine until fifty numbers are inserted.
What is the fiftieth number to be inserted?

Maths Games – Example Problem 1.1

Example Problem 1.1 - Green

Lily and Billy each have 10 jellybeans.
Billy doesn't like black jellybeans though, so he gives his three black jellybeans to Lily.
How many more jellybeans does Lily now have than Billy?

Example Problem 1.1 - Yellow

Lily and Billy each have the same number of jellybeans.
Billy doesn't like black jellybeans though, so he gives his three black jellybeans to Lily.
How many more jellybeans does Lily now have than Billy?

Example Problem 1.1 - Orange

Marty has 6 more magazines than Jen has.
After he gives 10 magazines to Jen, how many more magazines will Jen have than Marty?

Maths Games Example Solution 1.1 - Yellow

Lily and Billy each have the same number of jellybeans.

Billy doesn't like black jellybeans though, so he gives his three black jellybeans to Lily.

How many more jellybeans does Lily now have than Billy?

Strategy 1: Build a Table, and Find a Pattern

Suppose Lily and Billy both start with 10 jellybeans.

After Billy gives 3 jellybeans to Lily:

- Lily has 10 + 3 = 13 jellybeans, and
- Billy has 10 – 3 = 7 jellybeans.

Lily now has 13 – 7 = 6 more jellybeans than Billy.

Before		After		
Lily	Billy	Lily	Billy	Difference
10	10	13	7	13 – 7 = 6

Suppose Lily and Billy both start with 23 jellybeans.

After Billy gives 3 jellybeans to Lily:

- Lily has 23 + 3 = 26 jellybeans, and
- Billy has 23 – 3 = 20 jellybeans.

Lily now has 26 – 20 = 6 more jellybeans than Billy.

Before		After		
Lily	Billy	Lily	Billy	Difference
10	10	13	7	13 – 7 = 6
23	23	26	20	26 – 20 = 6

We can see that it doesn't matter how many jellybeans they had to begin with.

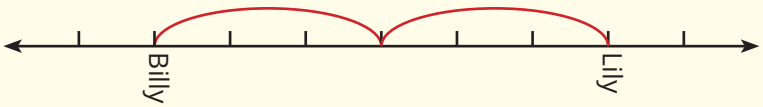
After Billy gives **3** jellybeans to Lily, Lily has **6** more jellybeans than Billy.

Strategy 2: Draw a Diagram

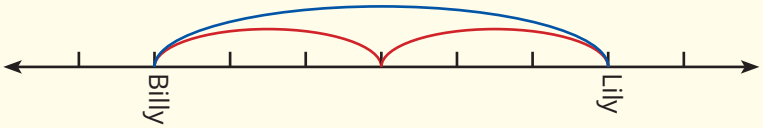
Lily and Billy have the same number of jellybeans.

Billy gives Lily 3 jellybeans.

After doing so, Billy has 3 fewer, and Lily has 3 more.



Afterwards, how many more jellybeans does Lily have than Billy?



Lily now has **6** more jellybeans than Billy.

Answers **1.1 - Green:** 6 **1.1 - Orange:** 14

1.1 - Yellow: 6

Maths Games – Example Problem 1.2

Example Problem 1.2 - Green

Jon has interlocking blocks to build a tower.
He has blue, yellow, and green blocks.
He repeats the pattern "blue, yellow, green" over and over again.
What colour block would be at the 17th position?

Example Problem 1.2 - Yellow

Jon has interlocking blocks to build a tower.
He has blue, yellow, green, and red blocks.
He repeats the pattern "blue, yellow, green, red" over and over again.
What colour block would be at the 39th position?

Example Problem 1.2 - Orange

A list of numbers beginning with 28 is 28, 34, 40, 46, ...
Notice that each number after the first number is 6 more than the previous number.
How many numbers less than 100 are in this list?
Include the four numbers already listed.

Maths Games Example Solution 1.2 - Yellow

Jon has interlocking blocks to build a tower.

He has blue, yellow, green, and red blocks.

He repeats the pattern "blue, yellow, green, red" over and over again.

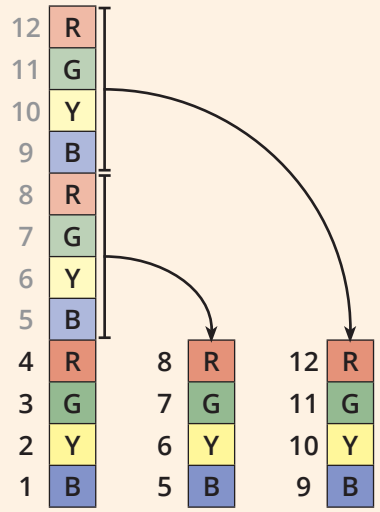
What colour block would be at the 39th position?

Strategy: Find a Pattern

Jon starts by placing the first four blocks:

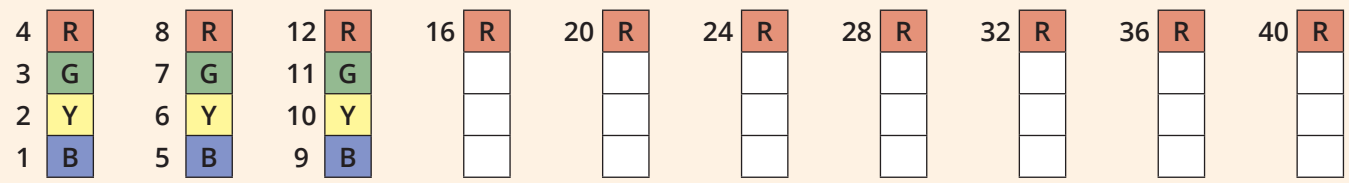
blue, yellow, green, red.

This pattern continues for every **4** blocks.

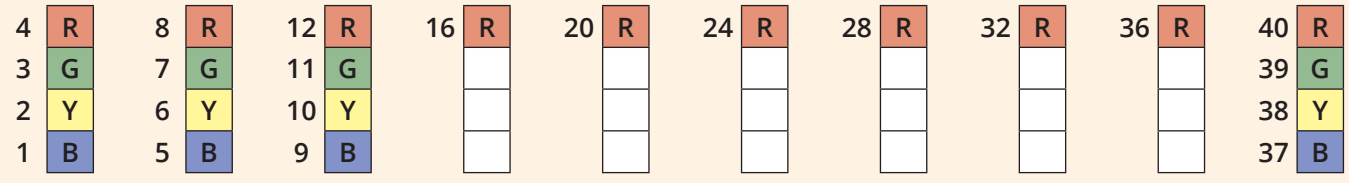


By separating the single tower into short towers of **4** blocks each, we can see that there is a red block for each multiple of **4**.

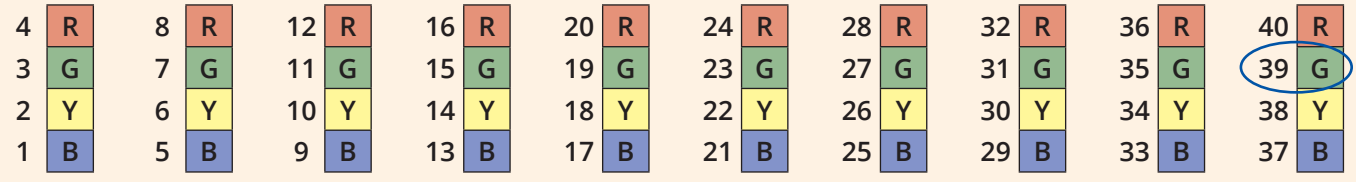
Why might this happen?



Continuing with multiples of **4**, it looks like we'll have a red block at position **36**, and another at position **40**.



We can see that Jon's pattern of blue, yellow, green, red must also appear in positions **37-40**.



The **39th** block will be **Green**.

Answers

1.2 - Green: Yellow

1.2 - Orange: 12

1.2 - Yellow: Green

Maths Games – Example Problem 1.3

Example Problem 1.3 - Green

Suppose today is Tuesday.
In all, how many Fridays are there in the next 18 days?

Example Problem 1.3 - Yellow

Suppose today is Tuesday.
In all, how many Fridays are there in the next 53 days?

Example Problem 1.3 - Orange

The 25th of May, 2025, will occur on a Sunday.
On which day of the week will the 1st of May, 2025 occur?

Maths Games Example Solution 1.3 - Yellow

Suppose today is Tuesday.

In all, how many Fridays are there in the next 53 days?

Before we get started, we need to work out what "in the next 53 days" means.

It may help to think of a smaller number of days.

For example, the next 2 days would be Wednesday and Thursday.

We will count tomorrow (Wednesday) as the first of the next 53 days.

Strategy 1: Build a Table, and Find a Pattern (1)

Let's draw a calendar.

We will begin counting from the Wednesday.

Can you see a pattern?

Since a week is 7 days, we should find multiples of 7 for one of the days of the week.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			1	2	3	4
5	6	7	8	9	10	11
12	13	14			17	
		21			24	
		28			31	
		35			38	
		42			45	
		49	50	51	52	53

Alternatively, we can recognise that every 7th day from today will be a Tuesday.

In 7 × 7 = 49 days from today, 7 weeks will have passed. Each of those 7 weeks includes a Friday.

Since the 49th day from today is a Tuesday, the 52nd day from today is another Friday.

We can see that there are **8** Fridays in the next **53** days.

Strategy 2: Build a Table, and Find a Pattern (2)

Since today is Tuesday, Day 3 will be a Friday.

The following Friday will be 7 days later, on Day 3 + 7 = 10.

By continuing to add 7 days each time, we will find that days 17, 24, 31, 38, 45 and 52 will all be Fridays.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			1	2	3	4
5	6	7	8	9	10	
					17	
					24	
					31	
					38	
					45	
					52	53

There are **8** Fridays in the next **53** days.

Answers

1.3 - Green: 3

1.3 - Yellow: 8

1.3 - Orange: Thursday

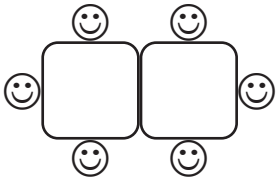
Maths Games – Example Problem 1.4

Example Problem 1.4 - Green

Four people can be seated at a single table.

If two tables are placed end to end, six people can be seated as shown in the diagram.

How many tables must be placed end to end to seat 10 people?

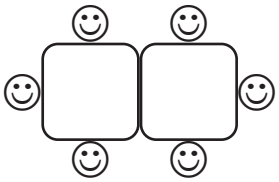


Example Problem 1.4 - Yellow

Four people can be seated at a single table.

If two tables are placed end to end, six people can be seated as shown in the diagram.

How many tables must be placed end to end to seat 22 people?



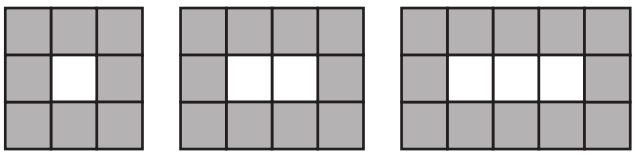
Example Problem 1.4 - Orange

I am making a pattern with square tiles.

I surround one row of white tiles with grey tiles.

The diagram shows what it looks like when I have 1, 2, and 3 white tiles.

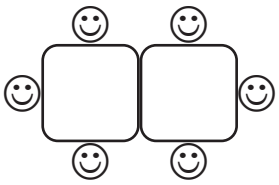
How many grey tiles will I need when I have a row of 20 white tiles?



Maths Games Example Solution 1.4 - Yellow

Four people can be seated at a single table.

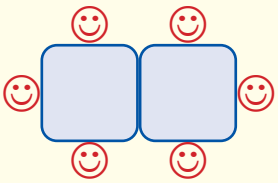
If two tables are placed end to end, six people can be seated as shown in the diagram.



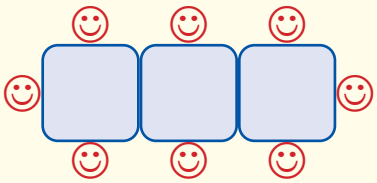
How many tables must be placed end to end to seat 22 people?

Strategy 1: Draw a Diagram, and Find a Pattern

6 people can sit at 2 tables.

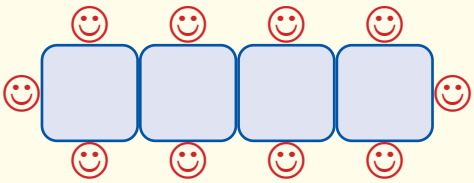


With 3 tables, we can seat 8 people.



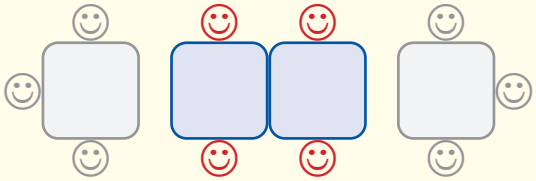
What changed when we went from 2 tables to 3 tables?

With 4 tables, we can seat 10 people.



What changed when we went from 3 tables to 4 tables?

Does it help to think of each new table being placed in the middle instead of at the end?



Is there a pattern with the number of people we can seat at the tables in the middle?

Each end table holds 3 people.

The two end tables together seat 6 people.

There are 22 - 6 = 16 people left.

So, all together, the interior tables need to hold 16 people.

Since each interior table holds 2 people, there must be 16 ÷ 2 = 8 interior tables.

In total, we have 2 end tables and 8 interior tables.

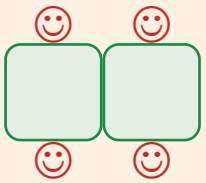
This means that 2 + 8 = **10** tables will seat 22 people.

Strategy 2: Find a Pattern

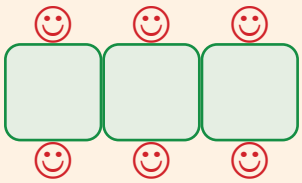
If we remove the 2 end seats, it might be easier to see a pattern.

So, with the end seats off: ☺ ☺

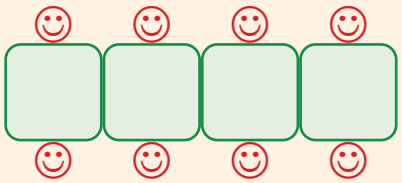
2 tables seat 4 people,



3 tables seat 6 people,



4 tables seat 8 people.



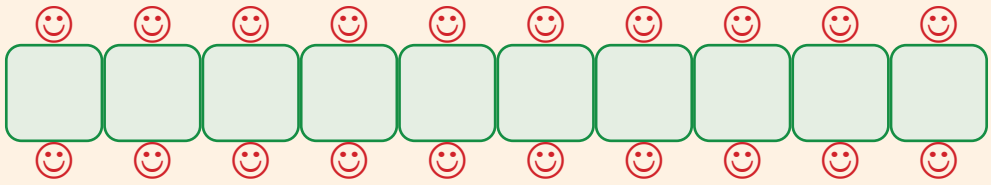
Can you see a pattern?

We have 22 people.

2 of them will be sitting on end seats.

So we have 22 - 2 = 20 people left to seat.

How many tables will we need for these 20 people?



After re-seating our two end seat people, that's 22 people in total.

So **10** tables will seat 22 people.

Answers

1.4 - Green: 4

1.4 - Orange: 46

1.4 - Yellow: 10

Maths Games – Example Problem 1.5

Example Problem 1.5 - Green

There are 30 students at a music camp.
For lunch one day, they buy bread rolls from a bakery.
The bakery sells the rolls in bags of 9.
They buy the smallest number of bags of rolls so that each student can have one roll for lunch.
After each student gets a roll, how many rolls will they have left over?

Example Problem 1.5 - Yellow

There are 140 students at a music camp.
For lunch one day, they buy bread rolls from a bakery.
The bakery sells the rolls in bags of 9.
They buy the smallest number of bags of rolls so that each student can have one roll for lunch.
After each student gets a roll, how many rolls will they have left over?

Example Problem 1.5 - Orange

What is the greatest number of Mondays that can occur in 45 consecutive days?

Maths Games Example Solution 1.5 - Yellow

There are 140 students at a music camp.

For lunch one day, they buy bread rolls from a bakery.

The bakery sells the rolls in bags of 9.

They buy the smallest number of bags of rolls so that each student can have one roll for lunch.

After each student gets a roll, how many rolls will they have left over?

Strategy 1: Build a Table

Let's build a table to find out how many bags of rolls they need to buy.

The number of rolls is a multiple of 9.

To save time, we can jump to an easy multiple of 9, such as $10 \times 9 = 90$.

We can then keep going, adding another bag of rolls each time, until we have enough rolls in total.

No. of Bags	No. of Rolls
1	$1 \times 9 = 9$
2	$2 \times 9 = 18$
...	...
10	$10 \times 9 = 90$
11	99
12	108
13	117
14	126
15	135
16	144

We now have enough rolls to give one to each student.

There are **140** students. Each student is getting one roll.

With 144 rolls, they will have $144 - 140 = 4$ rolls left over.

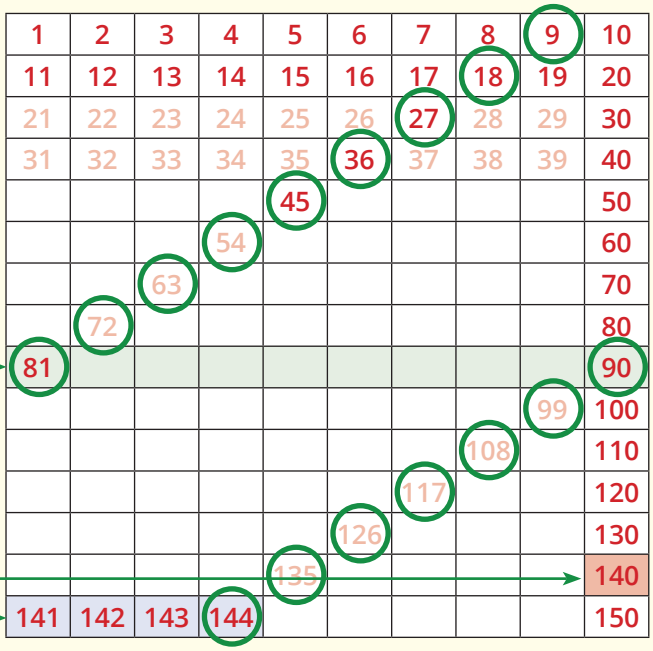
Strategy 2: Find a Pattern

It is often easier to think about numbers when we arrange them in tens.

We can lay out the rolls so that there are **10** rolls in each row.

If we circle the last roll from each bag, can you see a pattern?

Since we are adding 9 rolls each time, there are two circled on this line.



We have enough rolls to give one to each student.

Here are the ones we have left over.

After buying 144 rolls, and giving one roll to each student, there are $144 - 140 = 4$ rolls left over.

Answers

1.5 - Green: 6

1.5 - Orange: 7

1.5 - Yellow: 4

Maths Games – Example Problem 1.6

Example Problem 1.6 - Green

There are 20 steps in a staircase.
Archer is on step number 5. He runs up the steps two at a time.
Joey is on step number 20. He runs down the steps three at a time.
If both boys land on a new step every second, on which number step do they meet?

Example Problem 1.6 - Yellow

There are 50 steps in a staircase.
Archer is on step number 5. He runs up the steps two at a time.
Joey is on step number 50. He runs down the steps three at a time.
If both boys land on a new step every second, on which number step do they meet?

Example Problem 1.6 - Orange

Barry and Cecilia are both driving to Barry's house, 20 km away.
Barry drives at an average rate of 50 km per hour.
Cecilia drives at an average rate of 40 km per hour.
They both start at the same time, and neither person stops.
When Barry arrives home, how many minutes does he have to wait before Cecilia arrives?

Maths Games Example Solution 1.6 - Yellow

There are 50 steps in a staircase.

Archer is on step number 5. He runs up the steps two at a time.

Joey is on step number 50. He runs down the steps three at a time.

If both boys land on a new step every second, on which number step do they meet?

Strategy 1: Build a Table

To find the step where Archer and Joey meet, let's play out the boys' running patterns.

Archer starts on step number 5 and runs up the steps 2 at a time.

No. of Seconds	0	1	2	3	4	5	6	7	8	9	10
Archer's Position	5	7	9	11	13	15	17	19	21	23	25
Joey's Position											

Joey starts on step number 50 and runs down the steps three at a time.

No. of Seconds	0	1	2	3	4	5	6	7	8	9	10
Archer's Position	5	7	9	11	13	15	17	19	21	23	25
Joey's Position	50	47	44	41	38	35	32	29	26	23	20

After 9 seconds, both boys are on step number 23.

After 10 seconds, the boys have passed each other and are moving away from each other.

Therefore, the boys meet on step number **23**.

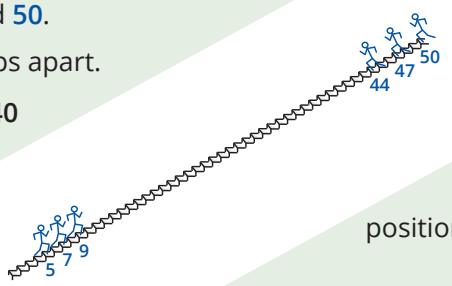
Strategy 2: Build a Table, and Find a Pattern

Archer and Joey start on steps 5 and 50.

At this time, they are 50 - 5 = **45** steps apart.

One second later, they are 47 - 7 = **40** steps apart.

Two seconds later, they are 44 - 9 = **35** steps apart.



No. of Seconds	0	1	2
Archer's Position	5	7	9
Joey's Position	50	47	44
Difference	45	40	35

It looks like the difference in position is decreasing by **5** steps each time.

Why might this be happening?

Let's follow the pattern until the boys meet.

We'll know we're there because the difference in position will be **0**.

No. of Seconds	0	1	2	3	4	5	6	7	8	9	10
Archer's Position	5	7	9								
Joey's Position	50	47	44								
Difference	45	40	35	30	25	20	15	10	5	0	
		-5	-5	-5	-5	-5	-5	-5	-5	-5	

Following the pattern, it seems that the boys should meet 9 seconds after starting:

Archer started on step 5 and went up 2 steps for 9 seconds, so Archer is now on step number 5 + (2 × 9) = 23.

Joey started on step 50 and went down 3 steps for 9 seconds, so he's now on step number 50 - (3 × 9) = 23

Therefore, after 9 seconds, both Archer and Joey are on step number **23**.

Answers

1.6 - Green: 11

1.6 - Orange: 6

1.6 - Yellow: 23

Maths Games – Example Problem 1.7

Example Problem 1.7 - Green

One hat and two shirts cost $8.
Two hats and one shirt cost $7.
Megan has exactly enough money to buy one hat and one shirt.
How much money does Megan have?

Example Problem 1.7 - Yellow

One hat and two shirts cost $21.
Two hats and one shirt cost $18.
Megan has exactly enough money to buy one hat and one shirt.
How much money does Megan have?

Example Problem 1.7 - Orange

In a stationery store, pencils have one price and pens have another price.
Two pencils and three pens cost 78c.
Three pencils and two pens cost 72c.
How much does one pencil cost?

Maths Games Example Solution 1.7 - Yellow

One hat and two shirts cost $21.

Two hats and one shirt cost $18.

Megan has exactly enough money to buy one hat and one shirt.

How much money does Megan have?

Strategy 1: Find a Pattern

We could represent the information in a table like this.

With 1 hat and 2 shirts, the cost is $21.

With 2 hats and 1 shirt, the cost is $18.

No. of hats	No. of shirts	Total cost
1	2	$21
2	1	$18

The table shows that, by

- Increasing the number of hats by 1, and
- Reducing the number of shirts by 1,

the total cost goes down by $3.

Continuing this pattern, we can see that 3 hats will cost $15.

Therefore, 1 hat must cost $15 ÷ 3 = $5.

No. of hats	No. of shirts	Total cost
1 (+1)	2 (−1)	$21 (−3)
2 (+1)	1 (−1)	$18 (−3)
3	0	$15

Working backwards, we can find the cost of 0 hats and 3 shirts.

0 hats and 3 shirts will cost $24, so 3 shirts costs $24.

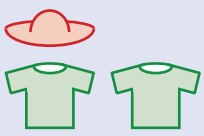
Therefore 1 shirt must cost $24 ÷ 3 = $8.

No. of hats	No. of shirts	Total cost
0 (+1)	3 (−1)	$24 (−3)
1 (+1)	2 (−1)	$21 (−3)
2 (+1)	1 (−1)	$18 (−3)
3	0	$15

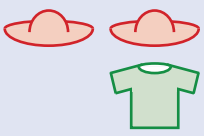
Since Megan has exactly enough money for 1 hat and 1 shirt, Megan has $5 + $8 = **$13**.

Strategy 2: Reason Logically

1 hat and 2 shirts cost $21.



2 hats and 1 shirt costs $18.



Adding both combinations together, we can see that 3 hats and 3 shirts would cost $21 + $18 = $39.

We can group these 3 hats and 3 shirts into 3 packages each of which contains 1 hat and 1 shirt.

Therefore 1 hat and 1 shirt costs $39 ÷ 3 = **$13**.

Answers

1.7 - Green: $5

1.7 - Yellow: $13

1.7 - Orange: 12c

Maths Games – Example Problem 1.8

Example Problem 1.8 - Green

The product of 1 × 3 × 5 × 7 × 9 is written as a counting number.
What is the last digit of that counting number?

Example Problem 1.8 - Yellow

The product of 1 × 3 × 5 × 7 × 9 × … × 99 is written as a counting number.
What is the last digit of that counting number?

Example Problem 1.8 - Orange

Michelle's Number Recycling Machine obeys exactly two rules:

1. *If an inserted number has exactly 1 digit, double the number.*
2. *If an inserted number has exactly 2 digits, compute the sum of the digits.*

The first number Michelle inserts is 1.
Then every answer she gets is inserted back into the machine until fifty numbers are inserted.
What is the fiftieth number to be inserted?

Maths Games Example Solution 1.8 - Yellow

The product of 1 × 3 × 5 × 7 × 9 × ... × 99 is written as a counting number.

What is the last digit of that counting number?

Strategy 1: Build a Table, and Find a Pattern

The product of **1 × 3 × 5 × 7 × 9 × ... × 99** must be a really big number.

However, the question is only asking for the last digit.

Let's try to work out the answer. As we do it, we will keep watching to see what happens to the last digit.

Working	1 × 3 = 3	3 × 5 = 15	$\begin{array}{r} 1\ 5 \\ \times\ 7 \\ \hline 1\ 0\ 5 \end{array}$ (3)	$\begin{array}{r} 1\ 0\ 5 \\ \times\ 9 \\ \hline 9\ 4\ 5 \end{array}$ (4)	$\begin{array}{r} 9\ 4\ 5 \\ \times\ 1\ 1 \\ \hline 9\ 4\ 5 \\ 9\ 4\ 5\ 0 \\ \hline 1\ 0\ 3\ 9\ 5 \end{array}$ (4)
Product so far	3	15	105	945	10395
Last digit	3	5	5	5	5 ←

The last digit has been **5** for a few results now.

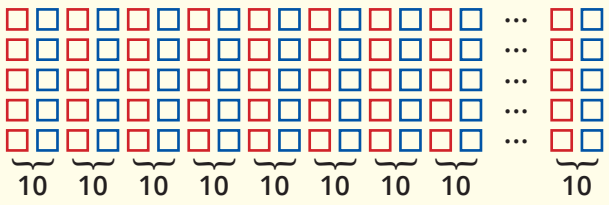
Why might this be the case?

Following the pattern, we can infer that the last digit of the product will be **5**.

Strategy 2: Reason Logically, and Draw a Diagram

It appears that multiplying any number by 5 results in a number that ends in either **5** or **0**.

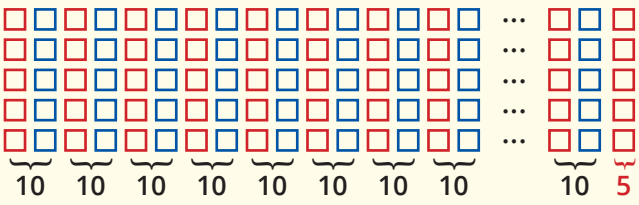
This makes sense because any multiple of 5 can be drawn as an array. The array could be like this:



resulting in a multiple of **10**.

This happens when there are an even number of 5s.

Alternatively, the array could be like this:

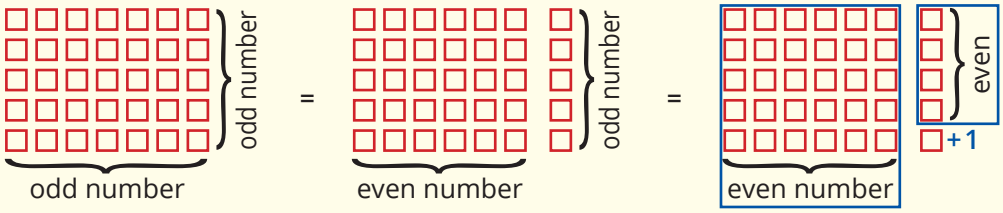


resulting in a multiple of **10**, plus another 5.

This happens when there are an odd number of 5s.

We also know that multiplying two odd numbers always makes an odd number.

We can represent this visually as follows.



1 × 3 × 5 × 7 × 9 × ... × 99 = 5 × (1 × 3 × 7 × 9 × ... × 99).

Since **(1 × 3 × 7 × 9 × ... × 99)** only multiplies odd numbers together, the result must be odd.

An odd multiple of 5 must end in **5**.

Therefore the product of **1 × 3 × 5 × 7 × 9 × ... × 99** must be a number that ends in **5**.

Answers

1.8 - Green: 5

1.8 - Orange: 16

1.8 - Yellow: 5

Answers

Set Green	
1.1	6
1.2	Yellow
1.3	3
1.4	4
1.5	6
1.6	11
1.7	$5
1.8	5

Set Yellow	
1.1	6
1.2	Green
1.3	8
1.4	10
1.5	4
1.6	23
1.7	$13
1.8	5

Set Orange	
1.1	14
1.2	12
1.3	Thursday
1.4	46
1.5	7
1.6	6
1.7	12c
1.8	16