



APSMO
2023 MATHS GAMES

IMPORTANT

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2023 MATHS GAMES

ORGANISATION AND PROCEDURES

For full details, see the Members' Area

- Maths Games papers are to be conducted under test conditions.

DO

- Supervise students at all times.
- Maintain silence.
- Provide blank working paper.
- Collect, mark and retain the papers.

DO NOT

- Print the papers prior to the scheduled date.
- Read the questions aloud to the students.
- Interpret the questions for students.
- Permit any discussion or movement around the room.
- Permit the use of calculators or other electronic devices.

- Papers should be scored by the PICO using the *Solutions and Answers* sheet provided.
- Original student answer sheets should be retained by the PICO until the end of the year.

ABSENT STUDENTS

- A student who is legitimately absent on the date of the Maths Games paper, may sit the paper on their return to school.
- If an absent student does not sit the paper on their return to school they should be marked as 'absent'.
- *Note: This policy differs from the Maths Olympiads Absent Student Policy which has additional requirements.*



APSMO

WEDNESDAY 6 SEPTEMBER 2023

MATHS GAMES
JUNIOR
4

Suggested Time: 30 Minutes

- 4A.** What is the value of the following?
 $(7 \times 8) - (6 \times 8) + (5 \times 6) - (4 \times 6) + (3 \times 4) - (2 \times 4)$

Hint: What is the difference between 9×10 and 8×10 ?

Write your answers in the boxes on the back.

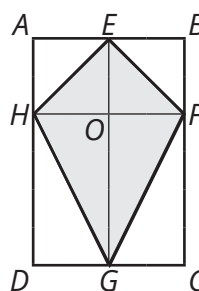
- 4B.** $ABCD$ is a rectangle with an area of 24 square centimetres.

Points E and G are midpoints of the sides on which they are located.

The line HF is parallel to the line AB .

What is the area of the kite $EFGH$?

Hint: Can you rearrange parts of the kite to make this easier to work out?



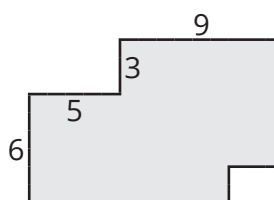
←
Keep your answers hidden by folding backwards on this line.

- 4C.** Cooper is stacking identical boxes in one large stack.
Each row has one less box than the row beneath it.
The stack has 9 boxes on the bottom row, and 1 box on the top row.
How many boxes are there in the stack?

Hint: Is there a convenient way to count the boxes?

- 4D.** The figure at the right is made by placing one rectangle on top of another.
All angles in the figure are right angles.
All lengths are given in centimetres.
What is the perimeter of the figure, in centimetres?

Hint: How wide is the figure?



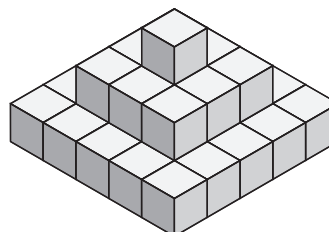
- 4E.** The pyramid at the right is made by stacking identical cubes on a flat table.

There are $5 \times 5 = 25$ cubes on the lowest layer, $3 \times 3 = 9$ cubes in the middle layer, and 1 cube on top.

The pyramid is spray-painted all around while on the table, so that the base has no paint on it.

How many of the cubes are painted on exactly two faces?

Hint: Can you see any cubes that would definitely be painted on exactly two faces?





**MATHS
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4A.

Student Name:

4B.

4C.

4D.

4E.

Fold here. Keep your answers hidden.



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Solutions and Answers

(Items in parentheses are not required)

4A: 18

4B: 12 (cm²)

4C: 45

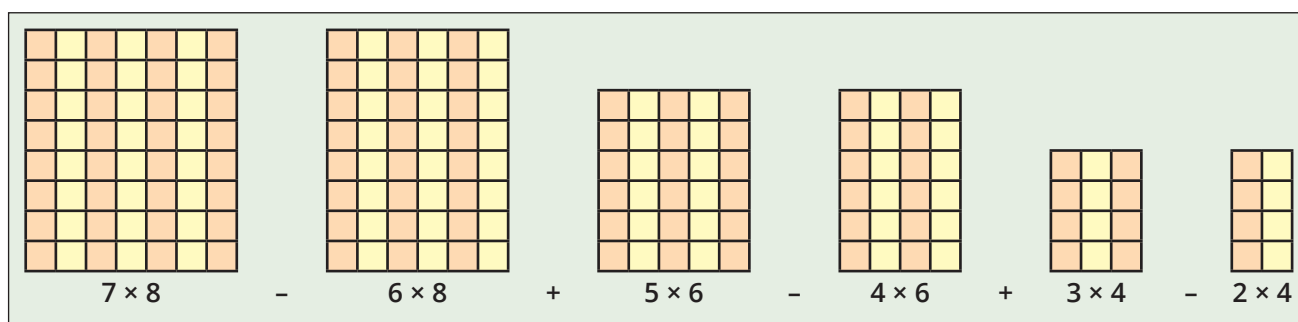
4D: 46 (cm)

4E: 16

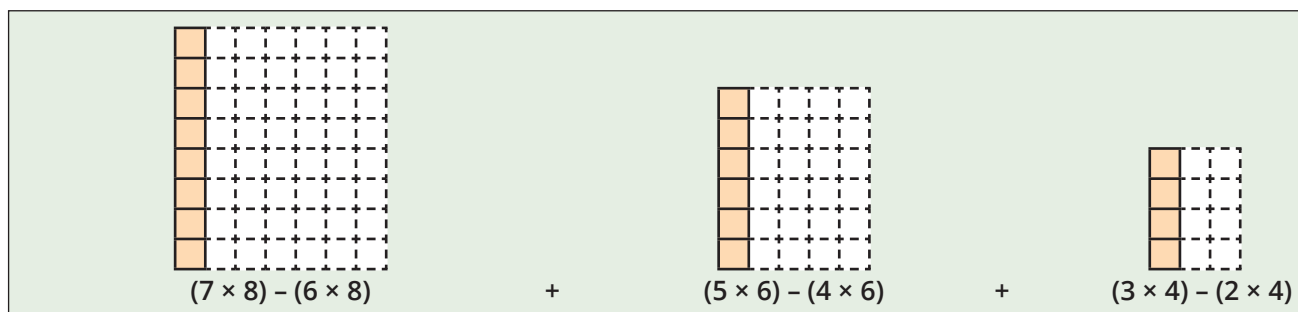
4A. The question is, what is the value of $(7 \times 8) - (6 \times 8) + (5 \times 6) - (4 \times 6) + (3 \times 4) - (2 \times 4)$?

Strategy 1: Convert to a More Convenient Form

To make this problem easier to think about, we can represent each multiplication as a diagram.



Each pair of products shares a factor, which is represented in the diagram as the height of the rectangle. We can use this to work out the difference between two consecutive terms.



The value of the expression is $8 + 6 + 4 = 18$.

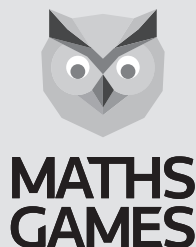
Strategy 2: Calculate the Result of Each Multiplication

We can solve each multiplication individually before subtracting and then adding them.

| | | | | | | | | | | |
|--------------|-----|--------------|-----|--------------|-----|--------------|-----|--------------|-----|--------------|
| 7×8 | $-$ | 6×8 | $+$ | 5×6 | $-$ | 4×6 | $+$ | 3×4 | $-$ | 2×4 |
| 56 | $-$ | 48 | $+$ | 30 | $-$ | 24 | $+$ | 12 | $-$ | 8 |
| 8 | | | $+$ | 6 | | | $+$ | 4 | | |

The value of the expression is $8 + 6 + 4 = 18$.

Follow-Up: What is the value of $9 \times 8 + 3 \times 8 + 8 \times 6 + 4 \times 6 + 7 \times 4 + 5 \times 4$? [216]



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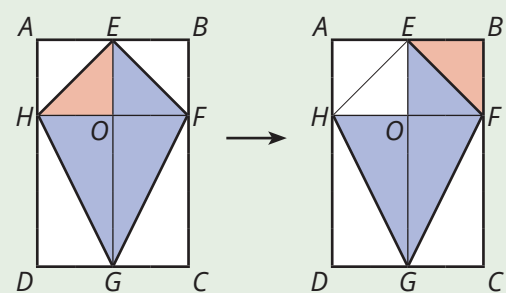
4B. The question is, What is the area of $EFGH$, in square centimetres?

Strategy 1: Convert to a More Convenient Form

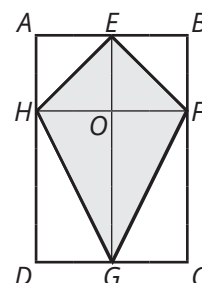
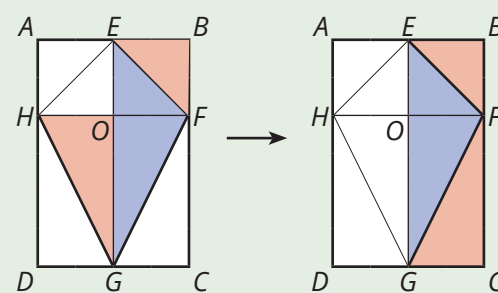
We know that the area of $ABCD$ is 24 square centimetres.

By rearranging parts of $EFGH$, we can work out how much of $ABCD$ it occupies.

We can begin by cutting out $\triangle EOH$, and rotating this area around E to fill in $\triangle EBF$.



We can also cut out $\triangle HOG$, and flipping and rotating this area to fill in $\triangle FCG$.



$EBCG$ is half of rectangle $ABCD$, so the area of $EBCG$ is $24 \div 2 = 12 \text{ cm}^2$.

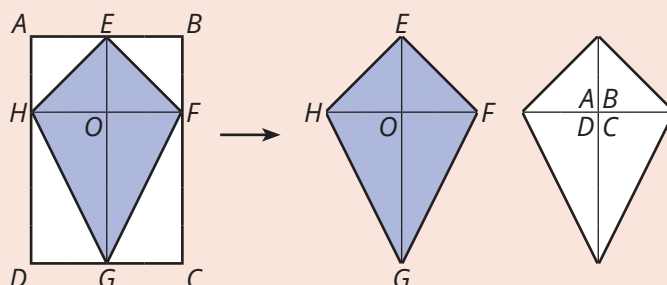
$EFGH$ has the same area as rectangle $EBCG$, so the area of $EFGH$ is 12 square centimetres.

Strategy 2: Divide a Complex Shape

We can rearrange the parts of $ABCD$ that are not covered by $EFGH$, to form a second kite that is the same size as $EFGH$.

Since both kites are the same size, they must each be half of the area of $ABCD$.

The area of kite $EFGH$ is $24 \div 2 = 12 \text{ cm}^2$.

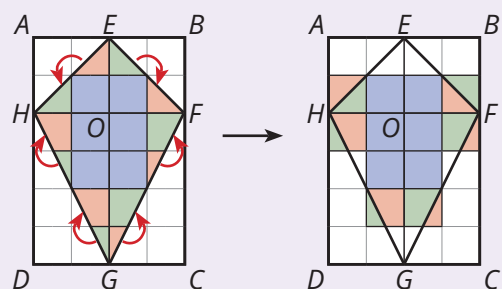


Strategy 3: Guess, Check and Convert to a More Convenient Form

Since $ABCD$ has an area of 24 square centimetres, we can guess some convenient measurements - e.g. 4 cm wide and 6 cm high.

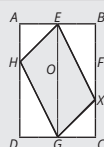
We can use these measurements to overlay a grid that comprises 24 squares.

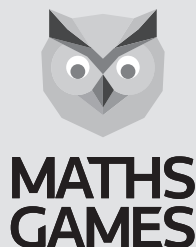
The area of kite $EFGH$ can then be rearranged to make up a total of 12 square centimetres.



Follow-Up: Suppose we mark a new point X on BC .

What is the area of quadrilateral $EXGH$? [12 cm^2]





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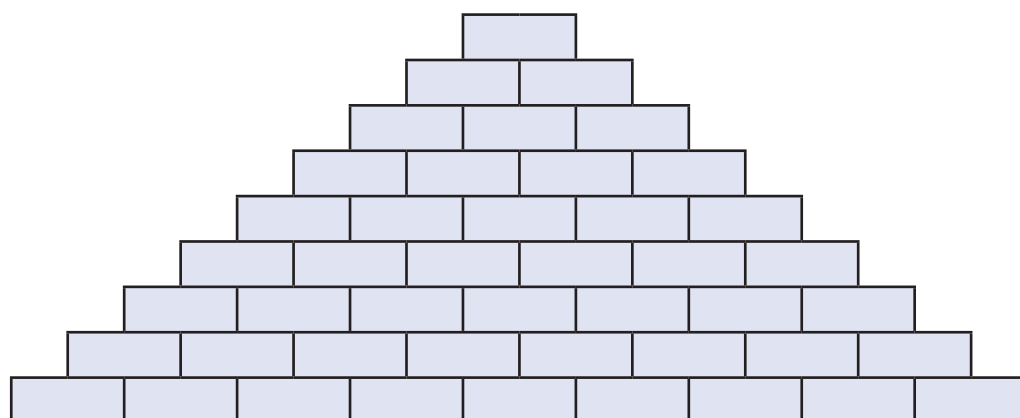
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4C. The question is, How many boxes are in the stack?

Strategy: Convert to a More Convenient Form

We can draw the stack of boxes, or use a table to list the number of boxes in each row.



| Row | Boxes |
|-----|-------|
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |
| 5 | 5 |
| 6 | 6 |
| 7 | 7 |
| 8 | 8 |
| 9 | 9 |

To find the total number of boxes, we can add up the number of boxes in each row.

Method 1: Group the numbers.

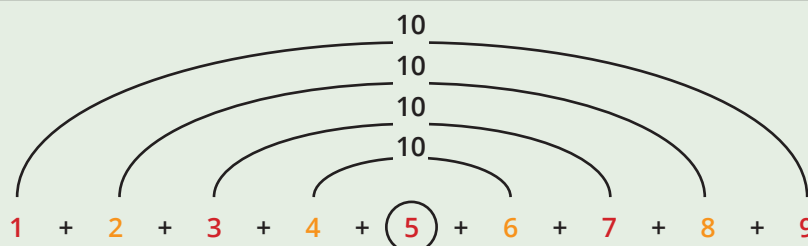
The numbers can be grouped to make it easier to add.

$$(1 + 9) + (2 + 8) + (3 + 7) + (4 + 6) + (5)$$

$$= 10 + 10 + 10 + 10 + 5$$

$$= 45.$$

Cooper's stack has 45 boxes.



Method 2: Duplicate the stack.

To find the number of boxes in the stack, we can make a copy of the stack and reverse the order of the rows, like this:

$$\text{Stack 1: } 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$$

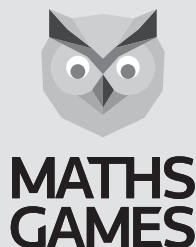
$$\text{Stack 2: } 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$$

All together, the two stacks would have $9 \times 10 = 90$ boxes, so one stack of boxes would have $90 \div 2 = 45$ boxes.

| | | | | | | | | | | | | | | | | |
|----|---|----|---|----|---|----|---|----|---|----|---|----|---|----|---|----|
| 9 | + | 8 | + | 7 | + | 6 | + | 5 | + | 4 | + | 3 | + | 2 | + | 1 |
| 1 | + | 2 | + | 3 | + | 4 | + | 5 | + | 6 | + | 7 | + | 8 | + | 9 |
| 10 | + | 10 | + | 10 | + | 10 | + | 10 | + | 10 | + | 10 | + | 10 | + | 10 |

9×10

Follow-Up: How many boxes would be in Cooper's stack if there were 12 boxes on the bottom row, and 1 box on the top row? [78]



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4D. The question is, What is the perimeter of the figure, in centimetres?

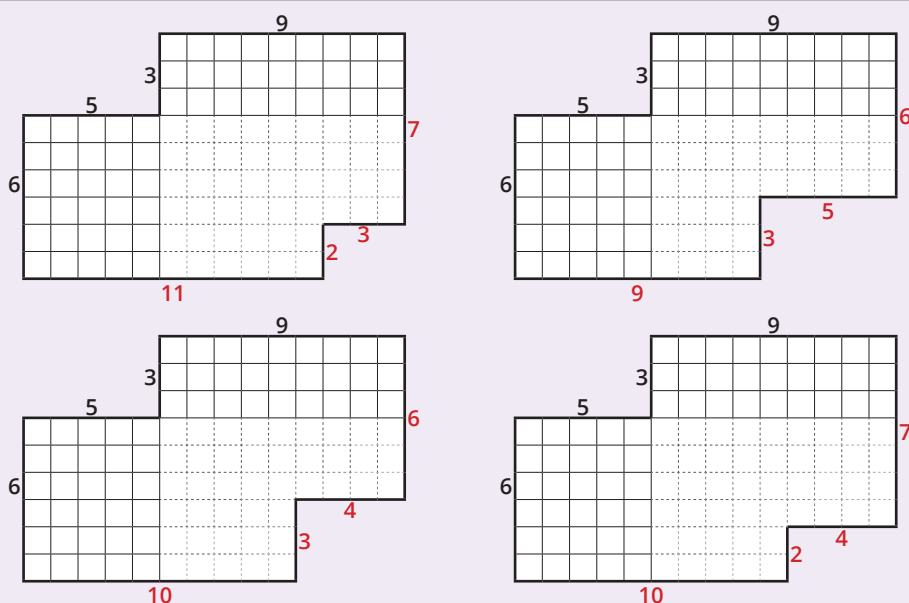
Strategy 1: Divide a Complex Shape, and Guess, Check and Refine

We can begin by sketching in a 1 cm grid pattern to divide the shape, based on the measurements that we know.

By extending the grid pattern to the other side, we can "guess" at the unknown measurements.

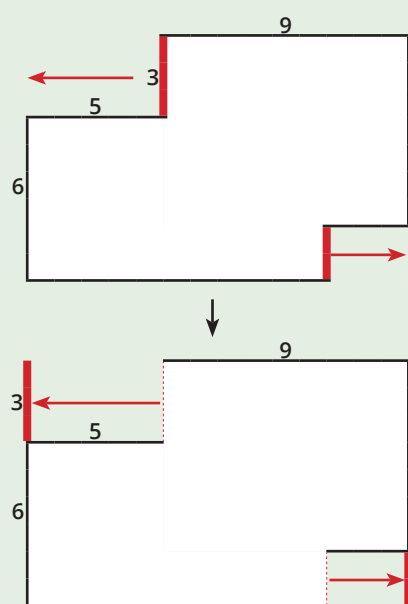
Four possible sets of measurements are shown at the right.

In each case, we can add all of the side lengths, and find that the perimeter is **46 centimetres**.

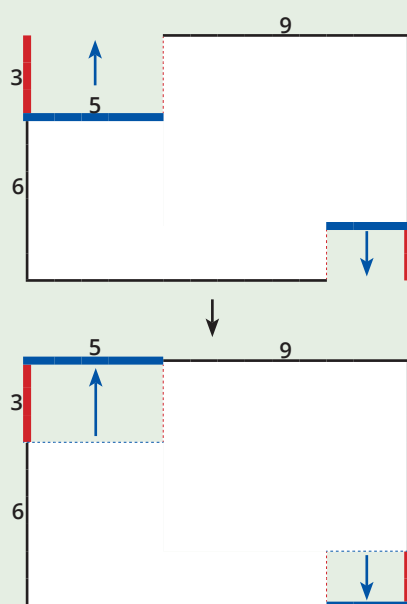


Strategy 2: Convert to a More Convenient Form

We can move the vertical sides of the shape outwards so that they are in a line.



Likewise, we can move the horizontal sides of the shape outwards so that they are in a line.



The sides can be rearranged to form a rectangle with side lengths $5 + 9 = 14\text{ cm}$, and $3 + 6 = 9\text{ cm}$.

The perimeter of the rectangle is $14 + 9 + 14 + 9 = 46\text{ cm}$.

Since the rectangle was constructed using the sides from the original figure, both shapes will have the same perimeter.

Therefore the perimeter of the figure is **46 cm**.

Follow-Up: There are two horizontal sides for which we do not have measurements. Suppose one of these two measurements is 10.5 cm. What would the other measurement be? [3.5 cm]



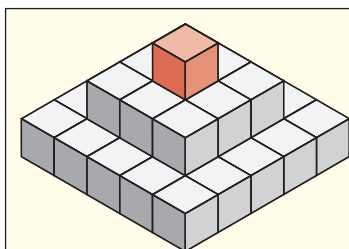
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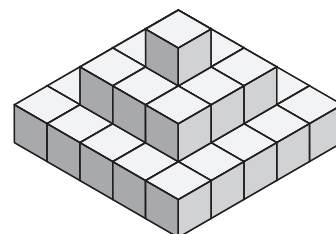
4E. The question is, How many of the cubes are painted on exactly two faces?

Strategy 1: Divide a Complex Shape

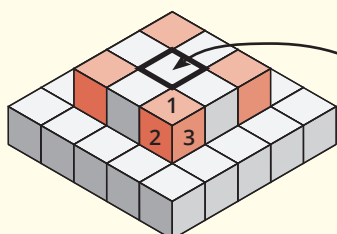


The 1 top-most cube has one exposed horizontal face on top, and four exposed vertical faces.

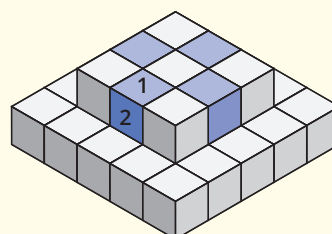
When the pyramid is painted, the top-most cube will have five faces painted.



In the second layer, each of the 4 corner cubes will have three faces painted.

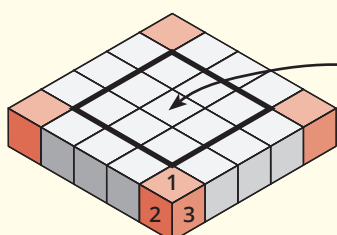


The centre cube will have no faces painted.

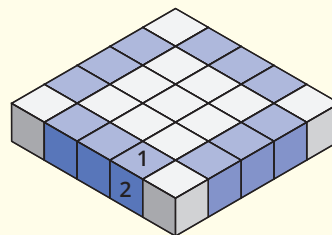


The 4 edge cubes will each have two faces painted.

In the third layer, each of the 4 corner cubes will have three faces painted.



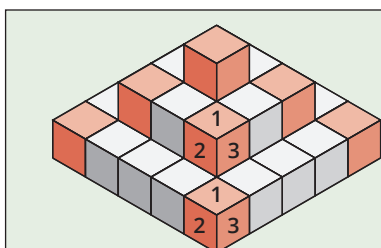
The 9 centre cubes will have no faces painted.



The 12 edge cubes will each have two faces painted.

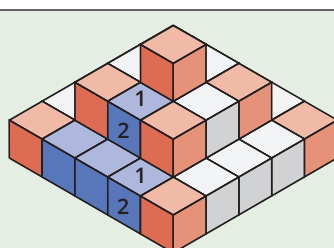
There would be $4 + 12 = 16$ cubes with two faces painted.

Strategy 2: Find a Pattern, and Divide a Complex Shape



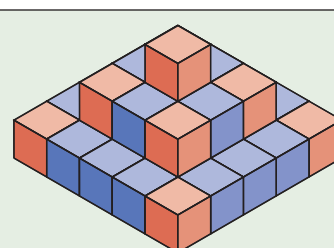
Each of the corner cubes has at least three exposed faces - one horizontal face on top, and at least two vertical faces.

When the object is painted, each of these cubes will have paint on at least three faces.



On the front-left side of the pyramid as seen from this angle, there are 4 cubes that each have two exposed faces.

Each of these cubes will have paint on exactly two faces.



This pattern of 4 cubes occurs 4 times as the object is rotated through one complete revolution.

There would be $4 \times 4 = 16$ cubes with two faces painted.

Follow-Up: If the pyramid had been 5 layers high, how many cubes would have three faces painted? [16]