APSMO

2023 MATHS GAMES

Important

The information contained in this file is ONLY for the use of registered participants of the 2023 APSMO Maths Games.

Organisation and Procedures

For full details, see the Members' Area

- Maths Games papers are to be conducted under test conditions.

DO	DO NOT
• Supervise students at all times. • Maintain silence. • Provide blank working paper. • Collect, mark and retain the papers.	• Print the papers prior to the scheduled date. • Read the questions aloud to the students. • Interpret the questions for students. • Permit any discussion or movement around the room. • Permit the use of calculators or other electronic devices.

- Papers should be scored by the PICO using the *Solutions and Answers* sheet provided.
- Original student answer sheets should be retained by the PICO until the end of the year.

Absent Students

- A student who is legitimately absent on the date of the Maths Games paper, may sit the paper on their return to school.
- If an absent student does not sit the paper on their return to school they should be marked as 'absent'.
- *Note: This policy differs from the Maths Olympiads Absent Student Policy which has additional requirements*.

APSMO MATHS GAMES

WEDNESDAY 26 JULY 2023

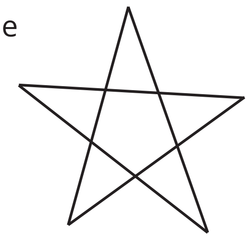
MATHS GAMES JUNIOR 3

Suggested Time: ***30 Minutes***

Write your answers in the boxes on the back.

Keep your answers hidden by folding backwards on this line. ←

3A. How many different triangles can be traced on the lines in this diagram?



Hint: You can make copies of the diagram, to help keep track of the triangles you find.

3B. Abby, Bron, Caleb, Dimity and Eli all have their birthday on February 29, a date which only occurs every four years.

When they met up for a joint birthday party, their ages were 4, 8, 12, 16 and 20.

Dimity was twice Bron's age.

Abby's age was the sum of Dimity's age and Caleb's age.

If Caleb was 8 years younger than Eli, how old was Caleb?

Hint: What values are possible for Dimity's age?

3C. In this magic square, the odd numbers from 1 to 17 are placed in the boxes so that the sum of the numbers in every row, column and diagonal is 27.

What number goes in the box that looks like this: □

		11
5		
	□	3

Hint: Is there a row, column, or diagonal that is nearly complete?

3D. Lucy has four t-shirts: blue, green, red, and purple.

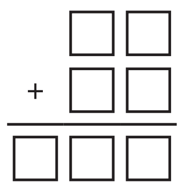
She is packing three t-shirts for a short stay at her grandfather's house.

How many different combinations of three t-shirts are there?

Hint: You could try counting in an organised way.

3E. In this correctly solved addition problem, the squares contain the following digit tiles:

0 1 2 3 4 5 6



What is the three-digit sum?

Hint: When adding two two-digit numbers, what is the greatest possible sum?

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JUNIOR
3

Student Name:

3A.

3B.

3C.

3D.

3E.

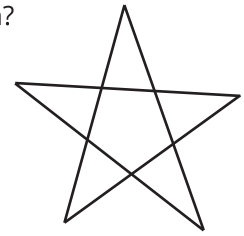
Fold here. Keep your answers hidden.

Solutions and Answers

(Items in parentheses are not required)

3A: 10	3B: 4	3C: 17	3D: 4	3E: 105

3A. The question is, How many different triangles can be traced on the lines in this diagram?

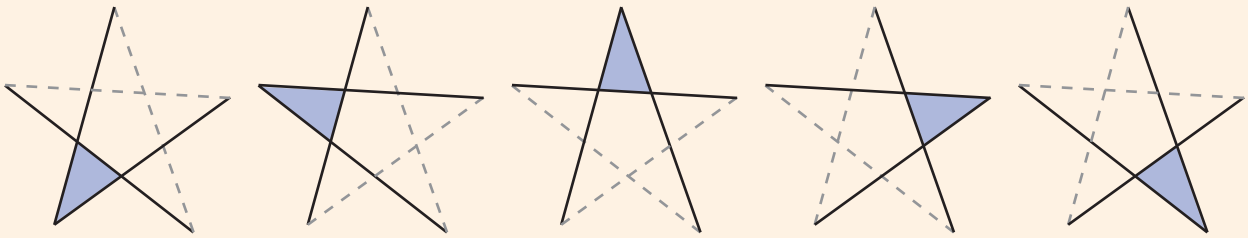


***Strategy:* Solve a Simpler Related Problem, and Make an Organised List**

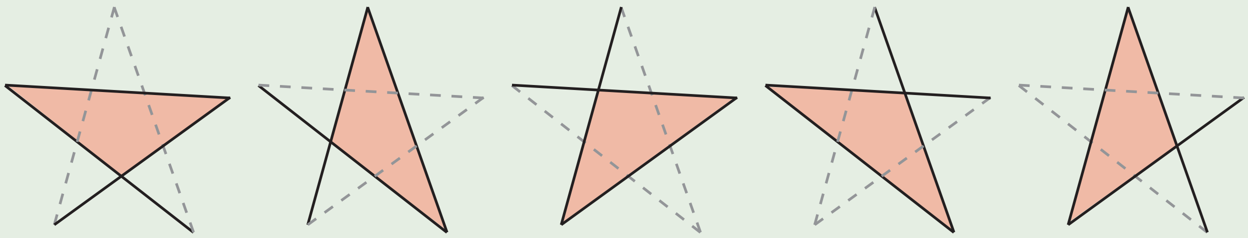
There are five lines in the diagram.

Since a triangle is made up of three lines, let's see what happens if we remove two of the lines.

If we remove two lines that cross over each other, one triangle remains.



If we remove two lines that meet at a point, one triangle remains.

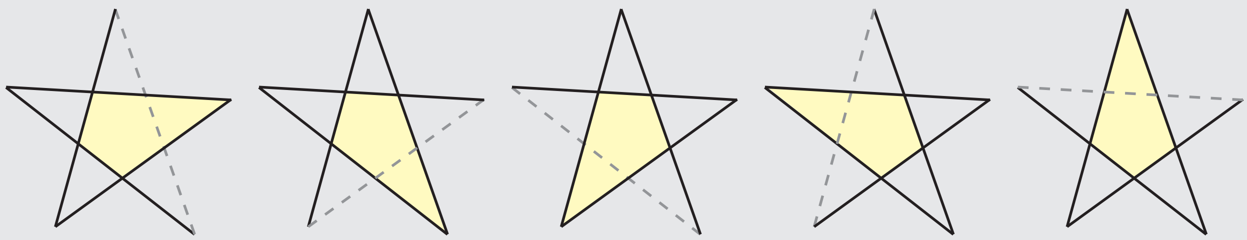


There are no two lines in this diagram that do not either:

- cross over each other, or
- meet at a point.

There are **10** different triangles that can be traced on the lines in this diagram.

Follow-Up: *How many different quadrilaterals can be traced on the lines in this diagram?* [5]



3B. The question is, How old was Caleb?

Strategy: **Eliminate All But One Possibility**

We have the following information:

- At their last birthday party, their ages were **4**, **8**, **12**, **16** and **20**.
- Dimity was twice Bron's age.
- Abby's age was the sum of Dimity's age and Caleb's age.
- Eli was eight years older than Caleb.

We can begin by finding the possible combinations for Dimity's age and Bron's age.

Bron's Age	4	8	12	16	20
Dimity's Age	8	16	24	32	40

Dimity cannot be **24**, **32**, or **40** years old, as these ages are not amongst the options.

This means that either:

- Bron was **4** and Dimity was **8**, or
- Bron was **8** and Dimity was **16**.

Bron's Age	4	8	12	16	20
Dimity's Age	8	16	24	32	40

Suppose Bron was **4** years old, and Dimity was **8**.

By listing possible values for Caleb's age, we can find the possible values for Abby's age.

Bron's Age	4	4	4	4	4
Dimity's Age	8	8	8	8	8
Caleb's Age	4	8	12	16	20
Abby's Age	12	16	20	24	28

In this scenario:

- Caleb can't be **4**, because Bron is **4**.
- Caleb can't be **8**, because Dimity is **8**.
- Abby can't be **24** or **28**.

Only one option remains.

Bron's Age	4	4	4	4	4
Dimity's Age	8	8	8	8	8
Caleb's Age	4	8	12	16	20
Abby's Age	12	16	20	24	28

Suppose Bron was **8** years old, and Dimity was **16**.

In this scenario, there is only one solution that makes sense for Abby's age.

Bron's Age	8	8	8	8	8
Dimity's Age	16	16	16	16	16
Caleb's Age	4	8	12	16	20
Abby's Age	20	24	28	32	36

Finally, we know that Eli was **8** years older than Caleb.

Only one of the two possible solutions has five different ages for the five people.

Caleb is **4** years old.

Bron's Age	4
Dimity's Age	8
Caleb's Age	12
Abby's Age	20
Eli's Age	20

Bron's Age	8
Dimity's Age	16
Caleb's Age	4
Abby's Age	20
Eli's Age	12

Follow-Up: *How old will Caleb be when he is half of Dimity's age?* [Caleb will be 12 years old when Dimity is 24 years old]

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3C. The question is, What number goes in the box that looks like this: □

		11
5		
	□	3

Strategy: **Eliminate All But One Possibility**

- The boxes contain the numbers **1**, **3**, **5**, **7**, **9**, **11**, **13**, **15** and **17**.
- Every row, column and diagonal adds up to give the answer **27**.

The sum of the three numbers in the right column is **27**.

We know that two of those values are **11** and **3**.

11 + □ + 3 = 27

11 + 13 + 3 = 27

		11
5		
	□	3

↓

		11
5		13
	□	3

There is now enough information to complete the middle row.

5 + □ + 13 = 27

5 + 9 + 13 = 27

		11
5		13
	□	3

↓

		11
5	9	13
	□	3

Finding the number in the centre gives us enough information to complete the left-right diagonal.

□ + 9 + 3 = 27

15 + 9 + 3 = 27

		11
5	9	13
	□	3

↓

15		11
5	9	13
	□	3

At this point, a number of different options are available.

Option 1: Complete the top row,

15 + 1 + 11 = 27

then the middle column.

1 + 9 + 17 = 27

15		11
5	9	13
	□	3

→

15	1	11
5	9	13
	□	3

→

15	1	11
5	9	13
	17	3

Option 2: Complete the left column,

15 + 5 + 7 = 27

then the bottom row.

7 + 17 + 3 = 27

15		11
5	9	13
	□	3

→

15		11
5	9	13
7	□	3

→

15		11
5	9	13
7	17	3

Option 3: Complete the right-left diagonal,

11 + 9 + 7 = 27

then the bottom row:

7 + 17 + 3 = 27

15		11
5	9	13
	□	3

→

15		11
5	9	13
7	□	3

→

15		11
5	9	13
7	17	3

The number that goes in the box that looks like this: □ is **17**.

Follow-Up: *This is a normal magic square. It uses the values 1 - 9, and every row, column, and diagonal adds up to 15.*
Can you make another normal magic square? How many different normal magic squares can you find?
[There are 8 normal magic squares; they are all flips and rotations of the example on the right.]

4	9	2
3	5	7
8	1	6

3D. The question is, How many different combinations of three t-shirts are there?

Strategy 1: **Solve a Simpler Related Problem, and Make an Organised List**

If Lucy only had to select one t-shirt, then there would be **4** ways to select that t-shirt.

B	G	P	R

To select two t-shirts, Lucy could start by selecting one shirt. As noted earlier, there are **4** ways to select one t-shirt.

There are **3** remaining t-shirts, so there are **3** ways to pick the second t-shirt.

There are **4 × 3 = 12** combinations in our list.

1st t-shirt	B	G	P	R
1st t-shirt combined with 2nd t-shirt	B G	G B	P B	R B
	B P	G P	P G	R G
	B R	G R	P R	R P

However, we can see that selecting Blue and Green has the same effect as selecting Green and Blue. In our list, each pair of t-shirts is counted twice. To indicate this, the duplicates are shaded in grey.

There are **12 ÷ 2 = 6** ways to select two t-shirts.

To select three t-shirts, Lucy can start with one of the **6** combinations of two t-shirts.

There are **2** remaining t-shirts, so there are **2** ways to pick the third t-shirt.

1st & 2nd t-shirts	B G	B P	B R	G P	G R	P R
combined with 3rd t-shirt	B G P	B P G	B R G	G P B	G R B	P R B
	B G R	B P R	B R P	G P R	G R P	P R G

Our list contains **2 × 6 = 12** combinations of three t-shirts.

However, we can see that B G P, B P G and G P B all refer to the same set of three t-shirts. In our list, each set of three t-shirts is counted three times.

There are **12 ÷ 3 = 4** ways to select three t-shirts.

Strategy 2: **Solve a Simpler Related Problem**

We'll begin, once again, with Lucy selecting just one t-shirt.

B	G	P	R

There would be **4** ways to select that t-shirt.

However, this time, let's suppose that Lucy puts that t-shirt back into her wardrobe.

She then packs the other three t-shirts.

With **4** different ways to select the shirt that she does not pack, there are **4** different ways to select the three shirts to be packed for Lucy's stay at her grandfather's house.

Follow-Up: *How many ways are there to select three t-shirts if Lucy owns 5 t-shirts?* [10 ways]

3E. The question is, What is the three-digit sum?

The digits we have available for use are: 0 1 2 3 4 5 6

□□ + □□ = □□□

Strategy: **Eliminate All But One Possibility**

We can start by trying to make the greatest possible sum.

Our highest-valued digits are 6 and 5, so we'll use them both in the tens place.

The next-highest digits are 4 and 3.

Using the given digits, the greatest sum we can make is 64 + 53 = 117.

64 + 53 = 117

Clearly, 117 is not the solution, because it uses two 1s.

However, by trying to find the greatest possible sum, we have worked out that the greatest possible value for the hundreds place is 1.

We now know that the sum must have 1 in the hundreds place. (If the sum had a leading 0, then it would not be a 3-digit number; in any case, there is no solution with 0 in the hundreds place.)

□□ + □□ = 1□□

With only one 1 and a sum that cannot exceed 117, the tens digit of the sum must be less than 1.

This means that the tens digit of the sum must be 0.

□□ + □□ = 10□

We now know that the tens digits of the two addends must either:

- add up to 10, or
- add up to 9, with regrouping from the ones place.

For regrouping to occur, the ones digits of the two addends must have a sum greater than 9.

□6 + □4 = 100   □6 + □5 = 101

By trying out both of the possible combinations, we can see that, if the sum of the ones digits was going to be greater than 9, then the ones digit of the sum will be either 0 or 1 – neither of which is possible (because we have already used our 0 and 1 tiles).

This means that the tens digits of the two addends must add up to 10.

The only two digits that can make this work are 6 and 4.

Note that it does not matter which digit is used for which addend.

6□ + 4□ = 10□

We're left with the digits 2, 3 and 5 to fill in the ones place values.

Compared to our earlier reasoning, this is a relatively straightfoward exercise.

The three-digit sum is **105**.

62 + 43 = 105

Follow-Up: *Suppose the digit tiles provided for this problem had been 1, 2, 3, 4, 5, 6, 8. What sums would be possible?*

[62 + 83 = 145, and 52 + 84 = 136]