Important

The information contained in this file is ONLY for the use of registered participants of the 2023 APSMO Maths Games.

Organisation and Procedures

For full details, see the Members' Area

- Maths Games papers are to be conducted under test conditions.

DO	DO NOT
• Supervise students at all times. • Maintain silence. • Provide blank working paper. • Collect, mark and retain the papers.	• Print the papers prior to the scheduled date. • Read the questions aloud to the students. • Interpret the questions for students. • Permit any discussion or movement around the room. • Permit the use of calculators or other electronic devices.

- Papers should be scored by the PICO using the *Solutions and Answers* sheet provided.
- Original student answer sheets should be retained by the PICO until the end of the year.

Absent Students

- A student who is legitimately absent on the date of the Maths Games paper, may sit the paper on their return to school.
- If an absent student does not sit the paper on their return to school they should be marked as 'absent'.
- *Note: This policy differs from the Maths Olympiads Absent Student Policy which has additional requirements*.

APSMO

WEDNESDAY 14 JUNE 2023

MATHS GAMES

MATHS GAMES JUNIOR 2

Suggested Time: ***30 Minutes***

Write your answers in the boxes on the back.

←

Keep your answers hidden by folding backwards on this line.

2A. Simon has one more book than Tilly.

Tilly has four more books than Roger.

If all three students shared their books equally, they would have eight books each.

How many books does Tilly have?

Hint: You could guess a number of books for Tilly, and see if it works.

2B. Jimmy's pencil is $\frac{3}{7}$ of the length of a new pencil.

A new pencil is 8 cm longer than Jimmy's pencil.

How long is Jimmy's pencil, in centimetres?

Hint: You could draw a diagram showing both pencils. How long would Jimmy's pencil be, compared to the new pencil?

2C. A bakery makes fruit tarts with strawberries, blueberries, kiwi fruit and peaches.

Each tart has exactly two different types of fruit on it.

How many different types of tart are possible?

Hint: You can list different types of tart in an organised way.

2D. Carl uses a \$50 note to buy some sheet music and receives \$21 change.

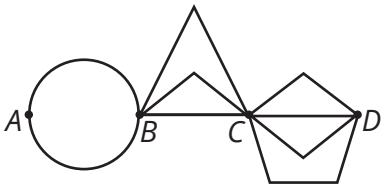
He then uses a \$20 note to buy some clarinet reeds and receives \$3 change.

If Carl now has \$54, how much money did he have before buying the sheet music and the reeds?

Hint: Try working backwards from the \$54 that Carl has at the end.

2E. Following only the paths shown, what is the number of different paths that go from *A* to *B* to *C* to *D* ?

You must touch each of those points exactly once.



Hint: How many different paths are there from *A* to *B*?

APSMO

WEDNESDAY 14 JUNE 2023

MATHS GAMES

JUNIOR

2

2A.

2B.

2C.

2D.

2E.

Student Name:

Fold here. Keep your answers hidden.

APSMO

WEDNESDAY 14 JUNE 2023

MATHS GAMES

JUNIOR 2

Solutions and Answers

(Items in parentheses are not required)

2A: 9	**2B:** 6 (cm)	**2C:** 6	**2D:** $100	**2E:** 24

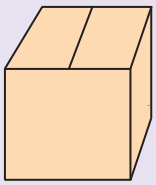
2A. The question is, How many books did Tilly start off with?

Strategy 1: **Work Backwards**

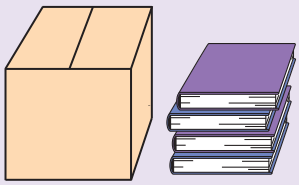
If Simon, Tilly and Roger shared their books equally, they would have **8** books each.

In total, they must have **8 × 3 = 24** books.

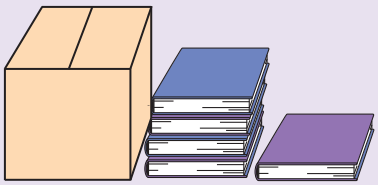
Let's suppose that Roger's books fit exactly in one box.



Tilly has 4 more books than Roger.



Simon has 1 more book than Tilly.

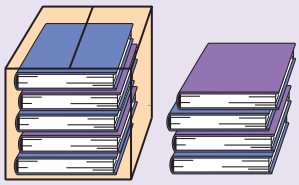


All together, Roger, Tilly and Simon have three boxes of books, plus another **4 + 5 = 9** books.

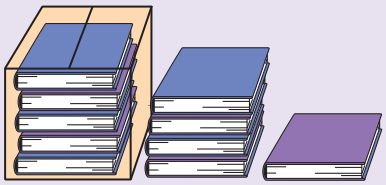
With **24** books in total, there must be **24 – 9 = 15** books in those three boxes.

Roger's books

Tilly's books



Simon's books



Each box would then contain **15 ÷ 3 = 5** books.

Tilly must have **5 + 4 = 9** books.

Strategy 2: **Build a Table, and Find a Pattern**

Let's build a table with possible numbers of books for Roger, Tilly and Simon.

We know that Tilly has **4** more than Roger, and Simon has **1** more than Tilly.

If Roger has **5** books, Tilly will have **9** books, and Simon will have **10** books, for a total of **24** books all together.

If those **24** books were shared equally, each student would have **8** books.

We can see that Tilly has **9** books.

Roger	Tilly	Simon	Total	Equal shares
1	5	6	12	4
2	6	7	15	5
3	7	8	18	6
4	8	9	21	7
5	9	10	24	8

Follow-Up: *If Roger, Tilly, Simon and Vicky all shared their books, they would have 7 books each. How many books does Vicky have?* [4]

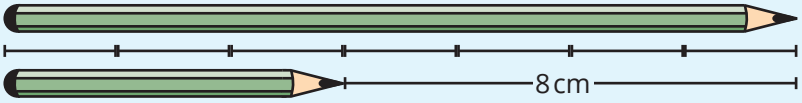
2B. The question is, How long is Jimmy's pencil, in centimetres?

Strategy 1: **Draw a Diagram, and Work Backwards**

We'll begin by dividing a new pencil up into **7** equal lengths.

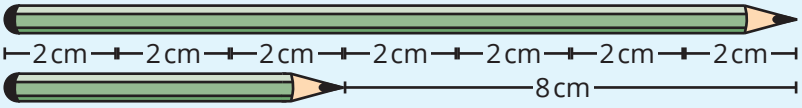
Jimmy's pencil is $\frac{3}{7}$ of the length of the new pencil.

The new pencil is 8 cm longer than Jimmy's pencil.

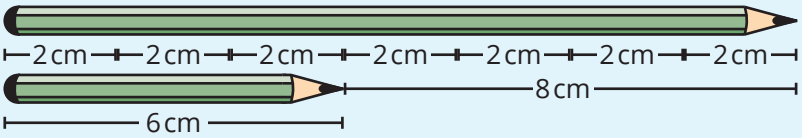


From the diagram, we can now see that **4** lengths is equal to **8 cm**.

This means that **1** length must be equal to **8 cm ÷ 4 = 2 cm**.



Therefore, Jimmy's pencil is **2 cm + 2 cm + 2 cm = 6 cm** long.



Strategy 2: **Guess, Check and Refine**

We know that Jimmy's pencil is is $\frac{3}{7}$ of the length of a new pencil.

Let's guess different multiples of **7**, to see if they make sense for the length of a new pencil.

Suppose a new pencil is **7** cm long.

- One-seventh of the length is **7 cm ÷ 7 = 1 cm**.
- Jimmy's pencil is $\frac{3}{7}$ of the length. **3 × 1 cm = 3 cm**.
- The difference between a new pencil and Jimmy's pencil would be **7 cm – 3 cm = 4 cm**.

The difference is too small. A new pencil is supposed to be **8 cm** longer than Jimmy's pencil.

Suppose a new pencil is **2 × 7 cm = 14 cm** long.

- One-seventh of the length is **14 cm ÷ 7 = 2 cm**.
- Jimmy's pencil is $\frac{3}{7}$ of the length. **3 × 2 cm = 6 cm**.
- The difference between a new pencil and Jimmy's pencil would be **14 cm – 6 cm = 8 cm**.

That matches our question.

Jimmy's pencil is **6 cm** long.

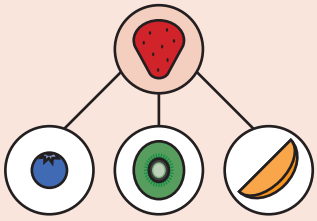
Follow-Up: *Jimmy buys a box of new pencils. Laid end to end, his pencils have a total length of 90 cm. How many pencils were there in the box of new pencils?* [6: 6 × 14 cm + 6 cm = 90 cm]

2C. The question is, How many different types of tart are possible?

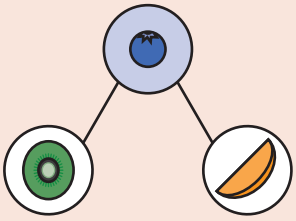
Strategy 1: **Make an Organised List**

Each tart has exactly two different types of fruit.

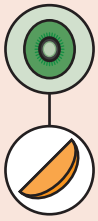
Strawberries can be paired once with each of the other three fruits.



Blueberries can be paired once with each of the remaining two.



That just leaves kiwi fruit to be paired with peaches.



We have:

- 🍓-🫐, 🍓-🥝, 🍓-🍑 (3 types)
- 🫐-🥝, 🫐-🍑 (2 types)
- 🥝-🍑 (1 type)

There are 3 + 2 + 1 = **6** possible different types of tart.

Strategy 2: **Build a Table, or Draw a Diagram**

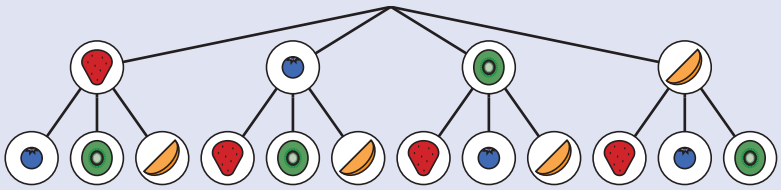
Each fruit is used in three different pairings.

For example,

- Strawberries could be paired with blueberries, kiwi fruit or peaches.
- Kiwi fruit could be paired with strawberries, blueberries or peaches.

These combinations can be represented in a table, or in a tree diagram.

	Strawberry	Blueberry	Kiwi fruit	Peach
Strawberry		*S-B*	*S-K*	*S-P*
Blueberry	*B-S*		*B-K*	*B-P*
Kiwi fruit	*K-S*	*K-B*		*K-P*
Peach	*P-S*	*P-B*	*P-K*	



There are **12** pairs of fruits.

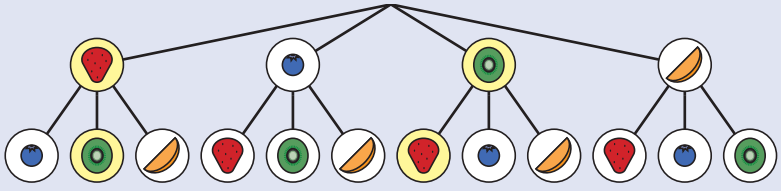
However, we can see that some of the options are effectively the same as others.

For example, the combination strawberries-kiwi fruit is the same as kiwi fruit-strawberries.

Every pairing in the table is counted twice.

In total, there are **12** ÷ **2** = **6** possible different types of tart.

	Strawberry	Blueberry	Kiwi fruit	Peach
Strawberry		*S-B*	*S-K*	*S-P*
Blueberry	*B-S*		*B-K*	*B-P*
Kiwi fruit	*K-S*	*K-B*		*K-P*
Peach	*P-S*	*P-B*	*P-K*	

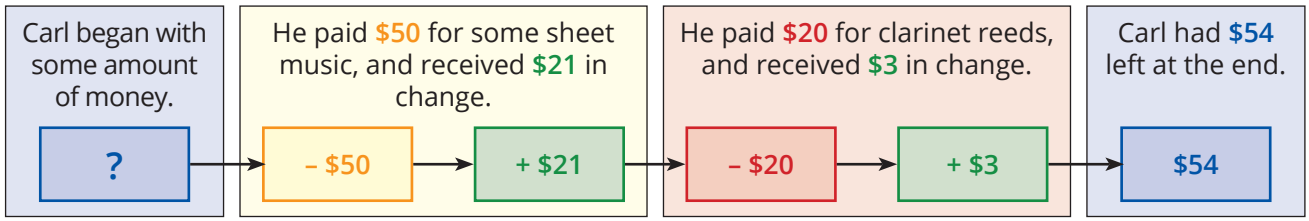


Follow-Up: *If the bakery made every possible combination (1, 2, 3 or 4 fruits), how many tarts would be possible?* [4 + 6 + 4 + 1 = 15]

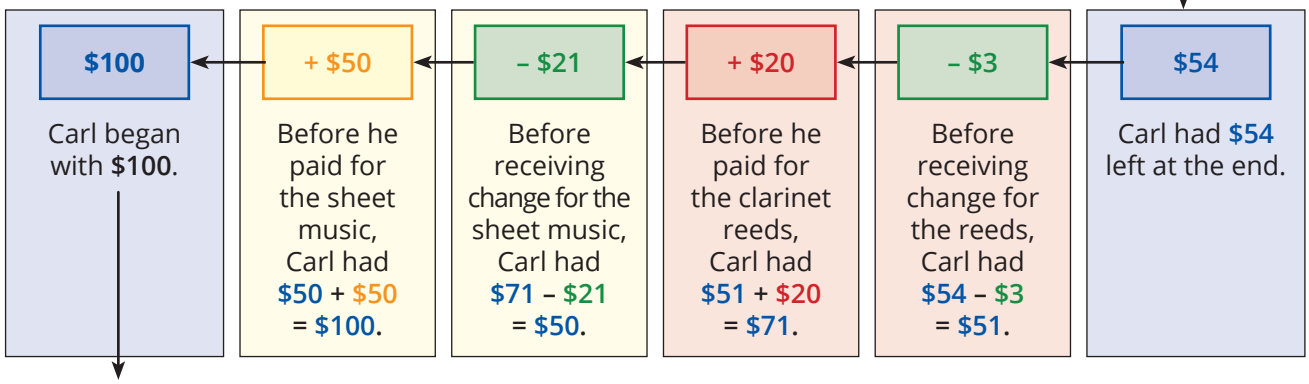
2D. The question is, How much money did Carl have before buying the sheet music and the reeds?

Strategy 1: **Work Backwards (1)**

We can follow all of Carl's transactions, and then work backwards to see how much he started with.



Now, we can work backwards to find the amount that Carl began with.



Carl began with **$100**.

Strategy 2: **Work Backwards (2)**

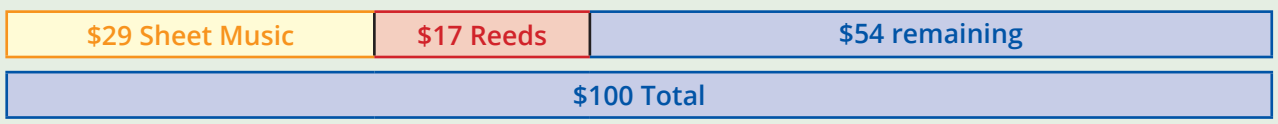
Carl uses a $50 note to buy some sheet music, and receives $21 change.

The sheet music must have cost $50 – $21 = $29.

He then uses a $20 note to buy some clarinet reeds, and receives $3 change.

The clarinet reeds must have cost $20 – $3 = $17.

Carl has $54 remaining after spending $29 on sheet music, and $17 on clarinet reeds.



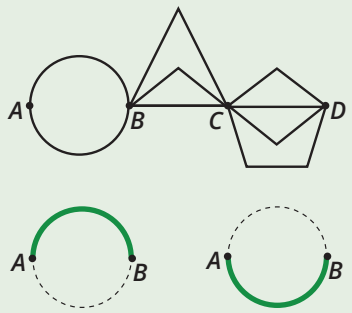
Before making his purchases, Carl must have had **$29 + $17 + $54 = $100**.

Follow-Up: *What is the smallest number of coins and notes that Carl can have after making his purchases? Possible denominations are $100, $50, $20, $10, $5, $2, $1, 50c, 20c, 10c, 5c.*
[4: If Carl started with two $50 notes, he could have a $50, $2, $1, $1 at the end.]

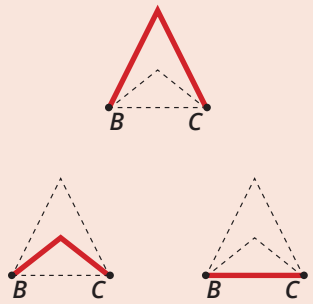
2E. The question is, What is the number of different paths that go from ***A*** to ***B*** to ***C*** to ***D***?

Strategy: **Make an Organised List**

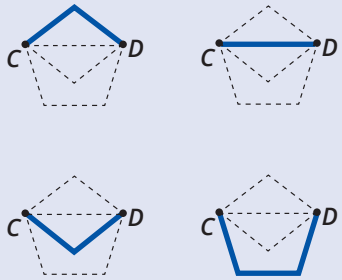
There are 2 paths from ***A*** to ***B***.



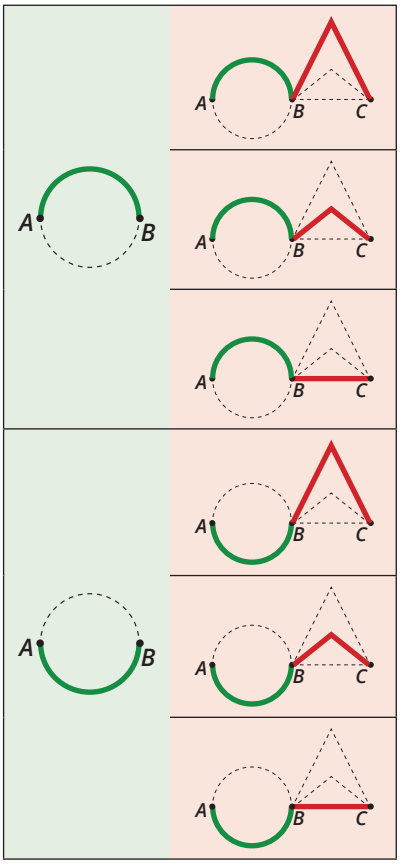
There are 3 paths from ***B*** to ***C***.



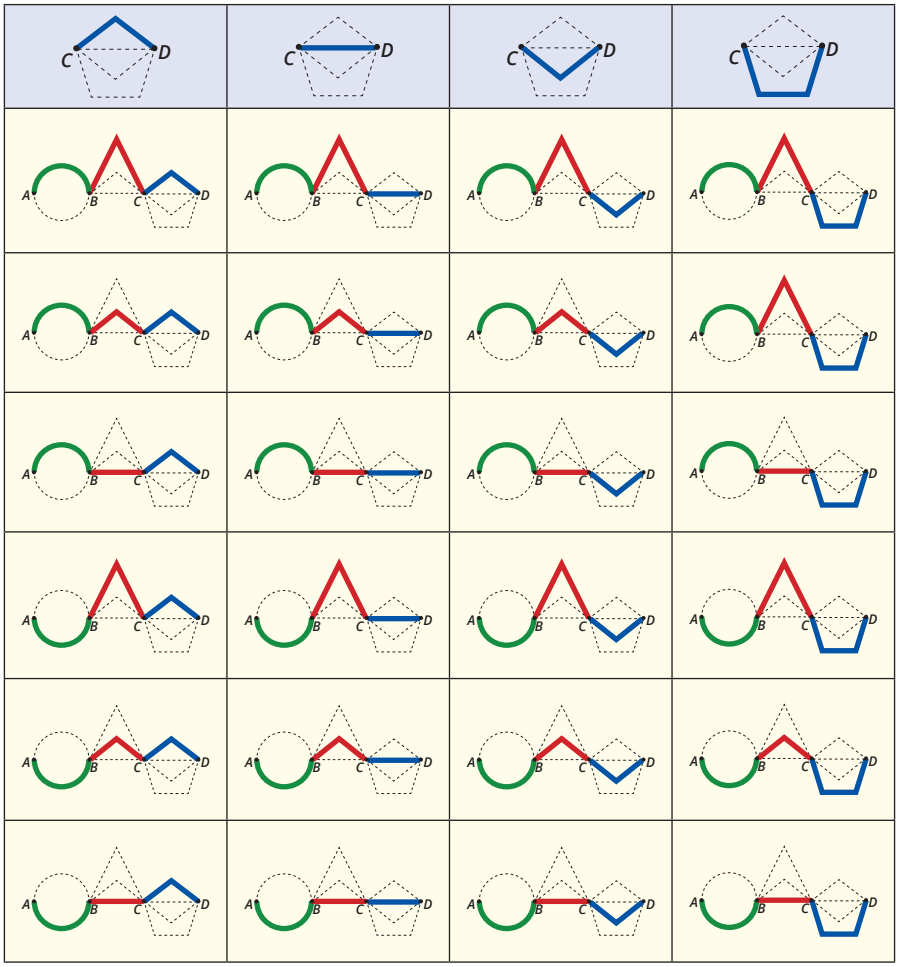
There are 4 paths from ***C*** to ***D***.



With 2 paths from ***A*** to ***B***, and 3 paths from ***B*** to ***C***, there are 2 × 3 = 6 paths from ***A*** to ***C***.



For each path from ***A*** to ***C***, there are 4 possible paths from ***C*** to ***D***.



There are 2 × 3 × 4 = **24** different paths from ***A*** to ***D***.

Follow-Up: *How many paths from A to D are possible if an extra "express" path is created that goes directly from A to C?* [28]