Important

The information contained in this file is ONLY for the use of registered participants of the 2023 APSMO Maths Games.

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APSMO
2023 MATHS GAMES

Organisation and Procedures

For full details, see the Members' Area

- Maths Games papers are to be conducted under test conditions.

DO	DO NOT
• Supervise students at all times. • Maintain silence. • Provide blank working paper. • Collect, mark and retain the papers.	• Print the papers prior to the scheduled date. • Read the questions aloud to the students. • Interpret the questions for students. • Permit any discussion or movement around the room. • Permit the use of calculators or other electronic devices.

- Papers should be scored by the PICO using the *Solutions and Answers* sheet provided.
- Original student answer sheets should be retained by the PICO until the end of the year.

Absent Students

- A student who is legitimately absent on the date of the Maths Games paper, may sit the paper on their return to school.
- If an absent student does not sit the paper on their return to school they should be marked as 'absent'.
- *Note: This policy differs from the Maths Olympiads Absent Student Policy which has additional requirements*.

MATHS GAMES

APSMO

WEDNESDAY 3 MAY 2023

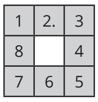
MATHS GAMES JUNIOR 1

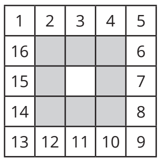
Suggested Time: ***30 Minutes***

Write your answers in the boxes on the back.

Keep your answers hidden by folding backwards on this line.

1A. Hugo is using square tiles that are all the same size.

He begins with one white tile,  and then surrounds it with a border of 8 grey tiles. 

He surrounds the grey tiles with a border of 16 white tiles. 

Hugo continues to alternate between grey and white borders.

How many tiles will Hugo need for the next white border?

Hint: You could draw more tiles around Hugo's pattern.

1B. Buying two bottles of water and a bottle of juice from a vending machine costs \$10.

Two bottles of juice and one bottle of water costs \$11.

How much does it cost to buy one bottle of water from this vending machine, in dollars?

Hint: Is a bottle of water more or less expensive than a bottle of juice?

1C. Jeremy and Kaleb are building a fence around a paddock.

They start at one corner and work around in opposite directions to each other.

Jeremy takes 30 minutes to build one metre of fence.

Kaleb takes 10 minutes to build one metre of fence.

The perimeter of the paddock is 80 metres long.

How many more metres of fence will Kaleb build than Jeremy?

Hint: How much of the fence will Jeremy build in one hour?

1D. 15 divided by 6 is 2 remainder 3.

In total, how many different counting numbers will leave a remainder of 3 when divided into 15?

Hint: You could build a table.

1E. In the next 16 days, there will be 3 Fridays.

How many Tuesdays were there in the past 38 days?

Hint: What day of the week might it be today?

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MATHS GAMES

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WEDNESDAY 3 MAY 2023

MATHS GAMES JUNIOR 1

1A.

Student Name:

1B.

1C.

1D.

1E.

Fold here. Keep your answers hidden.

Solutions and Answers

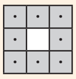
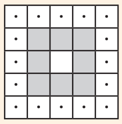
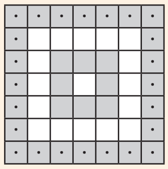
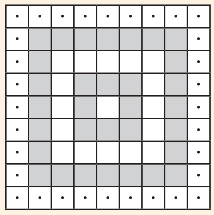
(Items in parentheses are not required)

1A: 32	**1B:** ($)3	**1C:** 40	**1D:** 3	**1E:** 6

1A. The question is, How many tiles will Hugo need for the next white border?

Strategy 1: **Build a Table, and Draw a Diagram**

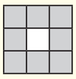
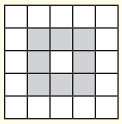
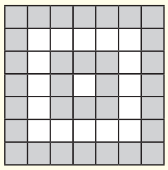
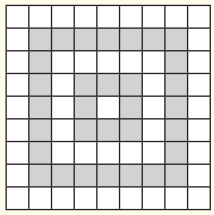
Let's use a table to record the number of tiles for each border.

Tile pattern				
Tiles in outside border	8	16	24	32

Hugo will need **32** tiles for the next white border.

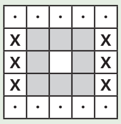
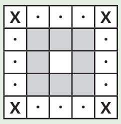
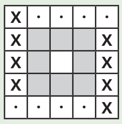
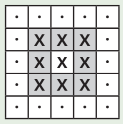
Strategy 2: **Build a Table, Draw a Diagram, and Find a Pattern**

It may help to consider the side length for each pattern of squares.

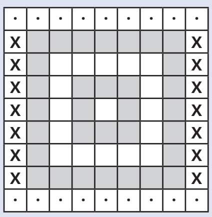
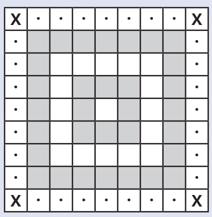
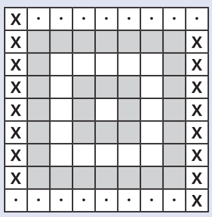
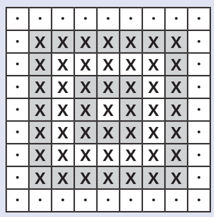
Tile pattern				
Side length of pattern	3	5	7	9

Every time Hugo adds another border, the side length increases by **2**.

We can then use many different methods to count the tiles in the outside border.
For example, for a side length of **5**:

			
5 + 3 + 5 + 3 = 16	4 × 3 + 4 = 16	4 × 4 = 16	(5 × 5) – (3 × 3) = 16

These patterns can then be applied to larger side lengths.
We want to find the number of tiles in the ouside border, for a side length of **9**.

			
9 + 7 + 9 + 7 = 32	4 × 7 + 4 = 32	4 × 8 = 32	(9 × 9) – (7 × 7) = 32

Regardless of the method we choose to use, we find that Hugo needs **32** tiles for the next white border.

Follow-Up: *How many tiles would there be in the white border after this one?* [48]

1B. The question is, How much does it cost to buy one bottle of water, in dollars?

***Strategy 1:* Build a Table, and Find a Pattern**

Let's guess that a bottle of water costs **$1**.
Since **2** bottles of water and **1** bottle of juice costs **$10**, **1** bottle of juice must cost **$10 – 2 × $1 = $8**.
If so, **2** bottles of juice and **1** bottle of water would cost **2 × $8 + $1 = $17**.

1 Water	1 Juice	2 Juice + 1 Water
$1	**$8**	**2 × $8 + $1 = $17**

If a bottle of water costs **$2**, **1** bottle of juice must cost **$10 – 2 × $2 = $6**.
2 bottles of juice and **1** bottle of water would cost **2 × $6 + $2 = $14**.

1 Water	1 Juice	2 Juice + 1 Water
$1	**$8**	**2 × $8 + $1 = $17**
$2	**$6**	**2 × $6 + $2 = $14**

Increasing the cost of the water by **$1** reduced the total cost for **2** bottles of juice + **1** bottle of water by **$3**.
We want to reduce the total cost for **2** bottles of juice + **1** bottle of water down by another **$3**, to **$11**.
Let's try increasing the cost of the water by another **$1**.

If a bottle of water costs **$3**, **1** bottle of juice must cost **$10 – 2 × $3 = $4**.
2 bottles of juice and **1** bottle of water would cost **2 × $4 + $3 = $11**.
That matches the question.

1 Water	1 Juice	2 Juice + 1 Water
$1	**$8**	**2 × $8 + $1 = $17**
$2	**$6**	**2 × $6 + $2 = $14**
$3	**$4**	**2 × $4 + $3 = $11**

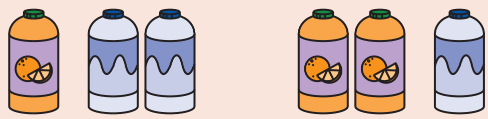
One bottle of water costs **$3**.

***Strategy 2:* Draw a Diagram and Reason Logically**

1 bottle of juice and **2** bottles of water costs **$10**.

2 bottles of juice and **1** bottle of water costs **$11**.

All together, **3** bottles of juice and **3** bottles of water would cost **$10 + $11 = $21**.



We can arrange **3** bottles of juice and **3** bottles of water into **3** equal groups, each containing **1** juice and **1** water.

If the **3** groups cost **$21** all together, one of the groups, with **1** juice and **1** water, must cost **$21 ÷ 3 = $7**.

1 bottle of juice and **2** bottles of water costs **$10**.

1 bottle of juice and **1** bottle of water costs **$7**.

Therefore, one bottle of water by itself must cost **$10 – $7 = $3**.

Follow-Up: *If it costs $24 for 2 bottles each of water, juice and soft drink, how much does 1 bottle of soft drink cost?* [$5]

1C. The question is, how many more metres of fence would Kaleb build than Jeremy?

***Strategy 1:* Build a Table, and Find a Pattern (1)**

Kaleb builds **1** metre of fence every **10** minutes.

Since there are **6** × **10** = **60** minutes in an hour, in one hour Kaleb would build **6** metres of fence.

Fence Length	1 m	1 m 1 m 1 m 1 m 1 m 1 m
Time	**10** mins	**6** × **10** = **60** mins (= **1** hour)

Jeremy builds **1** metre of fence every **30** minutes.

Since there are **2** × **30** = **60** minutes in an hour, in one hour Jeremy would build **2** metres of fence.

Fence Length	1 m	1 m 1 m
Time	**30** mins	**2** × **30** = **60** mins (= **1** hour)

Working together, in one hour Kaleb and Jeremy will build **6** metres + **2** metres = **8** metres of fence.

1 m 1 m 1 m 1 m 1 m 1 m 1 m 1 m

If Kaleb and Jeremy build **8** metres of fence in an hour, then in **10** hours they will build **10** × **8** = **80** metres of fence.

80 metres is the amount required to go around the paddock.

Time (hours)	1	2	3	...	10
Kaleb's Fence (m)	6	12	18	...	60
Jeremy's Fence (m)	2	4	6	...	20
Total Fence (m)	8	16	24	...	80

In **10** hours, Kaleb builds **60** metres of fence, and Jeremy builds **20** metres of fence.

So Kaleb would build **60** – **20** = **40** more metres of fence than Jeremy.

***Strategy 2:* Build a Table, and Find a Pattern (2)**

Jeremy builds **1** metre of fence in **30** minutes.

Kaleb builds **1** metre of fence in **10** minutes, so in **3** × **10** = **30** minutes he will build **3** × **1** = **3** metres of fence.

Jeremy's Fence (m)	1				
Kaleb's Fence (m)	3				
Total Fence (m)	4				
Difference (m)	2				

When Kaleb and Jeremy build **3** + **1** = **4** metres of fence together, Kaleb ends up building **3** – **1** = **2** metres more than Jeremy.

The difference is **half** of the total amount of fence built so far.

If we continue the table, we can see that the difference continues to be **half** of the total amount of fence built so far.

Why does this pattern occur?

Jeremy's Fence (m)	1	2	3	...	
Kaleb's Fence (m)	3	6	9	...	
Total Fence (m)	4	8	12	...	80
Difference (m)	2	4	6	...	40

When the **80** m fence is complete, Kaleb will have built **80** ÷ **2** = **40** metres more than Jeremy.

Follow-Up: *Jeremy and Kaleb agree to build half of the 120 m fence each. After he has finished his half, for how many hours does Kaleb need to wait until Jeremy has finished his half?* [20]

APSMO

MATHS GAMES

WEDNESDAY 3 MAY 2023

MATHS GAMES JUNIOR 1

1D. The question is, How many different counting numbers will leave a remainder of 3 when divided into 15?

***Strategy 1:* Build a Table**

We can try dividing **15** by every counting number that is less than, or equal to, **15**.

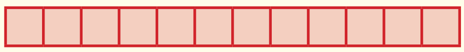
Divisor	Division and Remainder	Divisor	Division and Remainder	Divisor	Division and Remainder
1	15 ÷ 1 = 15 r. 0	6	15 ÷ 6 = 2 r. 3	11	15 ÷ 11 = 1 r. 4
2	15 ÷ 2 = 7 r. 1	7	15 ÷ 7 = 2 r. 1	12	15 ÷ 12 = 1 r. 3
3	15 ÷ 3 = 5 r. 0	8	15 ÷ 8 = 1 r. 7	13	15 ÷ 13 = 1 r. 2
4	15 ÷ 4 = 3 r. 3	9	15 ÷ 9 = 1 r. 6	14	15 ÷ 14 = 1 r. 1
5	15 ÷ 5 = 3 r. 0	10	15 ÷ 10 = 1 r. 5	15	15 ÷ 15 = 1 r. 0

There are **3** different counting numbers that leave a remainder of **3** when divided into **15**.

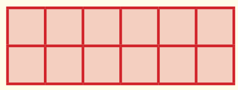
***Strategy 2:* Draw a Diagram**

If a number leaves a remainder of 3 when divided into **15**, then it must be a factor of **15 – 3 = 12**.

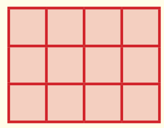
The factors of **12** are:



1 and 12,



2 and 6,

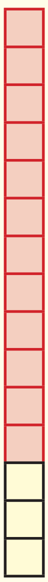


3 and 4.

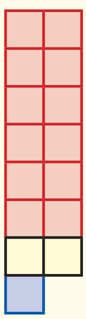
Let's try dividing **15** by all of the factors of **12**.

The remainder from a division must be less than the divisor.

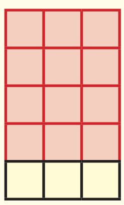
This means that, when we divide **15** by a number that is less than or equal to **3**, any remainder must be less than **3**.



15 ÷ 1 = 15
r. 0

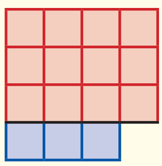


15 ÷ 2 = 7
r. 1

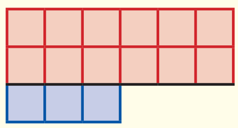


15 ÷ 3 = 5
r. 0

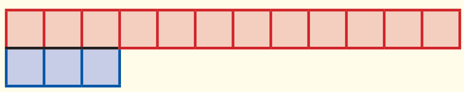
If we divide **15** by a factor of **12** that is greater than **3**, the remainder will be exactly **3**.



15 ÷ 4 = 3
r. 3



15 ÷ 6 = 2
r. 3



15 ÷ 12 = 1
r. 3

There are **3** different counting numbers that leave a remainder of **3** when divided into **15**.

Follow-Up: *How many numbers will leave a remainder of 3 when divided into 27?* [5 (4, 6, 8, 12, 24)]

1E. The question is, How many Tuesdays were there in the past 38 days?

"In the next 16 days" means 16 days starting from tomorrow.
"In the past 38 days" means 38 days, where the last of the 38 days was yesterday. **?**

Strategy: **Build a Table, and Find a Pattern**

There will be **3** Fridays in the next **16** days.

Let's draw a calendar, to find out how many days we need, to be able to fit in **3** Fridays.

To get **3** Fridays, we will need at least **15** days.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	

The extra (16th) day of the "next **16** days" could either be just before, or just after these **15** days.

If so, today must be either Wednesday or Thursday.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
			Today	16		

Sun	Mon	Tue	Wed	Thu	Fri	Sat
				Today		
						16

We can now figure out how many Tuesdays there were in the past **38** days.

Suppose today is a Wednesday. Let's count back.

We need to go back **1** day to get to the previous Tuesday.

Going back **1 + 7 = 8** days gets **2** previous Tuesdays, and so on.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
38	37	36				
		29				
		22				
		15				
		8				
		1	Today	16		

What if today is a Thursday?

We need to go back **2** days to get to the previous Tuesday.

Going back **2 + 7 = 9** days gets **2** previous Tuesdays, and so on.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
	38	37				
		30				
		23				
		16				
		9				
		2		Today		
						16

In both cases, we can see that there were **6** Tuesdays within the past 38 days.

Follow-Up: *How many Thursdays were there in the past 38 days?* [5]