APSMO

2022 : DIVISION S

WEDNESDAY 23 MARCH 2022

OLYMPIAD 1

MATHS OLYMPIAD

Total Time Allowed: ***30 Minutes***

Write your answers in the boxes on the back.

Keep your answers hidden by folding backwards on this line.

1A. How many square centimetres are equivalent to 1 square metre?

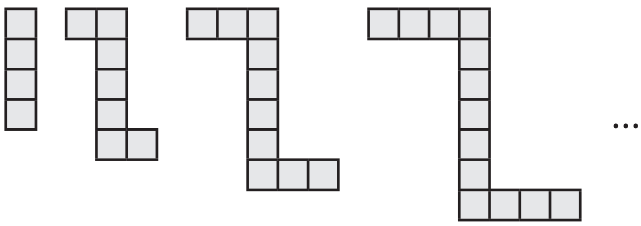
1B. The original price of a pair of jeans is $50.

Heather purchased a pair of these jeans after a 20% discount was applied, and had a further 10% discount applied to the already discounted price.

Pauline purchased a pair of jeans after a single 30% discount was applied to the original price.

How much more did Heather pay than Pauline, in dollars?

1C. Each figure in the sequence shown is made of identical square tiles.



If the pattern is continued, the Nth figure will consist of exactly 1000 square tiles.

Find N.

1D. A 4-digit "step up" number is a whole number in which the number formed by the leftmost two digits is 1 less than the number formed by the rightmost two digits.

For example, 1011 is a 4-digit "step up" number since 10 = 11 – 1.

How many 4-digit "step up" numbers have no repeated digits?

1E. A bookshelf holds 6 different textbooks, 5 different notebooks, and 23 different cookbooks.

How many different pairs of books can I select from the shelf, if the two books must be of different types?

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OLYMPIAD 1

1A.

1B.

1C.

1D.

1E.

Student Name:

Fold here. Keep your answers hidden.

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OLYMPIAD

1

Solutions and Answers

(Items in parentheses are not required)

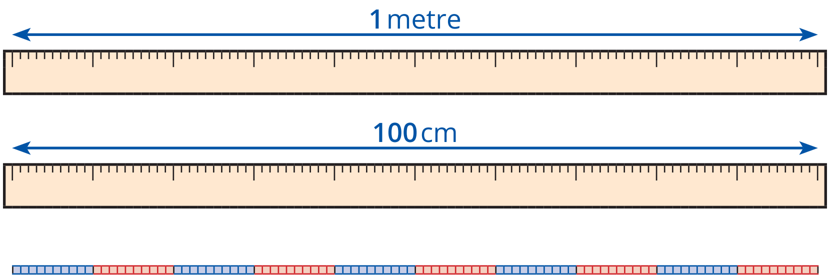
For teacher use only. Not for Distribution.

1A: 10000 (cm^2)	**1B:** ($)1	**1C:** 333	**1D:** 7	**1E:** 283

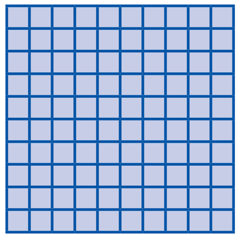
1A. The question is: How many square centimetres are equivalent to 1 square metre?

METHOD: *Draw a diagram, or model with concrete materials.*

Since **1** metre is equivalent to **100** centimetres, we can use **100** square centimetres to construct a metre-long stick that is **1** cm wide.



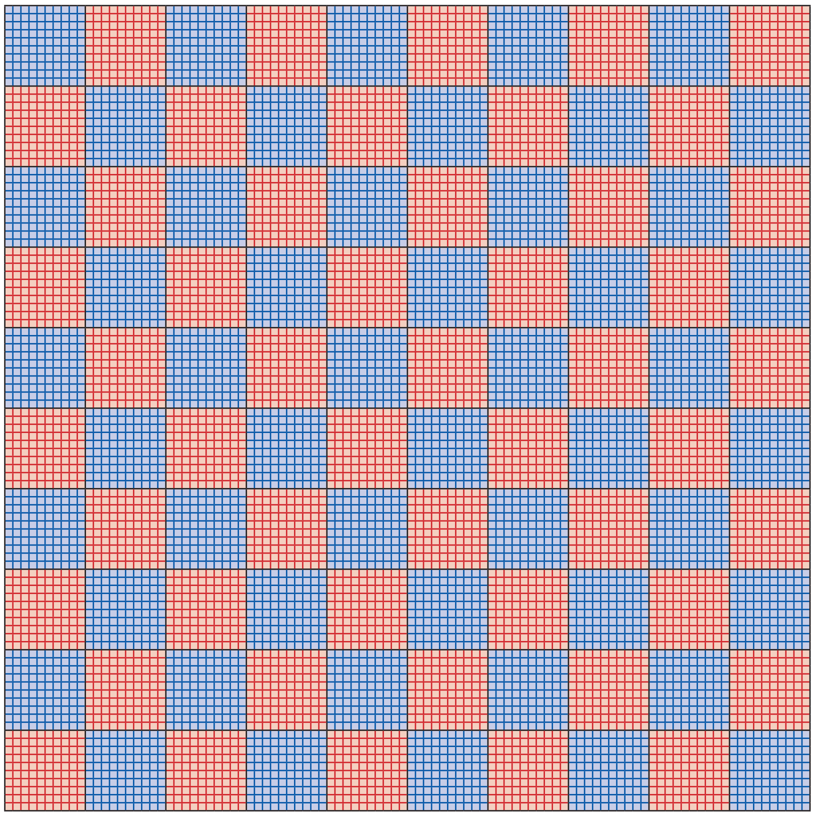
Alternatively, a square with **10** cm sides is also equivalent to **10** cm × **10** cm = **100** square centimetres.



To construct **1** square metre, we could either use:

- **100** of the **1** cm-wide sticks, or
- **100** squares that have **10** cm sides.

Therefore **1** square metre is equivalent to **100** × **100** = **10000** square centimetres.



Follow-Up: *How many cubic centimetres are equivalent to 1 cubic metre?* [1000000]

1B. The question is: How much more did Heather pay than Pauline?

METHOD 1 *Strategy:* *Draw a diagram.*

The original price of a pair of jeans is **\$50**.

When Heather went shopping, the jeans were discounted by **20%**.

Since **100%** of the price is **\$50**,

20% of the price is $\frac{20}{100}$ × **\$50** = $\frac{1}{5}$ × **\$50** = **\$10**.

Discounting by **\$10** means that the new price is **\$50** – **\$10** = **\$40**.

Heather then received a further **10%** discount off the already discounted price of **\$40**.

10% of **\$40** is $\frac{10}{100}$ × **\$40** = $\frac{1}{10}$ × **\$40** = **\$4**.

Discounting by **\$4** means that the final price is **\$40** – **\$4** = **\$36**.

Heather paid **\$36** for her pair of jeans.

When Pauline went shopping, the jeans were discounted by **30%**.

Since **100%** of the price is **\$50**,

30% of the price is $\frac{30}{100}$ × **\$50** = $\frac{3}{10}$ × **\$50** = **\$15**.

Discounting by **\$15** means that the new price is **\$50** – **\$15** = **\$35**.

Pauline paid **\$35** for her pair of jeans.

Heather paid **\$36 – \$35 = \$1** more than Pauline did, for her pair of jeans.

METHOD 2 *Strategy:* *Represent each discount as the percentage that is paid.*

20% off is the same as **100%** – **20%** = **80% of** the price.

Heather received a **20%** discount, and then a **10%** discount off the already discounted price.

10% off is the same as **100%** – **10%** = **90% of** the price.

Heather paid **80%** × **90%** × **\$50** = **0.8** × **0.9** × **\$50** = **\$36** for her pair of jeans.

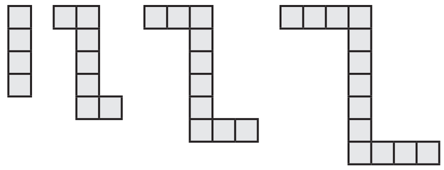
30% off is the same as **100%** – **30%** = **70% of** the price.

Pauline paid **70%** × **\$50** = **0.7** × **\$50** = **\$35** for her pair of jeans.

Heather paid **\$36 – \$35 = \$1** more than Pauline.

Follow-Up: *A calculator manufacturer needs to determine the LIST PRICE for the latest model, so that a 20% PROFIT can be made after they apply a 20% DISCOUNT off the LIST PRICE. It costs \$100 to construct one calculator. What should they use as their LIST PRICE?* [\$150]

1C. The question is: Find *N*, where the *N*th figure will consist of exactly 1000 square tiles.



METHOD 1 ***Strategy:*** *Convert to a more convenient form, and work backwards.*

We begin by listing the number of tiles that are used to construct each figure.

Figure	1	2	3	4	...	*N*
No. of Tiles	4	7	10	13	...	1000

The *N*th figure uses 1000 tiles.

Every figure requires 3 tiles more than the previous figure.

This means that we can express the pattern as growing by 3s.

Figure	1	2	3	4	...	*N*
No. of Tiles	4	7	10	13	...	1000
	3 + 1	2×3 + 1	3×3 + 1	4×3 + 1	...	999 + 1

+1 +1 +1

+3 +3 +3

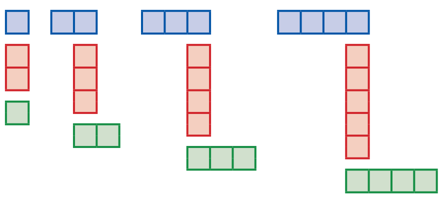
Working backwards, if we know the number of tiles in the figure, we can determine the corresponding Figure number by:

- Subtracting 1, and
- Dividing by 3.

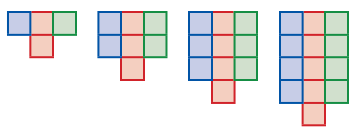
The figure that consists of 1000 tiles is figure number 999 ÷ 3 = **333**.

METHOD 2 ***Strategy:*** *Examine the construction of the figure, and use algebra.*

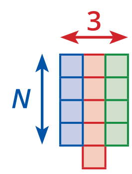
We can deconstruct the figures along the following lines.



The tiles for each figure can then be rearranged as follows.



From the diagram, we can see that the number of tiles for Figure *N* is equal to $3N + 1$.



We want to find the value of *N* where the number of tiles is 1000.

$$\begin{aligned} 3N + 1 &= 1000 \\ 3N &= 1000 - 1 \\ &= 999 \\ N &= 999 \div 3 \\ &= 333 \end{aligned}$$

The value of *N* is **333**.

Follow-Up: *Using the same sequence of shapes, what is the least value of N that would have a perimeter that is greater than 2022?* [337]

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OLYMPIAD 1

1D. The question is: How many 4-digit “step up” numbers have no repeated digits?

METHOD 1 ***Strategy:*** *Build a table, and eliminate numbers that do not satisfy the criteria.*

Since there is only one "step-up" number for every **2**-digit number, it is reasonable to just list every "step-up" number.

By eliminating all of the numbers that do have repeated digits, we can see that there are exactly **7** "step-up" numbers that have no repeated digits.

1011	2021	3031	4041	5051	6061	7071	8081	9091
1112	2122	3132	4142	5152	6162	7172	8182	9192
1213	2223	3233	4243	5253	6263	7273	8283	9293
1314	2324	3334	4344	5354	6364	7374	8384	9394
1415	2425	3435	4445	5455	6465	7475	8485	9495
1516	2526	3536	4546	5556	6566	7576	8586	9596
1617	2627	3637	4647	5657	6667	7677	8687	9697
1718	2728	3738	4748	5758	6768	7778	8788	9798
1819	2829	3839	4849	5859	6869	7879	8889	9899
1920	2930	3940	4950	5960	6970	7980	8990	

METHOD 2 ***Strategy:*** *Make an organised list.*

A "step-up" number is formed by putting together two **2**-digit numbers, where the second **2**-digit number exceeds the first **2**-digit number by **1**.

We can begin by listing some "step-up" numbers in an organised way.

First 2-digit number	10	11	12	13	14
"Step-up" number	1011	1112	1213	1314	1415

First 2-digit number	15	16	17	18	19
"Step-up" number	1516	1617	1718	1819	1920

If the ones value of the first **2**-digit number is in the range **0 - 8**, both of the **2**-digit numbers will have the same tens digit.

The first "step-up" number with no repeated digits is 1920.

The only way to create a "step-up" number with no repeated digits is by selecting the first **2**-digit number so that its ones value is **9**.

First 2-digit number	19	29	39	49	59
"Step-up" number	1920	2930	3940	4950	5960

First 2-digit number	69	79	89	99	
"Step-up" number	6970	7980	8990	99100	

(exceeds **4** digits)

We can see that 8990 has a repeated digit.

Therefore, by inspection, there are **7** "step-up" numbers that have no repeated digits.

First 2-digit number	19	29	39	49	59
"Step-up" number	1920	2930	3940	4950	5960

First 2-digit number	69	79	89	99	
"Step-up" number	6970	7980	8990	99100	

Follow-Up: *An enhanced “step-up” number is a 4-digit number such that the leftmost 2-digit number is any value less than the rightmost 2-digit number. For example, 2730 is included because 27 < 30. How many enhanced four-digit “step-up” numbers are there in total?* [4005]

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1E. The question is: How many different pairs of books can I select from the shelf, if the two books must be of different types?

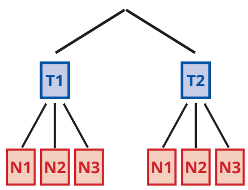
METHOD: *Draw a diagram or build a table, and solve a simpler related problem.*

Suppose there were just 2 different textbooks, 3 different notebooks, and 4 different cookbooks.

We could select 1 textbook, and 1 notebook.

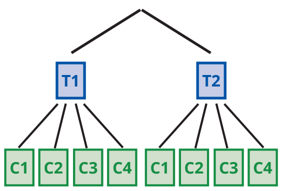
Using a representation such as a table or a tree diagram, we can see that the pairing can occur in 2 × 3 = 6 ways.

	T1	T2
N1	T1 N1	T2 N1
N2	T1 N2	T2 N2
N3	T1 N3	T2 N3



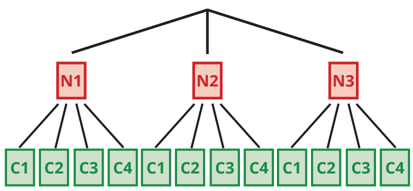
A pair comprising 1 textbook and 1 cookbook can occur in 2 × 4 = 8 ways.

	T1	T2
C1	T1 C1	T2 C1
C2	T1 C2	T2 C2
C3	T1 C3	T2 C3
C4	T1 C4	T2 C4



A pair comprising 1 notebook and 1 cookbook can occur in 3 × 4 = 12 ways.

	N1	N2	N3
C1	N1 C1	N2 C1	N3 C1
C2	N1 C2	N2 C2	N3 C2
C3	N1 C3	N2 C3	N3 C3
C4	N1 C4	N2 C4	N3 C4



With 2 different textbooks, 3 different notebooks, and 4 different cookbooks, there would be

(2 × 3) + (2 × 4) + (3 × 4)

= 6 + 8 + 12

= 26 different pairs of books, where the two books are of different types.

Using what we have noticed in the above pattern, for 6 different textbooks and 5 different notebooks, we see that there would be 6 × 5 = 30 different pairs comprising 1 textbook and 1 notebook.

	T1	T2	T3	T4	T5	T6
N1	T1 N1	T2 N1	T3 N1	T4 N1	T5 N1	T6 N1
N2	T1 N2	T2 N2	T3 N2	T4 N2	T5 N2	T6 N2
N3	T1 N3	T2 N3	T3 N3	T4 N3	T5 N3	T6 N3
N4	T1 N4	T2 N4	T3 N4	T4 N4	T5 N4	T6 N4
N5	T1 N5	T2 N5	T3 N5	T4 N5	T5 N5	T6 N5

With 6 different textbooks, 5 different notebooks, and 23 different cookbooks, there would be

(6 × 5) + (6 × 23) + (5 × 23)

= 30 + 138 + 115

= **283** different pairs of books, where the two books are of different types.

Follow-Up: *Suppose there are T different textbooks, N different notebooks, and C different cookbooks. T + N + C = 34. Find the greatest possible number of pairs of 2 books of different types.* [385]