APSMO

2022 : DIVISION J

WEDNESDAY 23 MARCH 2022

OLYMPIAD 1

Total Time Allowed: ***30 Minutes***

Write your answers in the boxes on the back.

Keep your answers hidden by folding backwards on this line.

1A. There are **6** cups, equally spaced, arranged in a circle in some order.

Each cup has a different colour: red, orange, yellow, green, blue and purple.

The blue cup is *not* next to the green cup.

The purple cup is next to the yellow cup.

The red cup is next to the green and purple cups.

Which cup is directly across from the orange cup?

1B. Calculate **4 × 5 × 6 × 25**.

1C. What is the sum of all the odd numbers from **1** to **19** inclusive?

1D. Oliver has a rectangular garden with an area of **28** square metres.

Its length and width are a whole number of metres.

The length of the garden is **3** metres longer than its width.

Oliver places **50** cm × **50** cm tiles together to make a continuous path surrounding the garden, along its edges.

How many tiles does Oliver need?

1E. Mara and Tara each have some pencils.

If Mara gives four pencils to Tara, they will have the same number of pencils.

If Tara gives two pencils to Mara, Mara will have **3** times as many pencils as Tara.

How many pencils does Mara have?

APSMO

MATHS OLYMPIAD

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OLYMPIAD 1

1A.

1B.

1C.

1D.

1E.

Student Name:

Fold here. Keep your answers hidden.

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OLYMPIAD 1

MATHS OLYMPIAD

Solutions and Answers

For teacher use only. Not for Distribution.

1A: Purple	**1B:** 3000	**1C:** 100	**1D:** 48	**1E:** 16

1A. The question is: Which cup is directly across from the orange cup?

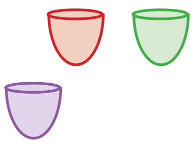
METHOD 1 ***Strategy:*** *Draw a diagram. Change the order of the statements to make it easier.*

The diagrams could also be mirror images or rotations of the ones drawn here.

Third statement:

The red cup is next to the green and purple cups.

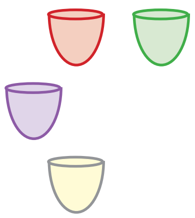
Draw this, knowing they form part of a circle.



Second statement:

The purple cup is next to the yellow cup.

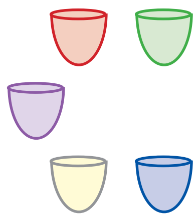
The purple cup is also next to the red cup, so it must be between the red and yellow cups.



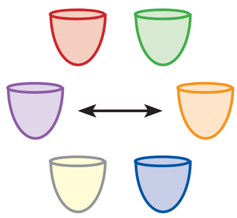
First statement:

The blue cup is *not* next to the green cup.

The only place left for the blue cup is next to the yellow cup.



The only place left for the orange cup is between the blue cup and the green cup.



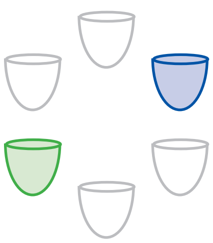
We can see that the cup directly across from the orange cup is the **purple** cup.

METHOD 2 ***Strategy:*** *Mark a place for each cup, then follow the statements in order.*

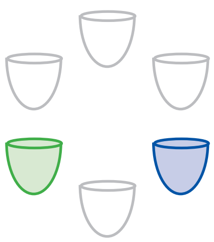
First statement:

The blue cup is *not* next to the green cup.

This leaves either one or two places between these cups.



OR

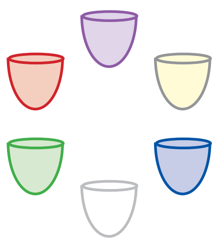


2nd and 3rd statements:

The purple and yellow cups are next to each other, and the red cup is next to the purple cup.

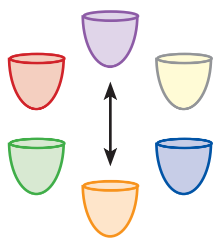
Only the second arrangement allows this, because it has **3** spaces together.

The red cup is next to the green and purple cups.



The orange cup must be in the one remaining place.

The cup directly across from the orange cup is the **purple** cup.



METHOD 3 ***Strategy:*** *Guess, check and refine.*

Place the cups in a circle in any order, then improve on this order by following the statements one at a time until all the statements are satisfied.

Follow-Up: *Abby, Bec, Cindy, Di and Eve are sitting in a circle in some order. Cindy is sitting between Abby and Eve. Abby is NOT next to Di. Who is sitting each side of Bec?* [Abby and Di]

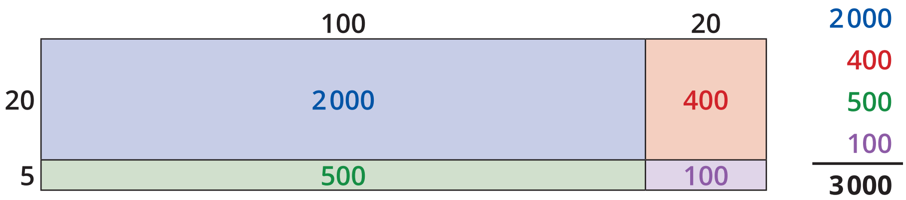
1B. The question is: Calculate **4 × 5 × 6 × 25**.

METHOD 1 ***Strategy:*** *Calculate from left to right.*

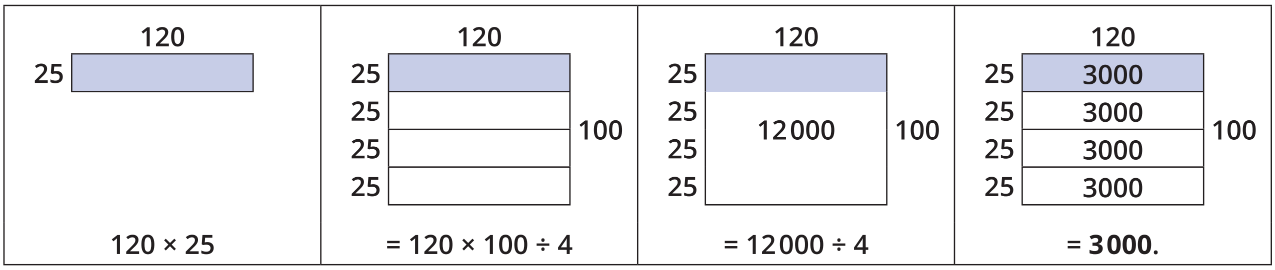
This calculation does not involve any grouping symbols, so you can make the calculation from left to right.

4	×	5	=	20
20	×	6	=	120
120	×	25	=	3000

120 × 25 could be calculated using long multiplication, or using the area method of multiplication, as illustrated below.



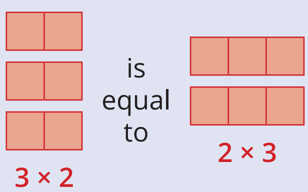
The last step of multiplying by **25** could also be made by realising that **25** is the same as **100 ÷ 4**.



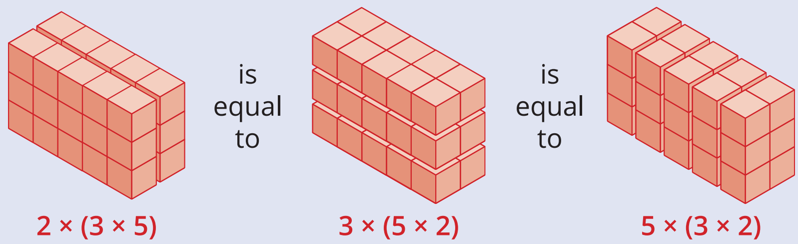
METHOD 2 ***Strategy:*** *Use the properties of multiplication.*

Multiplication is Associative and Commutative

When we multiply numbers, the order in which the numbers are multiplied does not matter.



This works even when multiplying more than **2** numbers.



Because our numeration system is a decimal (i.e. base **10**) system, it is easy to calculate using numbers that are powers of **10** (**10**, **100**, **1000** etc.)

Look for numbers that multiply to make a power of **10**.

In the expression, the first and last numbers are **4** and **25**, and we know that **4 × 25 = 100**.

Re-arranging the expression:

4 × 5 × 6 × 25 = **5 × 6 × (4 × 25)**
= **30 × 100**
= **3000.**

Follow-Up: *Evaluate 8 × 5 × 6 × 125.* [**30000**]

1C. The question is: What is the sum of all the odd numbers from **1** to **19** inclusive?

METHOD 1 ***Strategy:*** *Add all of the numbers.*

1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = **100.**

METHOD 2 ***Strategy:*** *Look for a pattern in the sum as each number is added.*

Start adding the odd numbers, one at a time.

The **1**st odd number is **1**.

The **2**nd odd number is **3**.
1 + 3 = 4.

The **3**rd odd number is **5**.
1 + 3 + 5 = 9.

The **4**th odd number is **7**.
1 + 3 + 5 + 7 = 16.

The **5**th odd number is **9**.
1 + 3 + 5 + 7 + 9 = 25.

Notice that as each odd number is added, the result is the next square number.

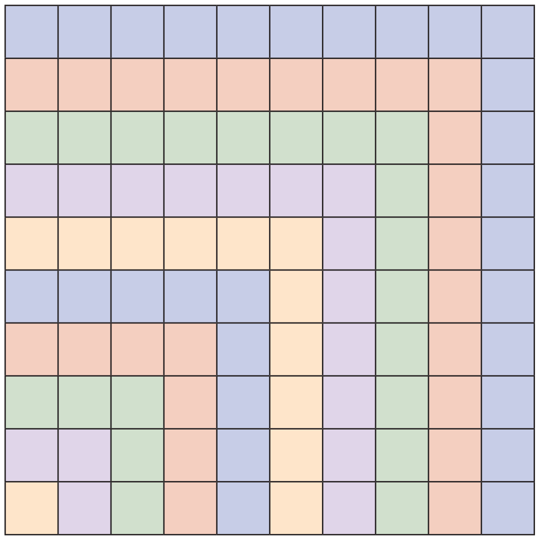
This pattern can be seen in the diagram below, where the odd number of tiles added each time is coloured differently from the previous odd number added.

Each time another group of tiles is added, a larger square is formed.

The number **19** is the **10**th odd number.

It follows that the sum of the odd numbers up to **19** is therefore **10** squared.

10 × **10** = **100**.



METHOD 3 ***Strategy:*** *Use the properties of addition.*

If you only have additions in an expression, changing the order of the numbers being added or grouping the additions differently does not change the value of the expression.

The pairs are joined by an arc, so each pair has the same sum.

This is known as the 'rainbow method'.

In this problem, each pair has a sum of 20.

1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19

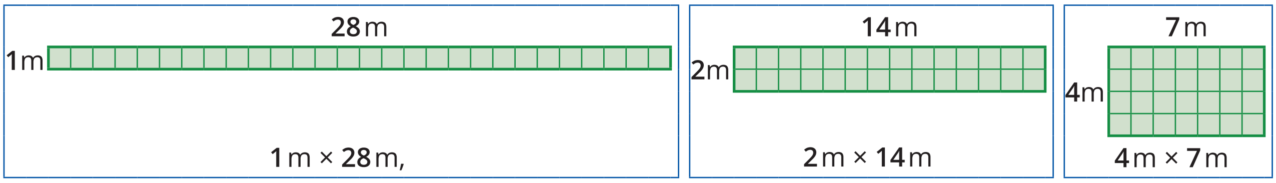
1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19
= (1 + 19) + (3 + 17) + (5 + 15) + (7 + 13) + (9 + 11)
= 20 + 20 + 20 + 20 + 20
= 100.

Follow-Up: *What is the sum of all the odd integers from **1** to **99** inclusive?* [**2500**]

1D. The question is: How many tiles does Oliver need?

METHOD 1 *Strategy:* *Draw a diagram and fit tiles around the perimeter.*

The garden area is **28** square metres and its length is **3** metres longer than its width.
With an area of **28** m² and the dimensions being a whole number of metres, the garden could be:



Since its length is **3** metres longer than its width, the garden is **4** metres wide and **7** metres long.

The square tiles have a side length of **50** cm, which is half a metre.

This means that for every metre of the perimeter, there are **2** tiles.

This makes 7 × 2 = 14 tiles along the length and 4 × 2 = 8 tiles along the width.

Then there are another 4 tiles in the corners to be drawn in.

Then count the number of tiles.

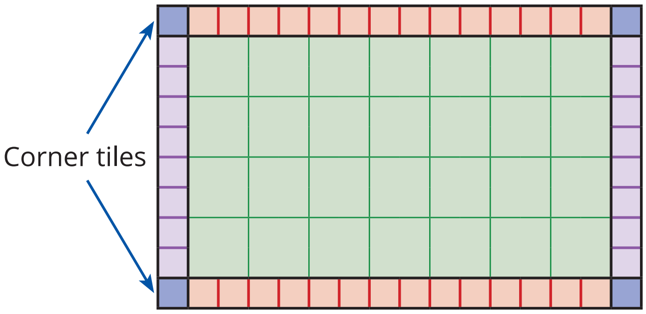
You could make the count easier by seeing the number of tiles as:

1 + 14 + 1 + 8 + 1 + 14 + 1 + 8 = **48**

or as (14 × 2) + (8 × 2) + 4 = **48**

or as (14 + 8) × 2 + 4 = **48**.

Oliver needs **48** tiles.

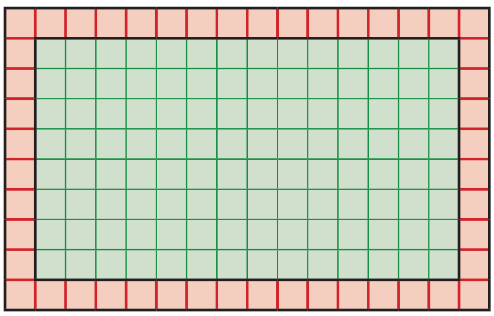


METHOD 2 *Strategy:* *Draw a diagram and subtract the garden area from the total area.*

Begin by drawing a diagram like the one shown in Method 1, and finding the dimensions to be **4** m × **7** m.

The area of the tiled path is the total area (garden plus path), less the area of the garden.

The tiles around the garden add **1** m to the width and **1** m to the length (one tile width of **0.5** m on each side of the length and one on each side of the width).



The dimensions of the total area are therefore **5** m × **8** m.

Path area = Total area − Garden area

Path area = (**5** × **8**) − (**4** × **7**) m² = **40** − **28** m² = **12** m².

Each tile has an area of **0.5** × **0.5** = **0.25** m².

Since **4** tiles fill a **1** m² area, Oliver will need **4** × **12** = **48** tiles for the path around the garden.

Follow-Up: *A square and a rectangle have the same perimeter. The length of the rectangle is **8** m longer than its width. If the area of the rectangle is **65** m², what is the area of the square?* [**81 m²**]

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OLYMPIAD 1

1E. The question is, How many pencils does Mara have?

METHOD 1 *Strategy:* *Create a table and use guess, check and refine.*

Guessing and checking pairs of numbers could take many guesses.
The process could be refined by using more information from the question.
If Mara gives **4** pencils to Tara, they each have the same number of pencils.
This means that originally, Tara had **4** fewer pencils and Mara had **4** more.
Mara must therefore have originally had **8** more pencils than Tara.
If Tara can give **2** pencils to Mara, and still have at least one pencil left, Tara must have at least **3** pencils.

Before		After		
Guess for Tara's pencils	Mara's pencils	Tara's pencils after giving **2** to Mara	Mara's pencils after getting **2** from Tara	Does Mara now have **3** times as many pencils as Tara?
3	3 + 8 = 11	3 – 2 = 1	11 + 2 = 13	No. **13** is not **1** × **3**.
4	4 + 8 = 12	4 – 2 = 2	12 + 2 = 14	No. **14** is not **2** × **3**.
5	5 + 8 = 13	5 – 2 = 3	13 + 2 = 15	No. **15** is not **3** × **3**.
6	6 + 8 = 14	6 – 2 = 4	14 + 2 = 16	No. **16** is not **4** × **3**.
7	7 + 8 = 15	7 – 2 = 5	15 + 2 = 17	No. **17** is not **5** × **3**.
8	8 + 8 = 16	8 – 2 = 6	16 + 2 = 18	Yes! **18** is **6** × **3**.

So, Tara must have **8** pencils and Mara must have **16** pencils.

METHOD 2 *Strategy*: *Apply algebraic thinking.*

Let M stand for the number of pencils Mara has, and let T stand for the number of pencils Tara has.

M $\quad$ T

If Mara gives Tara **4** pencils, they would both have the same number of pencils.

$M - 4 = T + 4$

If both of them then received another **4** pencils, they'd still have the same number of pencils.

$M = T + 8$

So, $M = T + 8$.
Instead of saying that Mara has M pencils, we can say that Mara has T + **8** pencils.

$T + 8$ $\quad$ T

If Tara gives away **2** pencils, and Mara gets **2** more pencils, then Mara has **3** times the number of pencils that Tara has.

$T + 8 + 2 = T - 2 + T - 2 + T - 2$

We have worked out that:	$T + 8 + 2$	=	$T - 2 + T - 2 + T - 2$
	$T + 10$	=	$3 \times T - 3 \times 2$
	$T + 10$	=	$3 \times T - 6$
Subtracting T from both sides:	10	=	$2 \times T - 6$
Adding **6** to both sides:	16	=	$2 \times T$
Dividing both sides by **2**:	8	=	T

T stands for the number of pencils Tara has, so we can see that Tara has **8** pencils.
Mara has T + **8** pencils.
Therefore Mara has **8** + **8** = **16** pencils.

Follow-Up: *Art has some money which is $**10** more than twice what Nick has. Nick gives Art $**10**. This results in Art having **4** times as much as Nick. How much did Nick start with?* [**$30**]